DIFFERENTIAL GEOMETRY - EXAM III - SPRING 2006

Name ____________________

Let α be a curve in $\mathbf{R^2}$ and $\phi = f(u,v)du + g(u,v)dv$ be a 1-form.
(a) Derive the computational rule for finding $\int_\alpha \phi$.

Let $\alpha : [-1,1] \to \mathbf{R^2}$ be the curve segment given by $\alpha(t) = (t, t^2)$.
(b) If $\phi = v^2 du + 2uvdv$, compute $\int_\alpha \phi$.

(c) Find a function f such that $df = \phi$ and check the Fundamental Theorem for Line Integrals in this case.

Let $X : \mathbf{R} \to M$ be a 2-segment defined on the unit square $R : 0 \leq u, v \leq 1$. If ϕ is the 1-form on M such that

$$\phi(X_u) = u + v \quad \text{and} \quad \phi(X_v) = uv,$$

verify Stokes's theorem by computing $\int\int_X d\phi$ and $\int_{\partial X} \phi$ separately.

Let $\mathbf{u_1}$ and $\mathbf{u_2}$ be orthonormal tangent vectors at a point p of M such that $S(\mathbf{u_1}) + S(\mathbf{u_2}) = 0$, and let $k := k(\mathbf{u_1})$. Calculate principal curvatures and principal vectors at p. Illustrate your answer with a picture.

Find the Gaussian curvature of the monkey saddle $M : z = x^3 - 3xy^2$, and express it in terms of $r = \sqrt{x^2 + y^2}$.
Graduate students only!!! Calculate the mean curvature as well.