

# VECTOR CALCULUS - EXAM I

Name \_\_\_\_\_

(5 pts) Find the angle  $\theta$  between the planes with equations

$$2x + 3y - z = -3 \quad \text{and} \quad 4x + 5y + z = 1$$

**(Extra Credit 10 pts)** Write symmetric equations of their line of intersection

(10 pts) Find the arc length of the curve

$$\mathbf{r}(t) = \left( \frac{t^2}{2}, \ln t, \sqrt{2} t \right) \quad \text{from } t = 1 \quad \text{to } t = 2$$

(10 pts) Find the curvature of the given plane curve at the indicated point

$$x = t - 1, \quad y = t^2 + 3t + 2, \quad \text{where } t = 2$$

(10 pts) Find the curvature  $k(t)$  of the space curve

$$\mathbf{r}(t) = (t, t^2, t^3)$$

(10 pts) Find tangential and normal accelerations for the curve

$$\mathbf{r}(t) = (t, t^2, t^3)$$

(10 pts) Suppose that the temperature  $\omega$  (in degrees Celsius) at the point  $(x, y)$  is given by

$$\omega(x, y) = 10 + (0.003)x^2 - (0.004)y^2.$$

In what direction  $\mathbf{u}$  should a bumblebee at the point  $(40, 30)$  initially fly in order to get warmer fastest? Find the directional derivative  $D_{\mathbf{u}}\omega(40, 30)$  in this optimal direction  $\mathbf{u}$ .

(5 pts) Write an equation of the plane tangent to the ellipsoid

$$2x^2 + 4y^2 + z^2 = 45 \quad \text{at the point} \quad (2, -3, -1)$$

(10 pts) Find a potential function for the conservative vector field

$$\mathbf{F}(x, y) = (6xy - y^3)\mathbf{i} + (4y + 3x^2 - 3xy^2)\mathbf{j}$$

(10 pts) Find  $\operatorname{div} \mathbf{F}$  and  $\operatorname{curl} \mathbf{F}$ , if the vector field  $\mathbf{F}$  is given by

$$\mathbf{F}(x, y, z) = (xe^y)\mathbf{i} + (z \sin y)\mathbf{j} + (xy \ln z)\mathbf{k}.$$

Also find  $\operatorname{curl} \mathbf{F}$  at the point  $(3, \frac{\pi}{2}, e)$