

*Profit = Revenue - Cost: A review of Basic Concepts*

Y = Total Profit

P = Price per Unit (e.g. per widget)

VC = Variable Cost per Unit (e.g. per widget)

z = Contribution Per Unit = Contribution Margin = P-VC

**Z** = Total Contribution = Q\*z = Q\*P - Q\*VC

FC = Fixed Cost

<p>Simplifying Assumption: Quantity Supplied = Quantity Demanded = Quantity Sold = Q</p> <p>TR Total Revenue = P*Q</p> <p>TVC = Total Variable Cost = VC*Q</p> <p>TC = Total Cost = TVC+ FC = VC*Q + FC</p> <p><b>Z</b> = Total Contribution = TR - TVC = P*Q- VC*Q = Q*(P-VC) = Q*z</p> <p>Total Profit = Y = TR - TC</p> <p>Total Profit = Y = TR - TVC - FC</p> <p>Total Profit = Y = <b>Z</b> - FC = z*Q - FC</p> <p>Total Profit = Y = P*Q - TC</p> <p>Total Profit = Y = P*Q - VC*Q - FC</p>	<p>D = Quantity Demanded</p> <p>S = Quantity Supplied</p> <p>Q = Quantity Sold = min(D,S)</p> <p>SV = Salvage Value</p> <p>LS = Cost of Lost Sale</p> <p>If D&gt;S, Y = P*Q - VC*S - LS*(D-S) - FC</p> <p>If D&lt;S, Y = P*Q - VC*S + SV*(S-D) - FC</p> <p>If D=S, the simplifying assumption holds.</p>
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Numeric Examples:

1a. Suppose  $Q=500$ .  $Y=\$0$ ,  $FC = \$40,000$ . Find  $Z$ . Use the Simplifying Assumption since  $Q$  is given: (Studying what happens when  $Y=\$0$  is called "Breakeven Analysis")

$$\$0 = Z - \$40,000$$

$$Z = \$40,000$$

1b. Suppose  $Q=500$ .  $Y=\$0$ ,  $FC = \$40,000$ . Find  $z$ . Use the Simplifying Assumption since  $Q$  is given:

$$Y = z \cdot Q - FC$$

$$\$0 = z \cdot 500 - \$40,000$$

$$\$0 + \$40,000 = z \cdot 500$$

$$\$40,000/500 = z.$$

$$z = \$80$$

$$\text{Check: } \$0 = \$80 \cdot 500 - \$40,000$$

$$\$0 = \$40,000 - \$40,000$$

$$\$0 = \$0$$

2a. Suppose  $Y=\$0$ . Find  $Q$  as a function of unknown  $P$ ,  $VC$ , and  $FC$

Use the Simplifying Assumption since  $Q$  is given:

$$Y = P \cdot Q - VC \cdot Q - FC$$

$$\$0 = (P - VC) \cdot Q - FC$$

$$\$0 + FC = (P - VC) \cdot Q$$

$$Q = \frac{FC}{P - VC}$$

2b. Now suppose price and variable cost are both tripled while  $Y$  remains zero. What happens to  $Q$ ?

$$\text{new } Q = \frac{FC}{3P - 3VC} = \frac{FC}{3(P - VC)} = \frac{1}{3} \cdot \frac{FC}{P - VC}$$

$$\text{new } Q = \frac{1}{3} \cdot \text{Old } Q$$

3a. Selling price per unit is  $\$50$ ; Variable cost per unit is  $\$10$ ; Salvage value per unit is  $\$4$ ; Cost of lost sales is  $\$2$  per unit. Find the profit ( $Y$ ) when 80 units are produced and the state of nature is a demand of 90 units. Do not include fixed costs.

$$P=\$50, VC=\$10, SV=\$4, LS=\$2, S=80, D = 90$$

$$\text{Since } D > S, Y = P \cdot Q - VC \cdot S - LS \cdot (D - S) - FC$$

$$Y = \$50 \cdot \min(80, 90) - \$10 \cdot 80 - \$2 \cdot (90 - 80)$$

$$Y = \$50 \cdot 70 - \$10 \cdot 80 - \$2 \cdot 10$$

3b. Selling price per unit is  $\$50$ ; Variable cost per unit is  $\$10$ ; Salvage value per unit is  $\$4$ ; Cost of lost sales is  $\$2$  per unit. Find the profit ( $Y$ ) when 80 units are produced and the state of nature is a demand of 70 units. Do not include fixed costs.

$$P=\$50, VC=\$10, SV=\$4, LS=\$2, S=80, D = 70$$

$$\text{Since } D < S, Y = P \cdot Q - VC \cdot S + SV \cdot (S - D) - FC$$

$$Y = \$50 \cdot \min(80, 70) - \$10 \cdot 80 - \$4 \cdot (80 - 70)$$

$$Y = \$50 \cdot 70 - \$10 \cdot 80 - \$4 \cdot 10$$

Note: in 3a and 3b,  $Y$  is really  $Z$  unless  $FC$  really is zero. Maybe it's a home business?

4. You own a network maintenance company and you have decided to expand your operations. You have consulted with several experts and have narrowed your choices down to two options. You can lease the need additional equipment, or you can purchase it. Your current operations will not be considered when making this decision. Q is the number of networks you will maintain, and you charge \$150 per network each month..

	Fixed Cost per month	Variable Cost per network per month	Contribution per network per month
Lease option	\$ 6,000	\$ 25	
Purchase option	\$ 8,000	\$ 15	

4a. Find the equation for profit for the lease option:

$$P = \$150, VC = \$25, FC = \$6,000$$

$$Y = P*Q - VC*Q - FC$$

$$Y = \$150*Q - \$25*Q - \$6,000$$

$$Y = \$125*Q - \$6,000$$

4b. Find the equation for profit for the purchase option:

$$P = \$150, VC = \$15, FC = \$8,000$$

$$Y = P*Q - VC*Q - FC$$

$$Y = \$150*Q - \$15*Q - \$8,000$$

$$Y = \$145*Q - \$8,000$$

4c. The lease option gives higher profit if the number of networks is small, the purchase option gives higher profit if the number of networks is large. What number of networks makes the two options give equal profit?

$$\$125*Q - \$6,000 = \$145*Q - \$8,000$$

$$-\$125*Q + \$8,000 = -\$125*Q + \$8,000$$

$$\$2,000 = \$10*Q$$

$$Q = \$2,000/\$10 = 200 \text{ networks}$$

Check:

$$\$125*200 - \$6,000 = \$145*200 - \$8,000$$

$$\$25,000 - \$6,000 = \$29,000 - \$8,000$$

$$\$19,000 = \$19,000$$

So if Q=200 networks, either option gives \$6,500 profit.

If Q < 200, profit < \$6,500 and leasing the new equipment is more profitable than purchasing it;

if Q > 200, profit > \$6,500 and purchasing the new equipment is more profitable than leasing it.