CALCULUS III - EXAM I - FALL 2000

Name _____

(10 pts) Find the distance between the center of the sphere

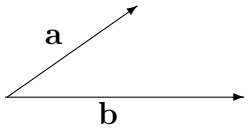
$$x^2 + y^2 + z^2 - 2x - 6z = 0$$

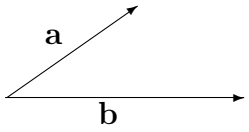
and the line

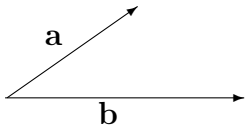
$$\frac{x+2}{2} = \frac{y}{1} = \frac{z+3}{2}.$$

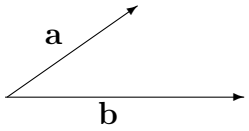
(5 pts) Where does the line $\frac{x+2}{2} = \frac{y}{1} = \frac{z+3}{2}$ intersect the plane $x-y+z-1 = 0$?

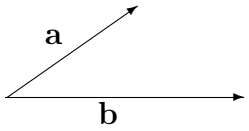
(10 pts) Draw $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $\mathbf{b}-\mathbf{a}$, $-\mathbf{a}-\mathbf{b}$, $\frac{1}{2}(\mathbf{a}+\mathbf{b})$:











(20 pts) Given the three points $P(1, 0, 2)$, $Q(2, 2, 1)$ and $R(0, 1, 4)$, let ΔPQR denote the triangle having P, Q and R as vertices. Find:

the area of PQR

the angle of ΔPQR at the vertex P

the equation of the plane containing the points P, Q and R

equations for the line through the point P which is perpendicular to the plane containing ΔPQR.

(10 pts) Find the volume of the parallelepiped whose edges are $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ with $A(1,2,3)$, $B(1,-1,2)$, $C(3,4,7)$, $D(2,4,0)$.

(10 pts) Compute $\mathbf{r}(t)$ from the following information:

$$\mathbf{r}''(t) = 2\mathbf{j} - (\sin t)\mathbf{k}, \quad \mathbf{r}'(0) = \mathbf{i} + \mathbf{j}, \quad \mathbf{r}(0) = \mathbf{i}.$$

(15 pts) The vertices of the triangle ABC are $A(1,1)$, $B(1,8)$, and $C(4,3)$. Points D and E are located on AC and BC, and $\frac{d(CD)}{d(CA)} = \frac{3}{5}$, $\frac{d(CE)}{d(CB)} = \frac{1}{2}$. Using the cross product find the area of the triangle DCE.