SURVEY OF CALCULUS - EXAM III

Name ________________________

(5 pts) A pharmacy has a uniform annual demand for 200 bottles of a certain antibiotic. It costs $10 to store one bottle for one year and $40 to place an order. How many times during the year should the pharmacy order the antibiotic in order to minimize the total storage and reorder costs?

$x$ the number of times the pharmacy places order
$y$ number of bottles in each order

Order cost: $40x$, Storage cost: $10\left(\frac{y^2}{2}\right)$

Total cost: $C(x) = 40x + 10\left(\frac{y^2}{2}\right) = 40x + 5y$

We also have $xy = 200 \implies y = \frac{200}{x} \implies C(x) = 40x + \frac{1000}{x}$

$C'(x) = 40 - \frac{1000}{x^2} = \frac{40x^2 - 1000}{x^2} = \frac{40(x^2 - 25)}{x^2} \implies x, y = \pm 5$

$x = -5$ does not work (x must be positive)

$C''(x) = \frac{2000}{x^3} > 0$ for any $x > 0 \implies C(5) = 400$ is the min. cost.

(5 pts) Calculate the indefinite integral

$$\int \left(\frac{12}{x^5} - \frac{1}{\sqrt{x^2}}\right) \, dx.$$

$$= \int\left(12x^{-5} - x^{-\frac{1}{2}}\right) \, dx = \frac{12x^{-4}}{-4} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{3}{x^4} - 3\sqrt{x} + C$$
(5 pts) In 2002, U.S. consumption of renewable energy was 5.89 quadrillion Btu (that is, \(5.89 \times 10^{15}\) Btu). Since the 1960s, consumption has been growing at a rate given by

\[ f'(t) = 0.004t + 0.062, \]

where \(t\) is years after 1960s. Find \(f(t)\) and estimate U.S. consumption of renewable energy in 2020.

\[
\begin{align*}
\int f(t) &= \int (0.004t + 0.062) \, dt = 0.004 \frac{t^2}{2} + 0.062t + C \\
&= 0.002t^2 + 0.062t + C \\
f(42) &= 0.002 \cdot 42^2 + 0.062 \cdot 42 + C = 5.89 \\
C &= -0.242 \\
f(60) &= 0.002 \cdot 60^2 + 0.062 \cdot 60 - 0.242 = 10.645 \text{ (quadrillion Btu)}
\end{align*}
\]

(5 pts) Integrate by a substitution

\[
I = \int \frac{x}{(5-2x^2)^3} \, dx.
\]

\[
\begin{align*}
U &= 5-2x^2 \\
dU &= -4x \, dx \Rightarrow x \, dx = -\frac{dU}{4} \\
I &= \int \frac{1}{U^5} \frac{dU}{-4} = -\frac{1}{4} \int U^{-5} \, dU = -\frac{1}{4} \frac{U^{-4}}{-4} + C = \frac{1}{16} U^{-4} + C \\
&= \frac{1}{16 (5-2x^2)^4} + C
\end{align*}
\]
(5 pts) Compute the Riemann sum $S_n$ for the function $f(x) = x^2 - 5x - 6$ by partitioning $[0, 3]$ into three subintervals of equal length, and taking $c_1 = 0.2$, $c_2 = 1.5$, and $c_3 = 2.8$.

$$S_n = S_3 = \Delta x \cdot f(c_1) + \Delta x \cdot f(c_2) + \Delta x \cdot f(c_3)$$
$$= f(0.2) + f(1.5) + f(2.8) = -6.96 - 11.25 - 12.16$$
$$= -30.37$$

(5 pts) Evaluate the integral

$$\int_{-2}^{-1} \frac{x^2 + 1}{x} \, dx.$$ 

$$= \int_{-2}^{-1} \left( \frac{x^2}{x} + \frac{1}{x} \right) \, dx = \int_{-2}^{-1} \left( x + \frac{1}{x} \right) \, dx = \left[ \frac{x^2}{2} + \ln |x| \right]_{-2}^{-1}$$
$$= \frac{1}{2} \left( (-1)^2 - (-2)^2 \right) + \ln 1 - \ln (-1) - \ln 2$$
$$= \frac{1}{2} (-1 - 4) - \ln 2 = \boxed{-\frac{3}{2} - \ln 2}$$
(5 pts) The data in the table describe the distribution of wealth in a country:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
<th>0.80</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.12</td>
<td>0.31</td>
<td>0.54</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

(A) Use quadratic regression to find the equation of a Lorenz curve for the data.
(B) Use the regression equation and numerical integration routine to approximate the Gini index of income concentration.

\[ y = ax^2 + bx + c \]
\[ a = 0.3125 \]
\[ b = 0.4175 \]
\[ c = -0.015 \]

\[ \int_0^1 \ln(x-y1, x, 0, 1) \cdot 2 \]
\[ 0.10416 \]

(5 pts) Calculate the definite integral

\[ I = \int_6^7 \frac{\ln(t-5)}{t-5} \, dt. \]

\[ U = \ln(t-5) \]
\[ U(6) = 2 \ln(6-5) = 2 \ln 1 = 0 \]
\[ U(7) = 2 \ln(7-5) = 2 \ln 2 \]

\[ I = \int_6^7 U \, du = \frac{U^2}{2} \bigg|_6^7 = \frac{\ln^2 2}{2} \]