SURVEY OF CALCULUS - EXAM I

Name _______________________

(5 pts) In the table below, the amount of the U.S. minimum wage is listed for selected years.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>WAGE</td>
<td>$1.15</td>
<td>$1.40</td>
<td>$2.00</td>
<td>$3.10</td>
<td>$3.35</td>
<td>$3.80</td>
<td>$4.25</td>
<td>$4.75</td>
<td>$5.15</td>
</tr>
</tbody>
</table>

Find an exponential regression model of the form \( y = a \cdot b^x \), where \( y \) represents the U.S. minimum wage \( x \) years after 1960. Round \( a \) and \( b \) to four decimal places. Graph the model. According to this model, what will the minimum wage be in 2005? In 2010?

\[
y = a \cdot b^x
\]

\[
a = 1.1538
\]

\[
b = 1.0425
\]

$7.51 in 2005

$9.25 in 2010

(5 pts) Let \( f(x) = \frac{x^2 - 3x - 10}{x+2} \). Find \( \lim_{x \to -2} f(x) \).

\[
\lim_{x \to -2} \frac{x^2 - 3x - 10}{x+2} = \frac{4 + 6 - 10}{-2 + 2} = \frac{0}{0}
\]

\[
= \lim_{x \to -2} \frac{(x+2)(x-5)}{(x+2)} = \lim_{x \to -2} (x-5) = -7
\]
(5 pts) If $4,000$ is invested at 7% compounded annually, how long will it take for it to grow to $6,000$, assuming no withdrawals are made? Compute answer to the next higher year if not exact.

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

$r = 0.07, \ m = 1, \ P = 4,000, \ A = 6,000$

$$6,000 = 4,000 \left(1 + \frac{0.07}{1}\right)^{t} = 4,000 \cdot 1.07^t$$

$$\frac{3}{2} = 1.07^t$$

$$\ln \frac{3}{2} = t \ln 1.07$$

$$t = \frac{\ln 3 - \ln 2}{\ln 1.07} = 5.9928 \ldots$$

6 years

(5 pts) Evaluate the following limits

(A) $$\lim_{x \to 5} \frac{x - 5}{|x - 5|}$$

$$\lim_{x \to 5} \frac{x - 5}{|x - 5|} = \begin{cases} 1, & x > 5 \\ -1, & x < 5 \end{cases}$$

(B) $$\lim_{x \to 5} \frac{x - 5}{|x - 5|}$$

$$\lim_{x \to 5} \frac{x - 5}{|x - 5|} = \lim_{x \to 5^+} \frac{x - 5}{|x - 5|}$$

$$\lim_{x \to 5^+} \frac{x - 5}{|x - 5|} = 1$$

(C) $$\lim_{x \to 5} \frac{x - 5}{|x - 5|}$$

$$\lim_{x \to 5} \frac{x - 5}{|x - 5|} = \lim_{x \to 5^-} \frac{x - 5}{|x - 5|}$$

$$\lim_{x \to 5^-} \frac{x - 5}{|x - 5|} = -1$$

$$\lim_{x \to 5} \frac{x - 5}{|x - 5|} = \text{DNE}$$
(5 pts) A company is planning to manufacture a new blender. After conducting extensive market surveys, the research department estimates a weekly demand of 600 blenders at a price of $50 per blender and a weekly demand of 800 blenders at a price of $40 per blender. Assuming the demand equation is linear, use the research department’s estimates to find the revenue equation in terms of the demand $x$.

\[
\begin{array}{c|c}
x & p \\
600 & 50 \\
800 & 40 \\
\end{array}
\]

\[
p(x) = a + bx
\]

\[
\begin{cases}
50 = 0 + b \cdot 600 \\
40 = a + b \cdot 800 \\
\end{cases}
\]

\[
a = 50 - 600b \\ 40 = 50 - 600b + 800b \\
-10 = 200b, \quad b = -\frac{10}{200} = -0.05 \\
a = 50 + 600 \cdot 0.05 = 50 + 30 = 80
\]

\[
p(x) = 80 - 0.05x \quad \Rightarrow \quad R(x) = x \cdot p(x) = 80x - 0.05x^2
\]

(5 pts) Solve the inequality and express the answer in interval notation:

\[
x + 5 = 0 \\
x = -5
\]

\[
f(x) = \frac{x^2 - 4x}{x + 5} > 0.
\]

\[
x^2 - 4x = 0 \\
x = 0 \quad x_2 = 4
\]

\[
\begin{array}{c|c|c|c}
- & -5 & + & 0 & - & 4 & + & >
\end{array}
\]

\[
f(-6) = \frac{-6(-6-4)}{-6+5} = + < 0
\]

\[
f(1) = \frac{1(1-4)}{1+5} = + < 0
\]

\[
f(-1) = \frac{-1(-1-4)}{-1+5} = + > 0
\]

\[
f(5) = \frac{5(5-4)}{5+5} = + > 0
\]

Answer: \((-5, 0) \cup (4, \infty)\)
(5 pts) Find \( \frac{df}{dx} \left[ \frac{1}{x^4} - 3\sqrt{x} \right] \).

\[
f'(x) = 4(x^{-4})' - 5\left(x^{\frac{1}{3}}\right)' = 4\left(-4x^{-5}\right) - 5\left(\frac{1}{3}x^{-\frac{2}{3}}\right)
= -\frac{16}{x^5} - \frac{5}{3} \frac{3}{x^2}
\]

(5 pts) The market research department of a company recommends that the company manufacture steam irons. After suitable test marketing, the research department presents the demand equation \( p(x) = 20 - \frac{x}{50} \), where \( x \) is the number of irons retailers are likely to buy per week at $p. The financial department provides the cost equation \( C(x) = 3,600 + 2x \), where $3,600 is the estimated fixed costs and $2 is the estimated variable costs. Use the graph of the revenue and cost equations to find the break-even points.

\[
p(x) = 20 - \frac{x}{50} = \implies R(x) = xP(x) = 20x - \frac{x^2}{50}
\]

\[
C(x) = 3,600 + 2x
\]

\[
x(20 - \frac{x}{50}) = 0 \implies x = 0, x = 1000
\]

\[
(300, 4200)
(600, 4800)
\]

\[
R(500) = R_{max} = 500(20 - \frac{500}{50}) = 5000
\]

\[
C(0) = 3,600; C(1000) = 3,600 + 2000 = 5,600
\]

Break-even pts: \( C(x) - R(x) \)

\[
3600 + 2x = 20x - \frac{x^2}{50} \implies \frac{x^2}{50} - 18x + 3600 = 0
\]

\[
x^2 - 900x + 180000 = 0
\]