DIFFERENTIAL EQUATIONS - EXAM II

Name ____________________

1. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$. Fully justify your answer.

(a) $f_1(x) = \sin x, \quad f_2(x) = \cos x.$

(b) $f_1(x) = e^{3x}, \quad f_2(x) = e^{3(x-1)}.$

(c) $f_1(x) = 2x, \quad f_2(x) = x - 1, \quad f_3(x) = x^2.$

2. Find an interval centered about $x = 0$ for which the given initial-value problem has a unique solution.

$$(1-x)y'' - (\tan x)y = e^x, \quad y(0) = 1, \quad y'(0) = 0.$$

3. The function $y_1(x) = x^{-1}$ is a solution of the given homogeneous linear differential equation. Use reduction of order to find a second solution $y_2(x)$, and the general solution of the given DE on the interval $(0, \infty)$. Write the fundamental set of solutions for the given DE on $(0, \infty)$.

$$2x^2y'' + 3xy' - y = 0.$$

4. Find the general solution of the given homogeneous linear differential equations with constant coefficients.

(a) $y'' + 5y' + 6y = 0.$

(b) $y'' - 6y' + 9y = 0.$

(c) $y'' - 4y' + 5y = 0.$

5. Solve the given differential equation by the method of undetermined coefficients

$$y'' - 3y' - 4y = 2e^{-t}.$$

6. Solve the given differential equation by variation of parameters

$$y'' + y = \sec^2 x.$$