

MMCS - FINAL EXAM

Name _____

(7 pts) Solve the following system by Gaussian elimination or indicate the nonexistence of solutions.

$$\left. \begin{array}{rcl} 3x + y - 2z & = & 11 \\ -4x + 2y + z & = & -13 \\ 5x - 3y + 7z & = & 8 \end{array} \right\}$$

(7 pts) Make up a specific system (with numbers) of 3 equations with 4 variables that has no solution, or explain why no such example can exist.

Make up a specific system (with numbers) of 3 equations with 4 variables that has infinitely many solutions, or explain why no such example can exist.

(7 pts) Find the inverse A^{-1} or show why A^{-1} cannot exist.

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -2 & 1 & 0 \\ 3 & -6 & 8 \end{bmatrix}$$

(7 pts) What kind of conic section (or pair of straight lines) is given by the quadratic form? Transform it to principal axis.

$$3x_1^2 + 22x_1x_2 + 3x_2^2 = 0$$

Express the initial vector $x^T = [x_1, x_2]$ in terms of the new coordinate vector $y^T = [y_1, y_2]$.

Find the angle of the rotation. Draw the picture.

(7 pts) If a cage contains 100 mice, five of which are male, what is the probability that exactly two male mice will be included if 15 mice are randomly selected?

(7 pts) Suppose that 1% of all tires manufactured in a certain plant will last more than 80000 miles. If you buy 4 such tires, what is the probability that at least 2 of them will last more than 80000 miles?

(7 pts) A one person barber shop has a mean weekly gross income of $\mu = \$680$ and an exact standard deviation $\sigma = \$60$. What is the probability that next week's gross income will be \$750 or more?

(7 pts) Determine a 95% confidence interval for the mean μ of a normal population by using a sample size of 17 that has a sample mean $\bar{x} = 50.60$ and a sample standard deviation s of 0.95.

(7 pts) Suppose you are trying to buy a three year old luxury car. Before you go shopping, you look up the average "blue book" value for such cars, and the book says the average price should be \$38,500. You think the average price in Georgia is different from \$38,500, but you have no idea whether it is higher or lower than \$38,500. You look at sales records for the past few months, and learn that 20 of these cars have sold for an average price of \$39,900. National data says that the exact standard deviation is $\sigma = \$4000$. Is there enough evidence so say the real average is different from \$38,500? Use a significance level $\alpha = 1\%$.

(7 pts) Find the Fourier series for

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases} \quad \text{and} \quad f(x + 2\pi) = f(x).$$