Determine a function $g$ and an interval $[a, b]$ on which fixed-point iteration will converge to the smallest positive root of the equation

$$x^2 + 10 \cos x = 0$$

Estimate the number of iterations required to obtain an approximation accurate to within $10^{-3}$ and perform the calculations.
Use Newton’s method to find a solution for the following problem:

\[ f(x) := e^x + 2^{-x} + 2 \cos x - 6 = 0, \quad x \in [1, 2] \]

Stop your iterations when \( \max \{|x_n - x_{n-1}|, |f(x_n)|\} < 10^{-4} \).
(a) Use the following values and five-digit rounding arithmetic to construct the Hermite interpolating polynomial to approximate $\ln(e^{0.25} + 2)$

$$f(x) = \ln(e^x + 2)$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0986</td>
<td>0.33333</td>
</tr>
<tr>
<td>0.5</td>
<td>1.2944</td>
<td>0.45187</td>
</tr>
</tbody>
</table>

(b) (Graduate students only!!!) Determine an error bound for the approximation in part (a), and compare it to the actual error.
A clamped cubic spline $S(x)$ for a function $f(x)$ is defined on $[1, 3]$ by

$$S(x) = \begin{cases} 
S_0(x) = 1 + 3(x - 1) - 4(x - 1)^2 + 2(x - 1)^3, & 1 \leq x < 2, \\
S_1(x) = a + b(x - 2) + c(x - 2)^2 + d(x - 2)^3, & 2 \leq x \leq 3
\end{cases}$$

Given $f'(1) = f'(3)$, find $a$, $b$, $c$ and $d$. 
Use Composite Simpson’s Rule with $n = 4$ (two subintervals) to approximate the following IMPROPER integral:

$$\int_{1}^{2} \frac{\ln x}{(x - 1)^{1/5}} \, dx$$