Let $\alpha$ be a curve in $\mathbb{R}^2$ and $\phi = f(u,v)du + g(u,v)dv$ be a 1-form.

(a) Derive the computational rule for finding $\int_\alpha \phi$.

Let $\alpha : [-1,1] \rightarrow \mathbb{R}^2$ be the curve segment given by $\alpha(t) = (t,t^2)$.

(b) If $\phi = v^2du + 2uvdv$, compute $\int_\alpha \phi$.

(c) Find a function $f$ such that $df = \phi$ and check the Fundamental Theorem for Line Integrals in this case.
Let $X : \mathbb{R} \to M$ be a 2-segment defined on the unit square $R : 0 \leq u, v \leq 1$. If $\phi$ is the 1-form on $M$ such that

$$\phi(X_u) = u + v \quad \text{and} \quad \phi(X_v) = uv,$$

verify Stokes’s theorem by computing $\int f_X d\phi$ and $\int_{\partial X} \phi$ separately.
Let \( u_1 \) and \( u_2 \) be orthonormal tangent vectors at a point \( p \) of \( M \) such that \( S(u_1) + S(u_2) = 0 \), and let \( k := k(u_1) \). Calculate principal curvatures and principal vectors at \( p \). Illustrate your answer with a picture.
Find the Gaussian curvature of the monkey saddle $M : z = x^3 - 3xy^2$, and express it in terms of $r = \sqrt{x^2 + y^2}$.

**Graduate students only!!!** Calculate the mean curvature as well.