MIXED STRATEGIES IN 2X2, TWO PERSON, ZERO SUM GAMES

Consider the following 2x2 two person zero sum game

<table>
<thead>
<tr>
<th></th>
<th>P(C1)=q</th>
<th>P(C2)=1-q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>P(R1) = p</td>
<td>R1</td>
<td>a</td>
</tr>
<tr>
<td>P(R2) = 1-p</td>
<td>R2</td>
<td>c</td>
</tr>
<tr>
<td>EU of column</td>
<td>pa + (1-p)c</td>
<td>pb + (1-p)d</td>
</tr>
</tbody>
</table>

If there's no saddle point, then

Row's best strategy is to choose $p$ so that the payoff is equal regardless of which column the opponent chooses

$$pa + (1-p)c = pb + (1-p)d$$

which is equivalent to

$$pa - pb = (1-p)d - (1-p)c$$

$$p(a-b) = (1-p)(d-c)$$

$$p = \frac{d-c}{a-b} = \Omega(R1)$$

the odds in favor of Row playing R1, so the probability that Row will play R1 is

$$p = \frac{\Omega(R1)}{1+\Omega(R1)}$$

Col's best strategy is to choose $q$ so that the payoff is equal regardless of which row the opponent chooses

$$qa + (1-q)b = qc + (1-q)d$$

which is equivalent to

$$qa * qc = (1-q)d - (1-q)b$$

$$q(a-c) = (1-q)(d-b)$$

$$q = \frac{d-b}{a-c} = \Omega(C1)$$

the odds in favor of Col playing C1, so the probability that Col will play C1 is

$$q = \frac{\Omega(C1)}{1+\Omega(C1)}$$

Note that if these probabilities come out negative or greater than 1, it means that there is a saddle point and therefore a pure solution.