NUMERICAL ANALYSIS II
FINAL EXAM - SPRING 2003

Show your work/steps.

Name  ________________

Part 1. General questions. (60 pts)

In Part 1 indicate agree (X) or disagree (O) in blanc spaces ( ).

If you are given a system of linear equations, what methods can you use (choose all appropriate methods):

( ) Jacobi iterative method
( ) Gaussian elimination with pivoting
( ) Shooting Method
( ) Euler’s method
( ) finite differences
( ) The Adams Families
( ) Trapezoid Predictor - Corrector

Pivoting can be used

( ) for LU decomposition
( ) to solve a linear system \(Ax = b\) with \(\det(A) \neq 0\)
( ) to solve a linear system \(Ax = b\) with \(\det(A) = 0\)
( ) to avoid division by zero
( ) to increase an accuracy
( ) as a step of Runge-Kutta method

The accuracy of numerical solution to the linear system of equations is higher if

( ) the condition number of the system is a large number
( ) the condition number of the system is a small number
( ) the condition number of the system is zero
( ) the condition number of the system is a positive number less than 1
( ) the condition number of the system is a negative number
( ) the accuracy of numerical solution to the linear system of equations does not depend on the condition number
( ) The accuracy of second-order Runge-Kutta methods is less than the accuracy of Euler’s method

( ) If the coefficient matrix is very large and sparse, then Gaussian elimination is the best way to solve the linear system problem

( ) Runge-Kutta methods are single-step

( ) Adams-Bashforth methods require either the solution of a nonlinear equation or a predictor-corrector scheme

( ) The second-order Adams-Moulton method is the trapezoid method

( ) The Runge-Kutta methods can be used to generate starting values for Adams methods
Part 2. Problems.

1. (30 pts) Let

\[
A = \begin{pmatrix}
2 & 3 & 1 \\
0 & 1 & 2 \\
1 & 1 & 4
\end{pmatrix}
\]

Compute, directly from the definition, \( \text{cond}_\infty(A) \).
2. (30 pts) Beginning with $x_0 = 1$, write the first three iterations of Newton’s method for the equation $x^3 + x = 1$. 
3. (40 pts) (a) Given

\[ y' = f(t, y(t)), \quad y(t_0) = y_0, \quad t_0 \leq t \leq T, \quad (1) \]

write down the general form of second-order Runge-Kutta methods for solving (1).

(b) What conditions on the parameters will guarantee the accuracy of order two over the interval \([t_0, T]\)?

(c) Write down Runge-Kutta method of order two that corresponds to \(c_2 = \frac{1}{2}\). Identify this method.
(d) Use the method obtained in part (c) to solve the following system of ODEs:
\[ y_1' = -4y_1 - 2y_2 + \cos t, \quad y_1(0) = 0, \]
\[ y_2' = 3y_1 + y_2, \quad y_2(0) = -1, \]
on [0, 1] with \( h = \frac{1}{3} \).

(e) Summarize your results in the table:

<table>
<thead>
<tr>
<th>RK2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>t</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1/3</td>
</tr>
<tr>
<td>2/3</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>