

MATH MODELS FOR COMPUTER SCI - EXAM I

Name _____

Given

$$A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 4 & 6 \end{bmatrix}$$

(4 pts) Find the rank and a basis for the row space.

(4 pts) Find the rank and a basis for the column space.

(8 pts) Solve by Cramer's rule.

$$\left. \begin{array}{rcl} 2x & & -3z = 28 \\ 4x & -5y & +2z = 13 \\ & y & -5z = 29 \end{array} \right\}$$

Given

$$A = \begin{bmatrix} -5 & 0 & 15 \\ 3 & 4 & -9 \\ -5 & 0 & 15 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4 pts) Find $\hat{A} = P^{-1}AP$

(4 pts) Verify that A and \hat{A} have the same spectra.

(4 pts) Find eigenvectors \vec{y} of \hat{A} .

(2 pts) Show that $\vec{x} = P\vec{y}$ are eigenvectors of A .

(4 pts) What kind of conic section (or pair of straight lines) is given by the quadratic form? Transform it to principal axis.

$$13x_1^2 + 10x_1x_2 + 13x_2^2 = 72$$

(4 pts) Express the initial vector $x^T = [x_1, x_2]$ in terms of the new coordinate vector $y^T = [y_1, y_2]$.

(2 pts) Find the angle of the rotation. Draw the picture.