# Testing Simple Linear Regression (Chapter 3, Part 2)

3.5 Testing the Significance of the Slope and y-intercept 101

Standard Error of the Slope

Recall the standard error of $\bar{y}$ = standard deviation of residuals = s = $\sqrt{\frac{SSE}{n-2}}$

The standard error of the slope = $s_{b_1} = \frac{s}{\sqrt{SS_{xx}}} = \sqrt{\frac{SSE}{(n-2)SS_{xx}}}$

Testing the significance of the Slope

We can reject $H_0: \beta_1 = 0$ in favor of $H_1: \beta_1 \neq 0$ if |t| = $\left|\frac{b_1}{s_{b_1}}\right| > t_{\alpha/2}^{n-2}$

We can reject $H_0: \beta_1 = 0$ in favor of $H_1: \beta_1 > 0$ if t = $\frac{b_1}{s_{b_1}} > t_{\alpha}^{n-2}$

We can reject $H_0: \beta 1 = 0$ in favor of $H_1: \beta_1 < 0$ if t = $\frac{b_1}{s_{b_1}} < t_{\alpha}^{n-2}$

Recall that in Excel $t_{\alpha/2}^{n-2}$ =TINV(α,n-1) and $t_{\alpha}^{n-2}$ =TINV(2*α,n-2)

The p value is Pr[$t^{n-2} > \frac{b_1}{s_{b_1}}$] = Pr[$t^{n-2} < -\frac{b_1}{s_{b_1}}$]

A Confidence Interval for the Slope

We can be 1-$\alpha$ confident that $\beta_1$ is within $[b_1 \pm t_{\alpha/2}^{n-2} s_{b_1}]$

The Standard Error of the y-intercept

The standard error of the y-intercept, = $s_{b_0} = \sqrt{\frac{1}{n} + \frac{\bar{x}}{SS_{xx}}}$

Testing the significance of the Intercept

We can reject $H_0: \beta_0 = 0$in favor of $H_1: \beta_0 \neq 0$ if |t| = $\left|\frac{b_0}{s_{b_0}}\right| > t_{\alpha/2}^{n-2}$

3.6 Confidence and Prediction Intervals 108

Distance Value $\frac{1}{n} + \frac{(x_o - \bar{x})^2}{SS_{xx}}$

Sampling Distribution of $\bar{y}$ when $x=x_o$

mean = $\mu_{y|x_o} \simeq b_o + b_1 x_o$

Standard deviation =$\sigma\sqrt{\text{Distance}} \simeq s\sqrt{\text{Distance}}$

s = standard error = $\sqrt{\frac{SSE}{n-2}}$

Confidence Interval for $\mu_{y|x_o}$

We can be 1-$\alpha$ confident that $\mu_{y|x_o}$ is within $[\hat{y} \pm t_{\alpha/2}^{n-2} s\sqrt{\text{Distance}}]$

Prediction Interval for the "next" y whose $x=x_o$

We can be 1-$\alpha$ confident that the "next" y whose $x=x_o$ is within

$[\hat{y} \pm t_{\alpha/2}^{n-2} s\sqrt{1+\text{Distance}}]$

Note for Excel use =TINV($\alpha$,n-2) for $t_{\alpha/2}^{n-2}$

3.7 Simple Coefficients of Determination and Correlation 114

Simple Coefficient of Determination $r^2 = \frac{SSR}{SST} = \frac{SS_{yy}-SSE}{SS_{yy}}$

Simple Coefficient of Correlation $r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$

note that (coefficient of correlation)$^2$ = coefficient of determination.

r is negative if and only if $b_1$ is negative.

Significance Test for the Population Correlation

We can reject the null hypothesis that the population correlation $\rho$ ("rho") =0

in favor of the alternate hypothesis that $\rho \neq 0$

if the t statistic t = $\left| \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \right| > t_{a/2}^{n-2}$

The p-value is $\Pr\left(t^{n-2} \leq \left| \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \right|\right)$

3.8 An F test for the Model 121

We can reject the null hypothesis that the population regression coefficient $\beta_1 = 0$

in favor of the alternate hypothesis that $\beta_1 \neq 0$

if the F statistic F = $\frac{SSR}{SSE/(n-2)} = \frac{SST-SSE}{SSE/(n-2)} > F_{1-a}^{1,n-2}$

The p-value is $\Pr\left(F^{1,n-2} \geq \frac{SSR}{SSE/(n-2)}\right)$