$u = x^2+1 \quad u(1)=2$

$du = 2x\,dx \quad u(2)=5$

$$\int_1^2 \frac{dx}{x(x^2+1)^2} = \int_1^2 \frac{dx}{x} - \frac{1}{2}\int_2^5 \frac{du}{u} - \frac{1}{2}\int_2^5 \frac{du}{u^2} = \ln|x|\Big|_1^2 - \frac{1}{2}\ln|u|\Big|_2^5 - \frac{1}{2}\frac{u^{-1}}{-1}\Big|_2^5 = \boxed{\frac{3\ln 2}{2} - \frac{\ln 5}{2} - \frac{3}{20}}$$

CALCULUS II - EXAM II

Name ________________

(5 pts) Calculate the definite integral

$$\int_1^2 \frac{dx}{x(x^2+1)^2} = \int_1^2 \frac{dx}{x} - \int_1^2 \frac{x\,dx}{x^2+1} - \int_1^2 \frac{x\,dx}{(x^2+1)^2}$$

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}; \quad 1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$

$$1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$

$x^4: A+B=0$ | $x^2: 2A+B+D=0$ | $A=1$ | $D=-1$

$x^3: C=0$ | $x: C+E=0$ | $B=-1$ | $E=0$

| $1: A=1$ | $C=0$ |

(5 pts) Find indefinite integral

$$\int \frac{5x+1}{(x-1)^2(x+2)}\,dx = \int \frac{dx}{x-1} + 2\int \frac{dx}{(x-1)^2} - \int \frac{dx}{x+2}$$

$$\frac{5x+1}{(x-1)^2(x+2)} = \frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{B}{x+2}$$

$$5x+1 = A_1(x-1)(x+2) + A_2(x+2) + B(x-1)^2$$

$x=1: \quad 6 = 3A_2, \quad A_2 = 2$

$x=-2: \quad -9 = 9B, \quad B = -1$

$x=0: \quad 1 = -2A_1 + 2A_2 + B$

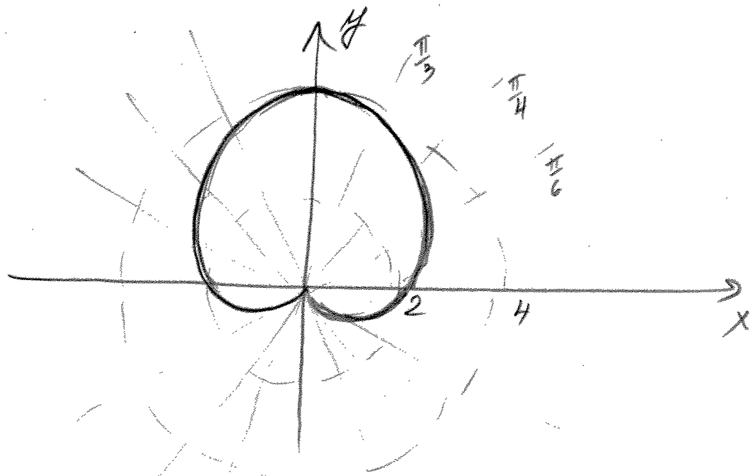
$1 = -2A_1 + 4 - 1, \quad 2A_1 = 2, \quad A_1 = 1$

$$\frac{5x+1}{(x-1)^2(x+2)} = \frac{1}{x-1} + \frac{2}{(x-1)^2} - \frac{1}{x+2}$$

$$= \boxed{\ln|x-1| - \frac{2}{x-1} - \ln|x+2| + C}$$

(5 pts) Sketch the graph of the function $r = 2(1 + \sin\theta)$.

Make a table and use y-axis symmetry



(5 pts) Find the area enclosed by the curve.

$$A = \frac{1}{2}\int_0^{2\pi} 4(1+\sin\theta)^2\,d\theta$$

$$= 2\int_0^{2\pi}\left(1 + 2\sin\theta + \frac{1-\cos(2\theta)}{2}\right)d\theta$$

$$= \int_0^{2\pi}(3 + 4\sin\theta - \cos(2\theta))\,d\theta$$

$$= \left(3\theta - 4\cos\theta - \frac{1}{2}\sin(2\theta)\right)\Big|_0^{2\pi} = \boxed{6\pi}$$

Let $x(t) = \cos^3 t,\ y(t) = \sin^3 t,\ t \in [0, \frac{\pi}{2}]$.

(5 pts) Find an equation in x and y for the line tangent to the curve at $t = \frac{\pi}{4}$.

$x'(t) = -3\cos^2 t \sin t, \quad y'(t) = 3\sin^2 t \cos t$

$x'(\frac{\pi}{4}) = -\frac{3\sqrt{2}}{4}, \quad y'(\frac{\pi}{4}) = \frac{3\sqrt{2}}{4}$

$(x(\frac{\pi}{4}), y(\frac{\pi}{4})) = (\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$

Tangent line:

$-(y - \frac{\sqrt{2}}{4}) = x - \frac{\sqrt{2}}{4}$

(5 pts) Compute the length of the graph.

$\sqrt{x'^2 + y'^2} = \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} = 3|\cos t \sin t|$

$L(c) = \int_0^{\frac{\pi}{2}} 3|\cos t \sin t|\, dt = \frac{3\sin^2 t}{2}\Big|_0^{\frac{\pi}{2}} = \boxed{\frac{3}{2}}$

(5 pt) Locate the centroid and determine the area of the surface generated by revolving the curve about the x-axis.

$\bar{x} L = \int_0^{\frac{\pi}{2}} \cos^3 t\ 3\sin t \cos t\, dt = \left[-\frac{3}{5}\cos^5 t\right]_0^{\frac{\pi}{2}} = \frac{3}{5}, \quad \bar{x} = \frac{2}{5}$

$\bar{y} L = \int_0^{\frac{\pi}{2}} \sin^3 t\ 3\cos t \sin t\, dt = \left[\frac{3}{5}\sin^5 t\right]_0^{\frac{\pi}{2}} = \frac{3}{5}, \quad \bar{y} = \frac{2}{5}$

$A_x = 2\pi \bar{y} L = \boxed{\frac{6\pi}{5}}$

State whether the sequence converges and, if it does, find the limit.

(2 pts)

$$\lim_{n\to\infty} \frac{n^6-1}{2n^5+n-6} \qquad DNE$$

$\text{degree}(n^6-1) = 6$

$\text{degree}(2n^5+n-6) = 5$

$6 > 5 \Rightarrow$ diverges

(3 pts)

$$\lim_{n\to\infty} \left\{2\ln(3n) - \ln(n^2+1)\right\}.$$

$$= \lim_{n\to\infty} \left[\ln(3n)^2 - \ln(n^2+1)\right] = \lim_{n\to\infty} \ln \frac{9n^2}{n^2+1} = \boxed{\ln 9}$$

$\frac{9n^2}{n^2+1} \xrightarrow[n\to\infty]{} 9$, since $\text{degree}(9n^2) = \text{degree}(n^2+1)$