Construct the Lagrange interpolating polynomial of degree at most two for the function

\[ f(x) = \sin(\ln x), \quad x_0 = 2.0, \; x_1 = 2.4, \; x_2 = 2.6, \]

and find a bound for the absolute error on the interval \([x_0, x_2]\).
Let $f(x) = 3 x e^x - e^{2x}$. Approximate $f(1.03)$ by the Hermite interpolating polynomial of degree at most three using $x_0 = 1$ and $x_1 = 1.05$. Compare the actual error to the error bound.
A clamped cubic spline $S$ for a function $f$ is defined on $[1, 3]$ by

$$S(x) = \begin{cases} 
S_0(x) = 3(x - 1) + 2(x - 1)^2 - (x - 1)^3, & 1 \leq x < 2, \\
S_1(x) = a + b(x - 2) + c(x - 2)^2 + d(x - 2)^3, & 2 \leq x \leq 3 
\end{cases}$$

Given $f'(1) = f'(3)$, find $a, b, c$ and $d.$
Use the most accurate three-point formula to determine each missing entry in the following table

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3</td>
<td>-0.27652</td>
<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>-0.25074</td>
<td></td>
</tr>
<tr>
<td>-0.1</td>
<td>-0.16134</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Approximate the integral using Trapezoid, Simpson’s and Midpoint rules

\[
\int_{1}^{1.5} x^2 \ln x \, dx
\]

For each rule find a bound for the error using the corresponding error formula, and compare this to the actual error.
Graduate students only!!! The quadrature formula
\[ \int_{-1}^{1} f(x) \, dx = c_0 f(-1) + c_1 f(0) + c_2 f(1) \]
is exact for all polynomials of degree less than or equal to 2. Determine \( c_0, c_1, \) and \( c_2. \)