DIFFERENTIAL EQUATIONS

REVIEW PROBLEMS FOR EXAM I

1. Find the solutions to the differential equation

\[ x' = \frac{2tx}{1+x} \]

having initial conditions \( x(0) = 0, \) \( x(0) = 1, \) \( x(0) = -2. \)

2. Find the general solution of

\[ x' = x \sin(t) + 2te^{-\cos(t)} \]

and the particular solution that satisfies \( x(0) = 1. \)

3. Find the general solution to

\[ x' = x \tan(t) + \sin(t) \]

and the particular solution that satisfies \( x(0) = 2. \)

4. Suppose a bottle of beer at 50°F is discovered on a kitchen counter in a 70°F room. Ten minutes later, the bottle is 60°F. If the refrigerator is kept at 40°F, how long had the bottle of beer been sitting on the counter when it was first discovered?

5. Solve

\[ dx + (x - e^{-y} \sin(y))dy = 0. \]

6. Draw phase diagram and describe the behavior as \( t \to -\infty, \) \( t \to +\infty \) of all solutions to the differential equation. Don’t solve the differential equation!

\[ x' = (x^2 - 1)(x - 2). \]

7. Draw phase diagram and describe the behavior as \( t \to -\infty, \) \( t \to +\infty \) of all solutions to the differential equation. Don’t solve the differential equation!

\[ x' = x^3 - 2x^2 + x. \]

8. A population obeying the logistic equation, begins with 1000 bacteria, then doubles itself in 10 hours. The population is observed to eventually stabilize at 20 000 bacteria. Find the number of bacteria present after 25 hours and the time it takes the population to reach one-half of its carrying capacity.

9. Solve Riccati equation

\[ y' + 2y^2 = t^{-2}, \quad y = t^{-1}. \]

\[ y' + y^2 = t^{-4}, \quad t^{-1} - t^{-2}. \]

10. Solve Bernoulli equation

\[ y' + \left(1 - \frac{3}{t}\right)y = \frac{y^{4/3}}{t}. \]