Linear Programming Fundamentals

By Thomas Whalen and Jeff Churchill

Spring 2010

A chicken farmer near Gainsville, Georgia would like to feed his flock properly, but as economically as possible. If he had 50, or even 5,000 chickens, he would undoubtedly go down to the local feed and seed emporium. With 500,000 chickens, a few cents per pound of feed adds up to serious money over a year's time. Nutritional requirements are well known (and don't include marigold petals). A wide variety of types and grades of grains and meals are capable of contributing important parts of these nutritional requirements. To complicate things further, most of these are commodity items whose market prices change daily and even hourly. What mix of various grades of corn, wheat, oat, millet, other grains, and supplements can he put through his feed mill to give him fat, healthy birds at least cost?

A major distributor of consumer electronics has 8 warehouses serving 325 retail outlets scattered around the contiguous United States. With the normal order/reorder cycle, supplies of the various products the company stocks in these warehouses can vary fairly dramatically. Similarly, the retail outlets experience their own fluctuations. In the fiercely competitive consumer electronics business, *no stock* equates to *no sale*. *Any* warehouse *can* ship to the Boise retail store to meet that store's need for six 19" SoSory modular color TV's if that warehouse has that many in stock. *Some* can do it a lot more cheaply than others. Bozeman might need 4 sets too. Reno, the nearest warehouse to both stores, has 8. Repeat this scenario across 7 more warehouses and 323 more stores. Take into account (at least), motor freight, UPS and its competitors, and air freight, all with differing rate structures among all pairs of points. Is there a way to work out a least cost shipping schedule each week in time for it to matter?

MacroSoft Software is preparing to "roll out" its new killer application, a *work processing* package named DRUDGE. It has identified the demographic characteristics of people who should be in the core market for DRUDGE, and has estimates of how many of these people live in each of the 17 major market areas they have targeted. They have also identified a long list of print and broadcast media in which they might advertise DRUDGE during the introduction. For each medium, they have information on the demographic "reach" in each of these market areas, and a list of advertising rates. No matter how powerful any one medium may be, they have set some criteria for maintaining a balance among them. With a $25,000,000 advertising budget for the rollout, how can they maximize their total exposure to targeted potential customers?

All of these situations, and many many more, have some things in common. In each case, there is a well defined goal; something that the decision maker wants to maximize (*e.g.*, exposures to qualified buyers) or to minimize (*e.g.*, the cost of shipping consumer electronics products). In each case, there are limitations that the decision maker is constrained to obey. The chicken farmer needs to meet nutritional requirements with an appropriate volume of a mix that the chickens won't refuse to eat. The more #2 red winter wheat he includes in his mix, the less he can use of other potential ingredients. The distributor must meet its retailers' orders, and can't ship the same stereo system to 2 different stores or leave a warehouse with a negative supply of Boogie Boxes. MacroSoft can't spend it all on TV (some "influentials" never watch), and a dollar they spend in InfoWorld isn't available to spend in PlayGirl. In all of these cases, there are tradeoffs. In all of these cases the relationships are really fairly simple, but there are **so many** to keep track of that it might seem mind boggling to the decision maker.

Linear Programming (**LP**) is a modeling approach for what is termed *constrained optimization*. As the term might seem to imply, constrained optimization involves selecting a provably best solution in a situation where you are constrained in some specific ways. There are a number of constrained optimization techniques, most of which fall under the heading *Mathematical Programming*. Mathematical programming encompasses Linear Programming, Integer Programming, Quadratic Programming, and more. Of all of these, the least complex and most widely applied modeling approach is Linear Programming.

Programming in this context does <u>*not*</u> imply computer programming, despite the fact that the first digital computer actually put into use was constructed so that George Dantzig, the "father" of Linear Programming, could solve large linear programming problems. The term "programming" is used here to imply developing a program of action in a decision situation.

The word *linear* is critical here. Linear programming solution methods depend on all relationships in the model being linear. "Linear" means that all variables appear in every equation or inequality with a coefficient that is (or represents) a numeric constant and with an exponent of 1. The coefficient may be positive, negative, 0, or 1. *If you graph a linear equation or inequality, it is always a straight line* (or, beyond 2 dimensions, a flat plane). Terms involving the equivalent of X^2, XY, $\frac{X}{Y}$, or $\frac{X_1+2X_2-5X_3}{4Y}$ are all examples of forms that you *never* see in a properly set up linear programming problem. If the real world problem requires non - linear relationships in order to describe the situation correctly, it calls for a model other than linear programming to solve the problem.

The basic form of a linear programming problem involves maximizing or minimizing a linear *objective function* subject to a set of linear *constraints*. The classic mathematical form for such a problem is:

$$
\begin{array}{llllll}
\text{Maximize } Z= & C_1X_1 + & C_2X_2 + & \ldots & C_nX_n & \\
 & & & \vdots & & \\
\text{Subject to:} & & & & & \\
 & a_{11}X_1 + & a_{12}X_2 + & \ldots & a_{1n}X_n & \leq b_1 \\
 & & & \vdots & & \\
 & a_{21}X_1 + & a_{22}X_2 + & \ldots & a_{2n}X_n & \leq b_2 \\
 & & & \vdots & & \\
 & \ldots & \ldots & \ldots & \ldots & \ldots \\
 & & & \vdots & & \\
 & a_{m1}X_1 + & a_{m2}X_2 + & \ldots & a_{mn}X_n & \leq b_m \\
 & & & \vdots & & \\
 & & & & \text{all } X_i & \geq 0
\end{array}
$$

This is only symbolic of a linear programming problem so that we can identify its components. It is not literally an LP problem as we will deal with them. Normally, our problems will not appear so starkly mathematical. You probably find that to be a relief.

This format symbolizes a typical *Maximization* problem with **n** *activity variables* (the X_i values) and with **m** constraints (the inequalities). The function Z is the *objective function*, and the C_i values are the *objective function coefficients*. When the problem has been solved, Z will be the total value of the objective function. We want Z to have the largest value possible.

The X_i values represent the activity variables. Usually, these days, we use meaningful variable names like TV or WHEAT instead of X_i. when we set up the problem. An activity variable is a measure of the quantity of some activity we will undertake as a result of our LP solution. Shipping a TV or buying a carload of wheat is an activity in this sense. The other name we often use for these variables is *decision variables*. When we solve an LP problem, our principal result is values for all of the X_i's. That set of values is, if we formulated and solved our linear programming problem correctly, the *optimal decision* or *optimal solution*. All variables (we will later see other kinds) are restricted to being zero or positive. This is very helpful in creating a way to solve these problems. It also usually makes good physical sense. You can't make, e.g., negative stereos. You might think of disassembling one to make its parts available for another product as making a "negative" stereo, but I challenge you to find a way to get the assembly labor back out of the stereo and use it in another product!

Assumptions behind linear programming

There are some very important assumptions underlying our use of linear programming. As with all models, we assume that our model is a close enough analogy to the real situation that we can let it help guide our decision making. We also have some assumptions that are specific to LP.

Linearity We assume that the underlying relationships we describe are linear. This means for example that there are no increasing or decreasing returns to scale. If we purchase parts or materials, we are not ignoring discounts that change with volume. This is simply the assumption that the relationships we describe as linear are *in fact* linear.

Additivity This assumption requires internal consistency in our units of measure. Translated into English, if we add apples to walnuts to marshmallows we may have Waldorf Salad but we do not have a viable LP objective function or constraint. For example, if one objective function coefficient is in $, another in ¥, another in F, another in £, and yet another in DM, well, it's all money isn't it? But how much <u>is</u> $100 + ¥1500 + 850F + £144 + 1900DM??? Or on the left hand side of a constraint if we have 2 hours/Widget $\times$ 4 hours + 3 Fidgets/hour $\times$ 6 Fidgets, just what does the resulting 26 mean?

Non Negativity We saw an implication of this assumption when we looked at the basic format for an LP problem. All variables are restricted to values greater than or equal to zero. This seldom poses a problem. In the rare case where we might have a legitimate need for a variable to take on a negative value, there is a trick for evading this requirement. You might see it if you take a more advanced course, but we won't consider it here.

Certainty Our solution methods become meaningless if we are uncertain of the values of the coefficients. The certainty assumption simply assumes that they aren't random variables. As we get deeper into LP, we will learn how we can avoid being overly restricted by this requirement.

Divisibility We must be willing to accept and implement solutions that are ***not*** whole numbers. 142.25 tons of wheat seldom bothers anybody, but 76.180375 trombones don't seem to lead very many students' parades, 12345.6789 table lamps seems a real curiosity, and $\frac{3}{4}$ of a nuclear submarine seems a chilling way to spend one's hard earned tax dollars. For wheat, we have no problem. The trombones are easier to cope with if we see this number as a *rate*, **not** a quantity. Another way of really saying about the same thing is to say that the .180375 trombone isn't scrap brass, but *work in process* that will be completed in the next operating period. If LP tells us to make 12345.6789 table lamps next year, there is no point in worrying. By the end of the year, many things will have happened to revise that, so it is really only a fairly approximate target to work toward. .6789 of a table lamp represents a very small amount of the total value involved anyway. In the case of the fractional submarine, we probably need to drop LP and try Integer Programming. It's hard to justify rounding to the nearest $10,000,000,000.

A Very Simple Linear Maximization Problem

Case 1: Widgets and Gadgets (Unpainted)

A widget takes 4 hours to assemble
A gadget takes 8 hours to assemble

If W is the number of widgets produced and G is the number of gadgets produced, the number of hours of assembly time required is 4W + 8G

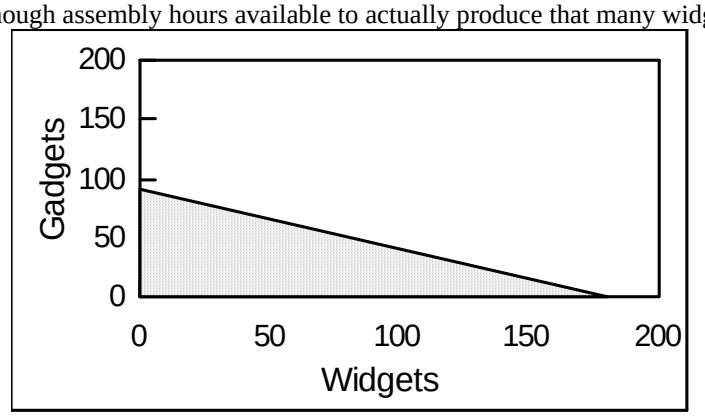
A maximum of 720 assembly hours are available per eight hour day since your factory can only accommodate 90 workers in the assembly area.

$4W + 8G \leq 720$

Each widget earns $50 contribution to profit and overhead. The contribution of gadgets varies from month to month; some months it is as low as $20 per gadget while other months it is as high as $110.

All widgets and gadgets are sold immediately at the going rate; none are kept in inventory.

In the following graph, the horizontal axis represents the number of widgets produced, and the vertical line represents the number of gadgets produced. Every point in the graph represents a "production schedule" calling for producing a number of widgets equal to the horizontal coordinate of the point, and a number of gadgets equal to the vertical coordinate of the same point. Points outside the shaded triangular area are "infeasable" in the sense that there are not enough assembly hours available to produce that many widgets and gadgets. The shaded area is called the "feasible region" because every point in its interior or its boundary represents a sensible production schedule in the sense that there *are* enough assembly hours available to actually produce that many widgets and gadgets.



QUESTION:
how many widgets and gadgets should you produce per day?

ANSWER: if the contribution per gadget is more than $100, produce 90 gadgets and no widgets. If the contribution per gadget is less than $100, produce 180 widgets and no gadgets.

(This problem is so simple that there's really no need for anything a powerful as linear programming, but it illustrates the barest essentials of an LP problem.)

Widgets and Gadgets (Painted)

You have acquired the facilities to paint your widgets and gadgets. (There is no longer any market for unpainted ones).
Your assembly constraints are as before: $4W + 8G \leq 720$ since each widget takes 4 hours to assemble, each gadget takes 8 hours to assemble, and the assembly department can accommodate up to 90 workers yielding 720 assembly hours per day.
Each widget takes 2 hours to paint, and each gadget takes 1 hour to paint. The paint department can accommodate up to 15 painters, yielding 120 painting hours per day. Thus, $2W + 1G \leq 120$
It is not necessary to use the full capacity if partial utilization is more profitable.

The net contribution for a painted widget is $100; the net contribution for a painted gadget varies, some months as low as $40 but other months as high as $210.

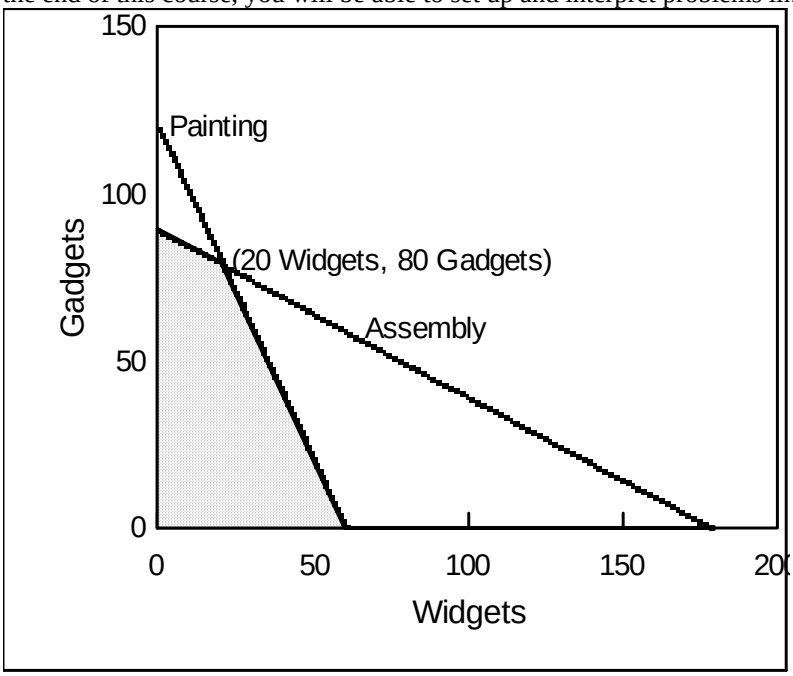
QUESTION:
How many widgets and gadgets should you produce per day?

ANSWER:
If the contribution per gadget is less than $50, make 60 widgets and no gadgets. (60 is the most widgets you can paint; there is some idle assembly capacity)

If the contribution per gadget is between $50 and $200, make 20 widgets and 80 gadgets. (This is the only way to completely utilize both assembly and painting capacity)

If the contribution per gadget is over $200, make 90 gadgets and no widgets. (90 is the most gadgets you can assemble; there is some idle painting capacity.)

This problem is a bit more complicated, though still far simpler than the ones LP was invented for. The graph below and the printouts on the following pages illustrate the solution when the going price for a gadget is $75. By the end of this course, you will be able to set up and interpret problems like this almost effortlessly!



Base Case with contrib per Gadget = $75

	Widgets	Gadgets		
Production Schedule	20	80	Value	Constraint
Contribution	$100.00	$75.00	$8,000.00	
Assembly	4	8	720	720
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Production Schedule Widgets	20.0000	0	$100.00	$50.00	$62.50
$C$2	Production Schedule Gadgets	80.0000	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$4	Assembly Value	720	$4.17	720	240	480
$D$5	Painting Value	120	$41.67	120	240	30

Effect of One Additional Assembly Hour

	Widgets	Gadgets		
Production Schedule	19.91667	80.16667	Value	Constraint
Contribution	$100.00	$75.00	$8,004.17	
Assembly	4	8	721	721
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$3	Production Schedule Widgets	19.9167	0	$100.00	$50.00	$62.50
$E$3	Production Schedule Gadgets	80.1667	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$5	Assembly Value	721	$4.17	721	239	481
$F$6	Painting Value	120	$41.67	120	240.5	29.875

Effect of One Additional Assembly Workstation

	Widgets	Gadgets		
Production Schedule	19.33333	81.33333	Value	Constraint
Contribution	$100.00	$75.00	$8,033.33	
Assembly	4	8	728	728
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$3	Production Schedule Widgets	19.3333	0	$100.00	$50.00	$62.50
$E$3	Production Schedule Gadgets	81.3333	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$5	Assembly Value	728	$4.17	728	232	488
$F$6	Painting Value	120	$41.67	120	244	29

Effect of One Additional Painting Workstation

	Widgets	Gadgets		
Production Schedule	25.33333	77.33333	Value	Constraint
Contribution	$100.00	$75.00	$8,333.33	
Assembly	4	8	720	720
Painting	2	1	128	128

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$3	Production Schedule Widgets	25.3333	0	$100.00	$50.00	$62.50
$E$3	Production Schedule Gadgets	77.3333	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$5	Assembly Value	720	$4.17	720	304	464
$F$6	Painting Value	128	$41.67	128	232	38

Effect of a Redundant Constraint

```
   MAX 100 WIDGETS + 75 GADGETS
SUBJECT TO  2)   4 WIDGETS + 8 GADGETS <=   720
            3)   2 WIDGETS +   GADGETS <=   120
            4)     WIDGETS +   GADGETS <=   101

OBJECTIVE FUNCTION VALUE  1) 8000.00000
```

Vari-able	Value	Reduced Cost	Current Coeff	Allowable Increase	Allowable Derease
Widgets	**20**	$0.00	$100.00	$50.00	$62.50
Gadgets	**80**	$0.00	$75.00	$125.00	$25.00

Row	Slack or Surplus	Dual Price	Current RHS	Allowable Increase	Allowable Derease
2	0	$4.17	720	12	480
3	0	$41.67	120	3	30
4	1	$0.00	101	Infinity	1

Effect of a Degenerate Constraint

```
   MAX 100 WIDGETS + 75 GADGETS
SUBJECT TO  2)   4 WIDGETS + 8 GADGETS <=   720
            3)   2 WIDGETS +   GADGETS <=   120
            4) WIDGETS +   GADGETS <=   100

OBJECTIVE FUNCTION VALUE  1)  8000.00000
```

Vari-able	Value	Reduced Cost	Current Coeff	Allowable Increase	Allowable Derease
Widgets	**20**	$0.00	$100.00	$50.00	$25.00
Gadgets	**80**	$0.00	$75.00	$25.00	$25.00

Row	Slack or Surplus	Dual Price	Current RHS	Allowable Increase	Allowable Derease
2	0	$0.00	720	Infinity	0
3	0	$25.00	120	80	0
4	0	$50.00	100	0	40

Effect of a "Small" Increase in Gadget Contribution

```
     MAX 100 WIDGETS + 195 GADGETS
SUBJECT TO   2)   4 WIDGETS + 8 GADGETS <=   720
             3)   2 WIDGETS + GADGETS <=   120

                    OBJECTIVE FUNCTION VALUE 1)     $17,600.00
```

Vari-able	Value	Reduced Cost	Current Coeff	Allowable Increase	Allowable Derease
Widgets	**20**	$0.00	$100.00	$290.00	$2.50
Gadgets	**80**	$0.00	$195.00	$5.00	$145.00

Row	Slack or Surplus	Dual Price	Current RHS	Allowable Increase	Allowable Derease
2	**0**	**$24.17**	**720**	**240**	**480**
3	**0**	**$1.67**	**120**	**240**	**30**

Effect of a "Large" Increase in Gadget Contribution

```
     MAX 100 WIDGETS + 205 GADGETS
SUBJECT TO   2)   4 WIDGETS + 8 GADGETS <=   720
             3)   2 WIDGETS + GADGETS <=   120

                    OBJECTIVE FUNCTION VALUE 1)     $18,450
```

Vari-able	Value	Reduced Cost	Current Coeff	Allowable Increase	Allowable Derease
Widgets	**0**	($2.50)	$100.00	$2.50	Infinity
Gadgets	**90**	$0.00	$205.00	Infinity	$5.00

Row	Slack or Surplus	Dual Price	Current RHS	Allowable Increase	Allowable Derease
2	**0**	**$25.625**	**720**	**240**	**720**
3	**30**	**$0.00**	**120**	**Infinity**	**30**

Sensitivity Analysis: "Allowable" Increases and Decreases

1. For an objective coefficient:

a. If the coefficient of one decision variable in the objective function increases or decreases BY an amount less than the allowable increase or decrease for that objective coefficient, (and everything else stays the same),

then the optimal solution will remain unchanged

but the optimal value will change.

b. If the coefficient of one decision variable in the objective function increases or decreases BY an amount equal to the allowable increase or decrease for that objective coefficient, (and everything else stays the same),

then there will be an infinite number of optimal solutions;

they will all have the same optimal value (different from the current one unless the allowable change is zero!)

c. If the coefficient of one decision variable in the objective function increases or decreases BY an amount greater than the allowable increase or decrease for that objective coefficient, (and everything else stays the same),

then there will be new optimal solution

and a new optimal value

2. For the RHS of a binding constraint:

a. If the right hand side of one binding constraint increases or decreases BY an amount less than or equal to the allowable increase or decrease for that constraint, (and everything else stays the same)

then there will be a new optimal solution

and the optimal value will increase or decrease by an amount equal to the product of constraint's dual price and the amount of change in the RHS

b. If the right hand side of one binding constraint increases or decreases BY an amount greater than the allowable increase or decrease for that constraint, (and everything else stays the same)

then there will be a new optimal solution

and the optimal value will increase or decrease by an unpredictable amount; rerun LINDO with the new RHS to find out.

3. For the RHS of a non-binding constraint:

a. If the right hand side of one non-binding constraint increases or decreases BY an amount less than or equal to the allowable increase or decrease for that constraint, (and everything else stays the same)

then the optimal solution will remain unchanged

and the optimal value will also remain unchanged

b. If the right hand side of one non-binding constraint increases or decreases BY an amount greater than the allowable increase or decrease for that constraint, (and everything else stays the same)

then there will be a new optimal solution and the optimal value will increase or decrease by an unpredictable amount; rerun with the new RHS to find out

"Allowable" Increases and Decreases

Quantity	Amount it Increases or Decreases BY	Optimal Decision	Change in Optimal Payoff
Objective Coefficient of Decision Variable "X"	Less then "Allowable"	Unchanged	Quantity of X times the change in its objective coefficient
	Equal to "Allowable"	An infinite number of solutions (including the old one)	Quantity of X times the change in its objective coefficient
	More than "Allowable"	Changed	Changed (Rerun to find the new Optimal Payoff)
Right-Hand-Side of Binding Constraint (Shadow Price Not = 0)	Less then "Allowable"	Changed	Sadow Price of the constraint times the change in its RHS
	Equal to "Allowable"	Changed	Shadow Price of the constraint times the change in its RHS
	More than "Allowable"	Changed	Changed (Rerun to find the new Optimal Payoff)
Right-Hand-Side of Non-Binding Constraint (Shadow Price = 0)	Less then "Allowable"	Unchanged	Unchanged
	Equal to "Allowable"	Unchanged	Unchanged
	More than "Allowable"	Changed	Changed (Rerun to find the new)Optimal Payoff)

Case 2: Beau Jarble's Maximization Problem

Beau Jarble has a small plant in which he makes a very limited line of inexpensive "mag" wheels for cars in the Secret Car Club of America (SCCA) Impound Touring category. All his wheels are 7" wide, and are drilled for either 4 or 5 wheel studs (for attaching the wheels). He purchases "blank" wheels cast to his design by a local foundry and finishes them in his little plant on weekends off from his regular job, using part time workers for labor. His part timers are all first rate machinists who earn $30 an hour.

The ITA model is made from a 13" diameter by 7" wide "A" casting and sells for $125. The ITS model, for heavier and more powerful cars, is made from the 14" by 7" "S" casting and sells for $145. His present stock of castings cost him $50 each for the A units and $57.50 each for the S units, but due to a drop in the price of aluminum he can get his next batch at $45 and $50. He has a good stock of castings on hand and considers the foundry to be utterly reliable. He does not anticipate being limited by the casting supply.

His wheels are highly regarded by both real racers and "cafe racers". While demand is certainly not unlimited, he figures he could sell all the ITA wheels he could possibly make. He does not think that he could sell more than 25 sets of ITS wheels per week. On the other hand, to maintain the prestige of his product, he never wants to sell more ITA than ITS.

The first operation in making a casting into a wheel is to use a milling machine to put a properly aligned flat surface on the back of the wheel where it mounts to the car. He has 3 milling machines and 3 operators who each work up to 10 hours per weekend. They are each able to "backface" 5 wheels per hour.

The next step is to drill the wheel mounting holes on a special drill press. An ITA wheel requires 4 holes, and an ITS wheel needs 5. His drill press operator can drill 60 holes/hour and is willing to work as many as 10 hours in a weekend. Once the wheels have their mounting holes drilled, they can go on a lathe to be trued and surface finished. Beau has 3 lathes and operators who each can work 10 hours on a weekend. It takes 18 minutes to perform all needed operations on an ITA, and 12 minutes on an ITS.

The last major operation is drilling the valve stem hole in each wheel. Beau does this himself on another drill press. It takes very little time, but requires a special stepped bit that costs $125 and is worn out after drilling 120 to 130 wheels. This weekend he has one new bit on hand, his old one is worn out, and he's ordered another that won't arrive before next Wednesday.

Beau does some final finishing and packaging himself. The materials for this work cost $12 for an ITA wheel and $ 19.50 for an ITS. His accountant takes $1/wheel depreciation on each of his machines, and figures other overhead at $10/wheel.

Beau would like to put together a practical production schedule for this weekend that would maximize profit for him. His friend Willy Wonky claims to know all about LP, and has offered to find Beau's best mix of wheels that way. Before you turn the page, why not get out a piece of paper and see if **you** can formulate this problem. I bet you can do better than Willy!

Willy went to work, and came back to see Beau after a couple of hours. First he showed Beau how he figured what to minimize:

	ITA	ITS
Casting	$50.00	$57.50
Backfacing	$6.00	$6.00
Drilling	$2.00	$2.50
Lathe	$9.00	$6.00
Stem	$1.00	$1.00
Final	$12.00	$19.50
Depreciation	$1.00	$1.00
Overhead	$10.00	$10.00
Accounting Cost	$91.00	$103.50

This, he said, set him up for this formulation of the problem:

```
Minimize Z=    91 ITA  +  103.5 ITS
Subject to:                   ITS    ≤    25     Demand
                - ITA  +      ITS    =     0     Prestige
                5 ITA  +    5 ITS    ≤    30     Backface
                4 ITA  +    5 ITS    ≤    60     Holes
               18 ITA  +   12 ITS    ≤    30     Lathe
                    ITA    25 ITS    ≤   100     Stem
               -
```

Beau knows better than this! Willy's formulation is wonky 9 ways from Sunday. There are a few mistakes that Willy didn't make, but most of the ones he missed, he only missed because Beau's problem didn't provide him the opportunity to commit the particular error.

We don't want to be as wonky as Willie, so let's interrupt this problem to look at a few basic principles for formulating LP problems. We won't hit everything, but what we will look at will be important.

Objective function

In linear programming problems, our activity variables must always be things on which the decision maker can act directly and which affect achievement of his or her goal. Willie at least got this right. It appears that his variable ITA was intended to represent the number of ITA wheels to make this weekend, and ITS represented the number of ITS wheels to make this weekend.

Correspondingly, the objective function should represent the effect on the variable that he or she wants to optimize from acting on those things. Willie apparently thought that variable was cost. That is seldom, if ever the case when we can directly identify a revenue with the activity variable. If Beau wants to minimize cost, it is easy. He can just go out of business. Voila! No cost! But, of course, also no revenue.

Generally, we minimize when there is a set of performance limitations we must stay within and wish to do so as cheaply or easily as possible. If Beau had a contract requiring him to deliver at least 100 wheels consisting of any mix of ITA and/or ITS models for $13,000, then it would be in his interest to find the least costly mix that was feasible for him to make this weekend. His decision would not affect revenue as long as he met the requirements of the contract. But here, every wheel is going to bring in either $125 or $145. It is <u>not</u> in Beau's best interests to ignore those revenues.

Beau wants to maximize his profit. More specifically, he wants to maximize the ***total positive increment to his economic well-being***. He can seriously mislead himself if he takes into account in his objective function anything that his decision cannot actually change. There is a rule we always try to apply when we attempt to measure the impact of a potential decision. We do not apply it <u>*only*</u> in linear programming, but we certainly do try always to use the rule in formulating LP problems. It defines ***Decision Costs*** and ***Decision Revenues***.

- ♦ Include a cost or revenue if it represents a future cash flow that arises as a direct consequence of the decision being made.

The selling price of a wheel is a future cash flow that Beau will collect only if he makes and sells that wheel. It qualifies.

Willie included some things that do not qualify. Accountants have some convenient *fictions* that may be useful for financial accounting purposes, but that have a strong barnyard odor when used for decision making purposes. These fictions are not <u>wrong</u>, but they are highly inappropriate (and therefore wrong) in most decision models. The *fictions* are:

1. *The correct cost to use is some form of historical cost*. For financial accounting this provides auditability and keeps the accountants from needing additional variance accounts. That is fine if it is your intent to measure *past* cash flow. From a decision making viewpoint, the value of an item is what it will cost to replace it so as to remain a going concern. If you do not intend to remain a going concern, estimated liquidation value, if any, is best.
2. *Buildings and equipment must be depreciated*. For financial accounting purposes, this allocates the cost of major items to the operating periods that they benefit. From a decision making viewpoint, if we use it this period and it will still be there to use next period, it costs us nothing. What we paid for the item in the past is a sunk cost, even if the accountant chooses to allocate it on a per item manufactured basis. Beau's accountant allocated $1.00 per wheel. He or she could have just as logically allocated 25¢ per hole!
3. *Overhead costs of manufacturing must be assigned to products*. Some consider this rule to be a little weird even for financial accounting purposes. These are period costs that will be incurred if the business is operating. Some are cash expenditures (the water bill, for example). Others are allocations like building depreciation. Even financial accountants know that, if a cost is directly traceable to increments of product, it should be a line item charge against that product. Overhead costs, by definition, do not arise from product mix decisions.

Good 'ol Willie managed to fall into all three of these traps. Let's get him out.

1. The wheel castings now on hand are a sunk cost. The only reason we want to treat them as costing something is that when Beau has used them up, he will need to buy more in order to stay in business. When he buys more, he'll pay the new price. So that's what they're worth.

2. Willie charged the accountant's equipment depreciation figure to the wheels. To whom should Beau write the depreciation check? This is not a decision cost. We exclude it.

3. What <u>are</u> we going to do with Willie? He even charged overhead. WE know better. Out it goes. We're making decisions, not keeping books.

When we're done, here's what we've got:

	ITA	ITS
Price	$125	$145
Casting	$50	$58
Backfacing	$6	$6
Drilling	$2	$3
Lathe	$9	$6
Stem	$1	$1
Final	$12	$20
Depreciation	$1	$1
Marginal Cost	$75	$85
Contribution Mrgin	$50	$80

You should trace this through - each item we included is a direct future cash consequence of making and selling a wheel.

Constraints

Let's take another look at the constraints for Beau's problem as Willie formulated them:

1	ITS		≤	25	Demand
2	ITS	- ITA	=	0	Prestige
3	5 ITS	+ 5 ITA	≤	30	Backface
4	5 ITS	+ 4 ITA	≤	60	Holes
5	12 ITS	+18 ITA	≤	30	Lathe
6	-25 ITS	ITA	≤	100	Stem

I'm going to try to translate this into its units of measure; in other words, attempt a dimensional analysis. It may reveal some problems.

1	*ITS*		ITA	≤	25 Sets of ITS
2	$+1\left(\frac{\text{prestige}}{\text{Per ITS}}\right)\times\text{ITS}$	+	$-1\left(\frac{\text{prestige}}{\text{Per ITA}}\right)\times\text{ITA}$	=	0 Prestige
3	$5(\frac{\text{ITS}}{\text{ITSPer Hour}})\times\text{ITS}$	+	$5(\frac{\text{ITA}}{\text{Per Hour}})\times\text{ITA}$	≤	30 hours Backface on Mill
4	$5(\frac{\text{holes}}{\text{Per ITS}})\times\text{ITS}$	+	$4(\frac{\text{holes}}{\text{Per ITA}})\times\text{ITA}$	≤	60 holes on drill press
5	$12(\frac{\text{munites}}{\text{Per ITS}})\times\text{ITS}$	+	$18(\frac{\text{munites}}{\text{Per ITA}})\times\text{ITA}$	≤	30 hours Lathe time
6	$-25\ \frac{\text{Sets of}}{\text{ITS}}\times\text{ITS}$	+	*ITA*	≤	100 Stem Holes

I've numbered the constraints to make them easier to refer to. The problems are now much easier to see and then to deal with. Taking each constraint in turn,

1. Incongruity arises here from comparing wheels and sets. Converting the RHS (Right Hand Side) to wheels will cure the problem.
2. Willie got this one correct dimensionally, but he fell into the common trap of over-constraining the solution. Beau didn't want to sell more ITA than ITS. Selling more ITS than ITA is just as good as selling the same number.
3. The left hand side evaluates to $\frac{\text{ITA}^2}{\text{hour}}+\frac{\text{ITS}^2}{\text{hour}}$, a nonsense quantity. It's pretty obvious that the coefficients should have been inverted.
4. The left hand side should be fine; it evaluates to *holes* drilled. The problem is in the RHS, where Willie forgot to multiply by the available number of hours of drill press time.
5. The left hand side evaluates to *minutes*, while the RHS is in *hours*. Either will work, but mixing them <u>won't</u> work worth a hoot.
6. Willie suffered from a couple of confusions here. He's back to mixing wheels and sets. Worse, for some reason he tried to combine 2 constraints into 1, causing both the ITS term and the RHS to be nonsense. He took one of Beau's simplest constraints and made it complicated.

Now that we know what the problems are, we can fix it pretty easily. Let's write the constraints the way we think they should be, then test them with dimensional analysis. If you got them right to start with, congratulations! If you are unsure, use dimensional analysis to test them <u>before</u> you continue reading. Write the constraints now, do the dimensional analysis check, then look at the rest of the page.

ITS		≤	100	Demand
ITS	- ITA	≥	0	Prestige
.2 ITS	+ .2 ITA	≤	30	Backface
5 ITS	+ 4 ITA	≤	600	Holes
.2 ITS	+ .3 ITA	≤	30	Lathe
ITS	+ ITA	≤	125	Stem

(I've written ITS before ITA here because I'm planning to graph ITS as the horizontal ("x") coordinate.) Before we examine this version of the constraints dimensionally, let's note a few things. If a decision variable does not affect a constraint, as in the first one, we may either omit it as you see here, or we may include it with a coefficient of 0. In both the first and the last constraint, no coefficients are *visible*. That <u>does</u> <u>not</u> <u>mean</u> that they aren't there. When we don't write a coefficient, a coefficient of 1 is implied. Some students whose math background is not as strong as it should be seem to think that a coefficient of 0 is implied. Not so. Finally, the second and fourth constraints are in hours. If you (correctly) formulated them in minutes, you aren't wrong but just made a different choice. You could, with difficulty, use seconds, fortnights, or decades.

Dimensionally, this is:

ITS		≤	*ITS*	Demand
$\left(\frac{\text{points}}{\text{per ITS}}\right)\times\text{ITS}$	$-\left(\frac{\text{points}}{\text{per ITA}}\right)\times\text{ITA}$	≥	*Points*	Prestige
$\left(\frac{\text{hours}}{\text{per ITS}}\right)\times\text{ITS}$	$+\left(\frac{\text{hours}}{\text{per ITA}}\right)\times\text{ITA}$	≤	*hours*	Backface
$\left(\frac{\text{holes}}{\text{Per ITS}}\right)\times\text{ITS}$	$+\left(\frac{\text{holes}}{\text{Per ITA}}\right)\times\text{ITA}$	≤	*holes*	Holes
$\left(\frac{\text{hours}}{\text{per ITS}}\right)\times\text{ITS}$	$+\left(\frac{\text{hours}}{\text{per ITA}}\right)\times\text{ITA}$	≤	*hours*	Lathe
$\left(\frac{\text{holes}}{\text{Per ITS}}\right)\times\text{ITS}$	$+\left(\frac{\text{holes}}{\text{Per ITA}}\right)\times\text{ITA}$	≤	*holes*	Stem

which resolves to

ITS	≤	*ITS*	Demand
points	≥	points	Prestige
hours	≤	*hours*	Backface
holes	≤	*holes*	Holes
hours	≤	*hours*	Lathe
holes	≤	*holes*	Stem

which isn't all that exciting, but it is at least clear that we don't have constraints that are obvious nonsense.

The corrected maximization problem

When we put together the corrections we have made, Beau's product mix problem is:

Maximize Z =	60 ITS	+50 ITA			
subject to:	ITS		≤	100 ITS	Demand
	ITS	- ITA	≥	0 *"points"*	Prestige
	.2 ITS	+ .2 ITA	≤	30 *hours*	Backface
	5 ITS	+ 4 ITA	≤	600 *holes*	Holes
	.2 ITS	+ .3 ITA	≤	30 *hours*	Lathe
	ITS	+ ITA	≤	125 *stemholes*	Stem

Willie didn't mention the non-negativity restriction that we saw when we looked at the mathematical form for an LP problem, and it isn't included here. Some people think that you should always write it. Maybe. On the other hand, it is inherent in the very fact that this is an LP problem, so we'll assume that while it is necessary to obey it, it is unnecessary to write it.

To find Beau a usable solution, our first responsibility is to see to it that any proposed solution is ***feasible***.

- ♦ A feasible solution is one that does not violate any constraints, including non negativity.

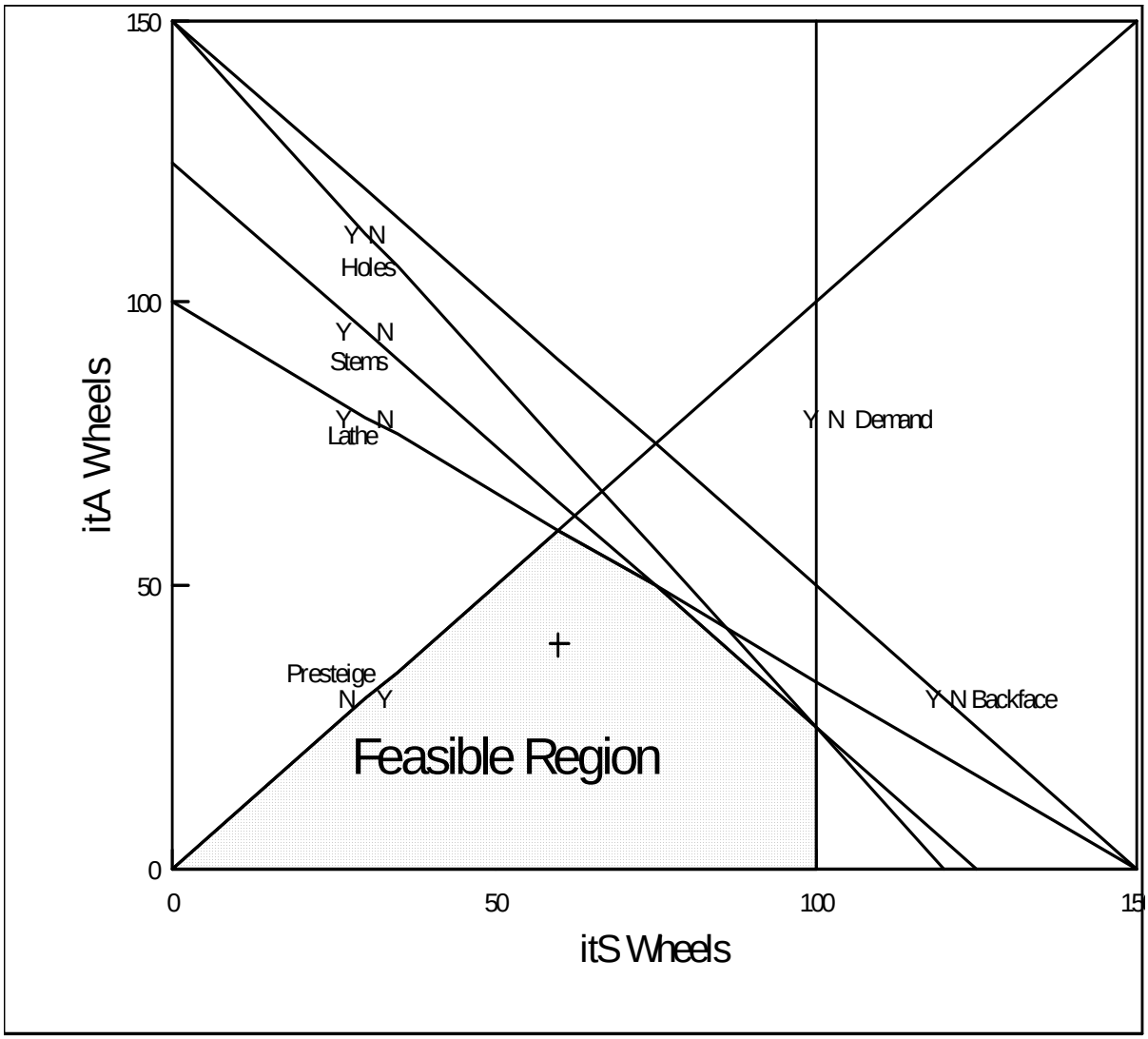
In this case, a feasible solution is a number of ITA and ITS wheels that

- ➢ includes either some or none of each product (Beau won't make wheels of antimatter!)
- ➢ includes no more than 100 ITS
- ➢ has at least as many ITS as ITA
- ➢ requires no more than 30 hours of milling machine time to backface the wheels
- ➢ requires the drill press operator to drill no more than 600 mounting holes in wheels
- ➢ needs no more than 30 lathe hours for truing and surface finishing wheels
- ➢ has Beau drilling valve stem holes in no more than a total of 125 wheels.

If you're sharp, you have already noticed that we are violating one of the assumptions of LP. Since the special stepped drill bit can drill 120 to 130 holes, but we split the middle and said 125, we are violating the ***certainty*** assumption. The possible risk here is that either at the end of the weekend he finds he could have made more wheels (with a different mix), or that he is stuck with up to 5 wheels that still need valve stem holes drilled. A more conservative approach would have used 120 on the RHS, but given up some profit potential. Later, we will see how to evaluate this. For now, let's just note that we sometimes knowingly violate this assumption, but should always examine the possible consequences.

FEASIBLE SOLUTIONS

In effect, our inequalities (including the implied $ITA \geq 0$ and $ITS \geq 0$) define a "space", which we call the ***feasible solution space***. We must select a point in that space for Beau, and to do that we'll have to know what points are members of that space. We're going to do that graphically. The graphical representation will help us to understand what LP does and how it works.



Beau Jarble's Constraints

I used **Y(es) and N(o)** to show which side of each constraint line was **infeasable.** The open space at the bottom (labeled Feasible Region) contains all combinations of ITA and ITS wheels Beau can make this weekend. Any point inside this space, or on a boundary, represents a combination that will not violate any constraint. Any point that is not in this region violates at least one constraint.

Beau's problem presents us with some offbeat situations that we sometimes see. First, we have a ***redundant constraint***. Looking at the graph, we can see that the valve stem drilling constraint ***dominates*** the backface constraint. The backface constraint lies entirely outside the feasible region that we would have if the special

valve stem drill were the only other constraint. In the olden days, when college students walked barefoot to class through raging blizzards, we suggested that people try to spot and eliminate such constraints before solving LP problems. Now that we have high speed electronic confusers to do the drudge work, we leave them in. If Beau's valve stem drill turned out to be the special titanium version that can drill 1000 holes, we can conceive of the possibility that backfacing might become a binding constraint.

The other offbeat situation that we have is that this turns out to be a ***degenerate*** problem. This says nothing about Beau's morals; that's a separate issue. Degeneracy can become a more complex issue when you have more than 2 activity variables, but it is a simple one here. In a 2 dimensional space, the intersection of 2 lines defines a point. But here, we have 3 constraint lines intersecting at ITA = 25, ITS = 100. This is one more line than is needed to define a point in 2 dimensions.

No big deal, you say. We **really** have that point nailed. Yes, but . . . At one time, the mathematicians could not prove that the Simplex Method (we'll see it later) would always find a solution to a degenerate problem. Even today, when someone designs a computer program to solve LP problems with the Simplex Method, he or she must take special precautions to deal with degeneracy.

Perhaps worse, we will see that an intersection on the boundary of the feasible region is a *candidate* to become the optimal solution. If this turns out to be the optimal solution (it will, though you have no way to know that yet without peeking), then that complicates something we do after the problem is solved. After we solve an LP problem, we usually have some interest in what is often called ***post-optimality analysis*** or ***sensitivity analysis***. Sensitivity analysis is just a formalized way of playing "what if?". It is often the most important reason for solving an LP problem, and sometimes is even the whole reason.

The most important (usually) kind of sensitivity analysis one might do on an LP solution is to ask "what would happen if I could change the RHS of this constraint?". Degeneracy greatly complicates answering this question. We must return to this issue after we have completed our solution.

Optimal solution

Somewhere in or on the edge of the feasible region is the very best combination of ITA and ITS wheels for Beau to make. Let's consider the point ITA = 40 and ITS = 60 (look for a little + near the top of the feasible region). Beau could make that combination, no sweat. He did just last weekend. Could it possibly be a candidate for *best* combination?

He could make 10 more ITA wheels, gaining $50 for each, without giving up a single ITS wheel. He could make 10 more ITS wheels, gaining $60 for each, without giving up a single ITA wheel. In fact, he could move to the point ITA = 50, ITS = 70 (10 more of *each*) and earn $1,100 more. If Beau were trying to minimize the cost of making wheels, he could make fewer of both kinds, thus reducing his total cost. This should lead us to the realization that whether you want to maximize or minimize,

- **at an interior point in the feasible space, you can always find another point that is better.**

For this reason, we can eliminate the entire inside of the feasible space and restrict our search to its boundaries. That means that somewhere along some constraint line (remembering that non-negativity constrains us) there must be the optimal point for which we are looking. That eliminates a whole lot of fruitless possibilities that we need not examine!

One way we might search is to set a target value for total contribution to overhead and profit and see what combinations of ITA and ITS might yield that contribution. That will generate a line that looks just like a constraint line, except that we'll make it a dashed line so we can tell it from a constraint line. If it is so far from the origin that it doesn't touch the feasible space, then we'll try a smaller number. If it is so close to the origin that it runs right through the middle of the feasible solution space, we'll try a larger number.

We call these lines ***isoprofit lines***. If we were minimizing cost, they would be ***isocost lines***. The generic term for isoprofit and isocost is ***isoquant***. (The prefix *iso* comes from a Greek word meaning equal or same.) If we are maximizing, as we are now, we want the highest total contribution isoprofit line that has *at least one* feasible point somewhere along it. If we are minimizing, then we would want the lowest total cost isocost line that has *at least one* feasible point. As we try different levels of profit or cost, we will see that the lines are all parallel to each other. That makes things easier for us.

We draw an isoprofit line very much like we do a constraint, except that we must invent the number on the right side of the equation. We find 2 intercepts, and join them with a straight line. For a first try, I usually start with the product of the objective function coefficients, because I know that will give me easy arithmetic in finding the intercepts. This time, I will start with a $50×$60 = $3000 isoprofit line.

$$50\ ITA + 60\ ITS = 3000$$
$$ITA = 0,\ ITS = 50$$
$$ITS = 0,\ ITA = 60$$

If you look at the graph (next page), this is obviously too low. Let's triple it and see what we get.
The process is the same; only the numbers change.

$$50\ ITA + 60\ ITS = 9000$$
$$ITA = 0,\ ITS = 150$$
$$ITS = 0,\ ITA = 180$$

This time I overshot and got no feasible points. Let's split the difference.

$$50\ ITA + 60\ ITS = 6000$$
$$ITA = 0,\ ITS = 100$$
$$ITS = 0,\ ITA = 120$$

That was a little too low, but it got me close enough that I could see that the highest isoprofit line had to pass right through the intersection at ITS=100, ITA = 25. I just moved my straightedge parallel to the $6000 isoprofit line and drew a partial final isoprofit line through that point. The total profit contribution at that point is

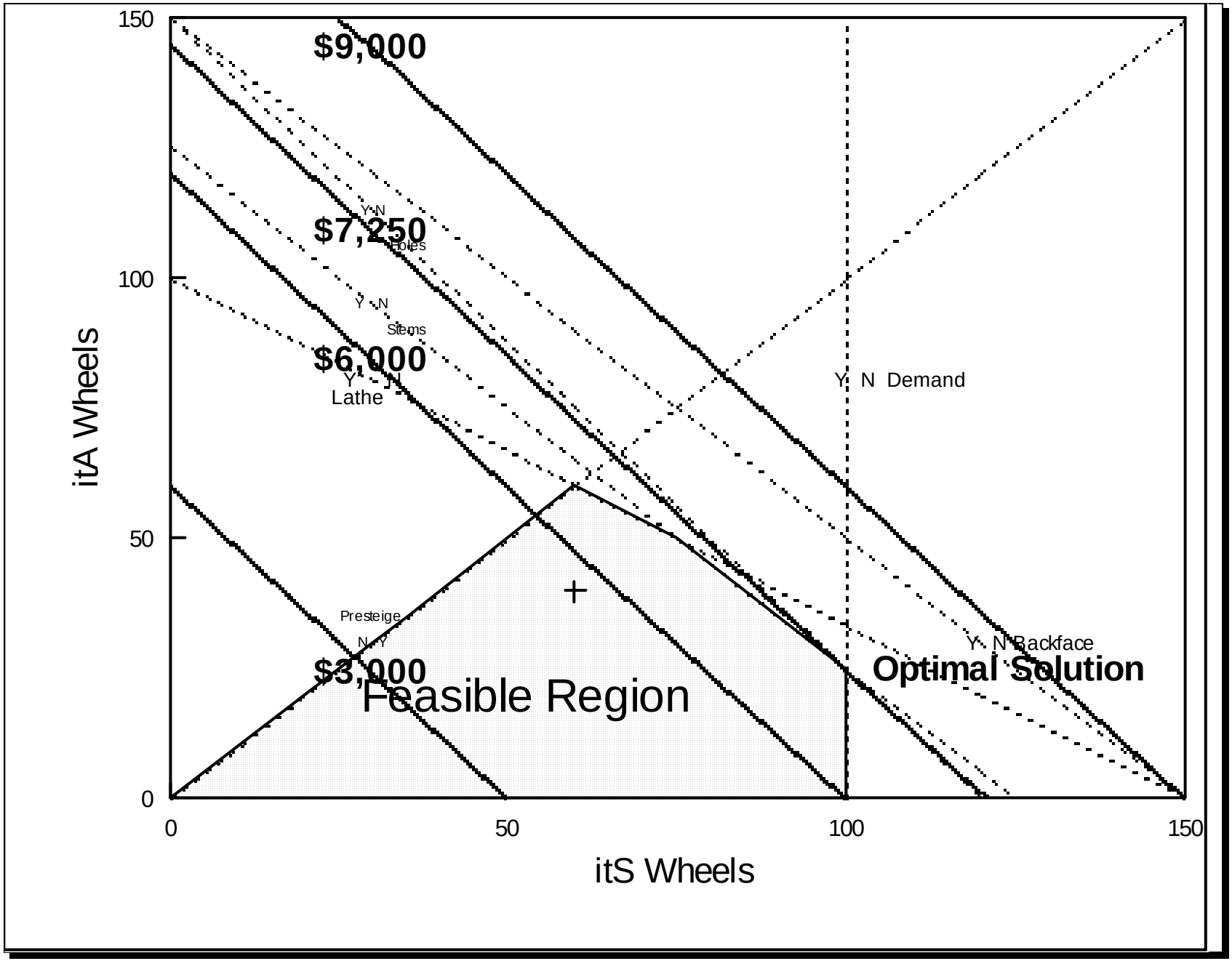
$$50\ ITA + 60\ ITS = profit$$
$$50 \times 25 + 60 \times 100 = 7250$$

We have found Beau's optimal solution. He should make 25 ITA wheels and 100 ITS wheels, for a total contribution of $7,250. Not bad for a weekend's work, but let's don't forget he has both overhead and a substantial investment. *It is very rare that we can legitimately call the final objective function value "profit".* Profit is an accounting term, and we disdain financial accounting principles for decision purposes.

You will note that we found Beau's best product mix at an intersection of constraints. We call this a ***corner point*** solution. The great breakthrough that made LP a practical problem solving tool instead of just an interesting mathematical structure was the proof of a theorem that permits us to focus on just the corner points. When you demystify the theorem, the essence of what it says is that

- **In a feasible linear programming problem, there is no solution superior to the best corner point solution.**

In Beau's feasible region, there are only 5 corner points. What this theorem says is that one of them *must* be the optimal solution. This is the cornerstone on which the Simplex method, which permits us to solve problems with many variables, is built.



Beau Jarble's Optimal Solution

Simplex Method

There are many versions of the Simplex Method, much as there are many dialects of the English language. They are all based on a more efficient search process than trial and error, thanks to the corner point theorem. In a typical version, it begins at the origin (all activity variables = 0). It then selects a constraint line to move along (remember now, an axis also acts as a ≥ 0 constraint), often by selecting the one with the most rapid rate of increase of the objective function. It moves out until another constraint prevents it from going any farther. This completes an "iteration".

It now looks for a feasible direction to move along another constraint line that will improve the objective function value. In larger problems, there may be many to choose from. It applies the same decision rule as before (e.g. most rapid increase of the objective). If two lines provide equal rates of improvement it will select between them at random. It follows this newly chosen constraint line until it again encounters an intersection. This completes another iteration. The Simplex Method continues to "iterate" until it encounters an intersection from which leaving along any constraint line worsens the value of the objective function. This intersection is always the optimal one.

In Beau's problem, such a typical Simplex Method would start at the origin, ITA = 0, ITS = 0. The objective function increases more rapidly along the ITS axis (in other words, the ITA ≥ 0 constraint line) than along the ITA axis (ITS ≥ 0 constraint line), so the next intersection considered will be ITA = 0, ITS = 100. This completes an iteration. From here, the most rapid rate of improvement is along the Demand constraint. The Simplex method will continue up the Demand constraint until it encounters another constraint. In this peculiar case, it encounters both the Hole Drilling and Stem Drilling constraints at ITA = 25, ITS = 100. Looking at the graph, you can see that any feasible direction away from this intersection carries the solution farther from the $7,250 isoprofit line. Since both available directions involve reducing the objective function, the Simplex Method identifies this as the optimal solution and stops.

This is as deep as we will probe into the inner workings of the Simplex Method. We will rely on Excel Solver to carry out the Simplex manipulations. Without going this far, we would lack even the foggiest idea of what was happening.

Degeneracy

A few pages back, we pointed out that ***degeneracy*** greatly complicates ***sensitivity analysis***, and we promised to return to that issue later. It is now later. *Sensitivity analysis* is simply the process of examining the shadow prices and the ranges over which they apply. It also includes objective function coefficient ranging. In the Widgets and Gadgets problem, which is ***not*** degenerate, the sensitivity analysis gave us no trouble beyond the fact that it was a new concept we had to wrestle with. Beau's problem, it greatly reduces the power of the sensitivity analysis.

The easiest way to spot degeneracy is to look for a constraint whose allowable increase or allowable decrease is zero. This can be seen in Beau's EXCEL printout for the constraints labeled Demand, Lathe, and Stem. Because of the problem's degeneracy, the shadow price is only good in one direction (the one whose allowable change is nonzero).

The point of all this? When we conduct sensitivity analysis, it is because we want or need to know the impact of a simple change in the problem situation. What is the effect of an increase or decrease in the available amount of one resource? What is the impact of a specific change in the value of one objective coefficient? In the absence of a degenerate solution, we can use the output to answer these questions as we did in the Widgets and Gadgets problem. *If the solution is degenerate, the only 100% reliable way to answer such questions is to change the quantity in which we are interested and rerun the problem.*

We have 3 questions that Beau might ask.

1. What would be the effect of more or less demand for ITS?
2. What would be the effect of more or less drill press time?
3. What would be the effect of long or short life of the special stem hole bit?

He could also ask about the other constraints, but they did not turn out to be ***binding constraints***, so we can confidently tell him that only large changes would have any effect.

Normally, a change in a right hand side value for a binding constraint has essentially the same effect, over a range, whether you decrease it or increase it. Because of degeneracy, that is not true here. With care and close attention, it is not too hard to pick out of a graph what will happen. When we reach the point of interpreting the results of solving with the Simplex Method, it is certainly far from a trivial problem. Since we do have a graph, let's see what happens when we change the right hand sides of Beau's active constraints.

Less Demand If demand for ITS wheels decreases, the demand constraint moves to the left, parallel to the current constraint. The optimal intersection would follow along the stem drilling constraint, with total contribution gradually decreasing, This is a normal response.	More Demand: The demand constraint moves to the right, parallel to itself. Since there is still an intersection at 100, 25 and the isoprofit line bisects it, the optimal solution does not change at all. The only effect is that demand is no longer a binding constraint. This results from the degeneracy.
Less Hole Drilling: The hole drilling constraint moves toward the origin, with the optimal intersection following down the demand constraint, with total contribution gradually decreasing. This is completely normal.	More Hole Drilling: If Beau had more hole drilling capacity available, the valve stem drilling constraint would prevent it from being used. The constraint would move parallel to itself away from the origin, but would no longer be binding. This comes from the degeneracy.
Less Stem Drilling: As with the mounting hole drilling constraint, the optimal solution would creep down the demand constraint, reducing the total contribution. We expect this effect from having less of an actively constraining resource.	More Stem Drilling: In this degenerate situation, the hole drilling constraint would prevent Beau from being able to take advantage of a longer lasting stepped bit for valve stems. As this constraint moves farther from the origin, parallel to itself, the hole drilling constraint prevents the optimal solution from adapting.

Solving Beau Jarble's Problem by Computer

On the next 2 pages you will find the EXCEL results and a summary similar to those for Widgets and Gadgets for this problem, giving (no surprise) the same solution we found graphically. In practice, the graphical method is only useful for introducing the basic concepts of linear programming; when we want to analyze a real problem, or even any but the simplest textbook problems, we use a computer, probably running the simplex algorithm or one of its variants.

Excel Output for Beau Jarble

ItA	ItS			
25	100			
50	60	7,250	Contribution	
0	1	100	Demand	100
0.2	0.2	25	Backface	30
4	5	600	Holes	600
0.3	0.2	27.5	Lathe	30
1	1	125	Stems	125
-1	1	75	Prestige	0
1	0	25	NNItA	0
0	1	100	NNItS	0

Microsoft Excel 5.0 Answer Report						
Target Cell (Max)						
	Cell	Name	Original	Final Value		
	$C$3	Contributio	0	7,250		
Adjustable Cells						
	Cell	Name	Original	Final Value		
	$A$2	ItA	0	25		
	$B$2	ItS	0	100		
Constraints						
	Cell	Name	Cell Value	Formula	Status	Slack
	$C$5	Demand	100	$C$5<=$E$5	Binding	0
	$C$6	Backface	25	$C$6<=$E$6	Not Binding	5
	$C$7	Holes	600	$C$7<=$E$7	Binding	0
	$C$8	Lathe	27.5	$C$8<=$E$8	Not Binding	2.5
	$C$9	Stems	125	$C$9<=$E$9	Binding	0
	$C$10	Prestige	75	$C$10>=$E$10	Not Binding	75
	$C$11	NNItA	25	$C$11>=$E$11	Not Binding	25
	$C$12	NNItS	100	$C$12>=$E$12	Not Binding	100

Microsoft Excel 5.0 Sensitivity Report							
Changing Cells							
			Final	Reduced	Objective	Allowable	
	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
	$A$2	ItA	25	0	50	10	2
	$B$2	ItS	100	0	60	2.5	10
Constraints							
			Final	Shadow	Constraint	Allowable	
	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
	$C$5	Demand	100	0	100	1E+30	**0**
	$C$6	Backface	25	0	30	1E+30	5
	$C$7	Holes	600	10	600	**0**	25
	$C$8	Lathe	27.5	0	30	1E+30	2.5
	$C$9	Stems	125	10	125	3.57	**0**
	$C$10	Prestige	75	0	0	75	1E+30
	$C$11	NNItA	25	0	0	25	1E+30
	$C$12	NNItS	100	0	0	100	1E+30

Summary of Output for Beau Jarble						Current RHS	Slack or Surplus	Dual Price	Allowablwe	
	Max	50 itA	+	60itS					Increase	Decrease
Demand	Su			itS	≤	100	0	0	Infinity	0
Backface	bj	.2 itA	+	.2 itS	≤	30	5	0	Infinity	5
Holes	ect	4 itA	+	5 itS	≤	600	0	10	0	25
Lathe		.3 itA	+	.2 itS	≤	30	2.5	0	Infinity	2.5
Stems	to	itA	+	itS	≤	125	0	10	3.57	0
Prestige		- itA	+	itS	≥	0	75	0	75	Infinity

Value	25		100
Reduced Cost	0		0
Current Coeff.	50		60
Allowable Increase	10		2.5
Allowable Decrease	2		10