Domain: \( x > 0 \) and \( p > 0 \) \( \Rightarrow \) \( \frac{500 - 0.5x}{0.5} > 0 \) \( \Rightarrow \) \( x < \frac{500}{0.5} = 1000 \)

SURVEY OF CALCULUS - EXAM II

Name

(5 pts) A company manufactures and sells \( x \) videophones per week. The weekly price-demand and cost equations are, respectively,

\[ p = 500 - 0.5x \quad \text{and} \quad C(x) = 20,000 + 135x. \]

(A) What price should the company charge for the phones, and how many phones should be produced to maximize the weekly revenue? What is the maximum weekly revenue?

(B) What is the maximum weekly profit? How much should the company charge for the phones, and how many phones should be produced to realize the maximum weekly profit?

\( (A) \) \( R(x) = xp = 500x - 0.5x^2; \quad R'(x) = 500 - x = 0 \quad \Rightarrow \quad x = 500 \) (phones)

\( (B) \) \( P(x) = R(x) - C(x) = 500x - 0.5x^2 - 20000 - 135x \)

\( = -0.5x^2 + 365x - 20000 \)

\( P'(x) = -x + 365 = 0 \quad \Rightarrow \quad x = 365 \)

\( \lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^3} = \left( \frac{0}{0} \right) = \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{3x^2} = \left( \frac{0}{0} \right) = \lim_{x \to 0} \frac{e^x - e^{-x}}{6x} = \lim_{x \to 0} \frac{e^x + e^{-x}}{6} = \frac{2}{6} = \frac{1}{3} \)
\[ x \text{ is the 1st number} \]
\[ y \text{ is the 2nd number} \]

(5 pts) Find two numbers whose difference is 30 and whose product is a minimum.

\[ x-y=30 \]
\[ f(x) = xy \rightarrow \min \]

\[ y = x-30 \Rightarrow f(x) = x(x-30) = x^2 - 30x \]
\[ f'(x) = 2x - 30 = 2(x-15) = 0 \Rightarrow x = 15 \]

\[ f' \bigg|_{x=15} = \frac{\text{min}}{15} \] (Global min, because it's only one critical pt)

The two numbers are:
\[ x = 15 \]
\[ y = 15 - 30 = -15 \]

(5 pts) Find the indicated derivative and simplify

\[ \frac{dy}{dx} \text{ for } y = \frac{10^x}{1+x^4}. \]

\[ \frac{dy}{dx} = \frac{10^x \ln 10 (1+x^4) - 4x^3 10^x}{(1+x^4)^2} \]

\[ = \frac{10^x (\ln 10 \cdot x^4 - 4x^3 + \ln 10)}{(1+x^4)^2} \]
(5 pts) Public awareness of a congressional candidate before and after a successful campaign was approximated by

\[ P(t) = \frac{8.4t}{t^2 + 49} + 0.1, \quad 0 < t < 24, \]

where \( t \) is time (in months) after the campaign started and \( P(t) \) is the fraction of people in the congressional district who could recall the candidate's (and later, congressman's) name. Find the critical values of \( P(t) \), the time intervals on which the fraction is increasing, the time intervals on which the fraction is decreasing, and the local extrema. Do not graph.

\[
P'(t) = 8.4 \left[ \frac{t^2 + 49 - 2t^2}{(t^2 + 49)^2} \right] = 8.4 \left[ \frac{49 - t^2}{(t^2 + 49)^2} \right]
\]

\[ t = 7 \quad (t = -7 \text{ is not in the domain}) \]

\[
P''(t) = \frac{8.4 \cdot 7}{(49 + 49)^2} \quad \text{is not} \quad \frac{2}{20} \]

\[
\begin{array}{c}
P''(t) = \frac{8.4 \cdot 7}{49 + 49} + 1 = 0.6 + 0.1 = 0.7 = 0.7 \quad \text{is a local maximum} \\
P \quad \text{is increasing on} \quad (0, 7) \quad \text{and decreasing on} \quad (7, 24)
\end{array}
\]

(5 pts) Compute the limit

\[
\lim_{x \to \infty} \frac{\sqrt{1 + x^2}}{x} = \lim_{x \to \infty} \frac{x \sqrt{\frac{1}{x^2} + 1}}{x} = \lim_{x \to \infty} \frac{\sqrt{\frac{1}{x^2} + 1}}{1} = 1
\]

3
(5 pts) A T-shirt manufacturer is planning to expand its workforce. It estimates that the number of T-shirts produced by hiring $x$ new workers is given by

$$T(x) = -0.25x^4 + 5x^3, \quad 0 \leq x \leq 15.$$ 

When is the rate of change of T-shirt production increasing and when is it decreasing? What is the point of diminishing returns and the maximum rate of change of T-shirt production? Graph $T$ and $T'$ on the same coordinate system.

$$T' = -3x^3 + 15x^2,$$
$$T'' = -6x + 30x = 0 \implies -3x(x-10) = 0, \quad x = 0, x = 10$$

1) $x = 10$ is the pt of diminishing returns
2) The maximum rate of change

$$T'(10) = 500$$

(5 pts) Find the indicated derivative and simplify

$$\frac{d}{dx} \log_3(4x^3 + 5x + 7).$$

$$= \frac{1}{(4x^3 + 5x + 7) \ln 3} \cdot (4x^3 + 5x + 7)'$$

$$= \frac{12x^2 + 5}{(4x^3 + 5x + 7) \ln 3}$$