Resource Allocation with Several Variables

Case 3: Hardcase Office Equipment

The Plant Manager for Hardcase Office Equipment Company needs to develop a short term production plan for next week. To develop this plan, she must know the needed materials and labor for each product. She must also know how much of each material, and how much labor, will be available. Since they always try to meet all firm orders, she must know what has been ordered. Finally, if LP is to be used, the contribution margin for each product must be developed. These facts are:

Product	Wood (board feet)	Plastic (sq. ft.)	Steel (lb.)	Assembly hours	Firm orders	Contrib. margin
Desks	0	6	9	2.5	75	$150
Chairs	3	1	1	1.2	120	$45
Tables	5	2	2	2.2	100	$100
File Cabinets	0	0	15	1.9	50	$40
Availability	2000	2400	2000	1000		

The Good Fairy did not bring her the contribution margins. She based them on the selling prices of the products, anticipated replacement costs of the materials (including some not mentioned here), and the cost of labor including employment taxes and benefits.

Another restriction that must be obeyed relates to the file cabinets, which are almost a commodity item. With little opportunity for product differentiation, they cannot be very profitably priced, and are offered only so that customers will see Hardcase as offering a complete product line. As a result of this, she never allows a production run to include more than 10% file cabinets.

THE FORMULATION

The basis for decision making is the number of each product to make and sell next week. There is nothing variable about the resources, so we should not use them as activity variables. Since the resource usage per unit is a fixed amount (unlike the typical blending problem), product - resource combinations (e.g., board feet of wood used in making desks) make no sense as activity variables. Thus our activity variables will be the number of DESKs, CHAIRs, TABLEs, and filing cabinets (CABNT) to schedule for next week's production.

The fact that she has provided contribution margins implies that we are going to try to maximize the contribution to profit. She is not in a situation where cost minimization makes sense. You would try to minimize costs where some minimum level of performance is required with revenues absent or unrelated to the decision being made. The ore-ordered quantities of the four products could decieve you into thinking this is the case here, but there's lots more money to be made selling on the open market if you maximize instead of minimizing! Thus, the objective function must be:

MAXimize Z = 150 DESK + 45 CHAIR + 100 TABLE + 40 CABNT

Now that we have defined her objective, we must consider the constraints within which she must operate. The most obvious constraints relate to resources. She should not plan to depend on imaginary resources. This implies our first four constraints:

	3 CHAIR	+5 TABLE		≤ 2000	board feet of genuine **tree wood**
6 DESK	+1 CHAIR	+2 TABLE		≤ 2400	sq. ft. of finest virgin **plastic**
9 DESK	+1 CHAIR	+2 TABLE	+15 CABNT	≤ 2000	lb. of cold rolled **steel**
2.5 DESK	+1.2 CHAIR	+2.2 TABLE	+1.9 CABNT	≤ 1000	hours of willing & **skilled labor**

A more troublesome constraint arises from the 10% restriction on file cabinets. This restriction says that the number of file cabinets produced must not exceed 10% of total production. This is a somewhat more complicated proportionality constraint than Beau Jarble had. If you forget about dimensional analysis, it is far too easy to blithely write

$$.10\ \text{CABNT} \le \text{DESK} + \text{CHAIR} + \text{TABLE} + \text{CABNT}$$ ☹ 💣

or even $$\text{CABNT} \le .1$$

both of which have been tried by students in the past to represent this restriction. Even without explicitly conducting dimensional analysis, the careful analyst can avoid this temptation by remembering that each variable represents a quantity of furniture units to produce and that the 10% represents a proportion of the total number of units to produce.

A little thought leads to the idea that we can represent this restriction with the inequality

$$\text{CABNT} \le .10\ (\text{DESK} + \text{CHAIR} + \text{TABLE} + \text{CABNT})$$

This works fine in modern programs such as Excel, but only because they convert it into standard form before solving the problem. Understanding the output, especially the sensitivity analysis, is facilitated by an understanding of how this is done. All it takes is a little light duty algebra. We begin by subtracting the RHS from both sides, and take it from there.

CABNT	$\le$	.10 (DESK + CHAIR + TABLE + CABNT)
- .10 (DESK + CHAIR + TABLE + CABNT)		- .10 (DESK + CHAIR + TABLE + CABNT)
or CABNT - .10 (DESK + CHAIR + TABLE + CABNT)	$\le$	0
or CABNT -.10 DESK -.10 CHAIR -.10 TABLE -.10 CABNT	$\le$	0
or -.10 DESK -.10 CHAIR -.10 TABLE + .90 CABNT	$\le$	0

This final form is not so terribly visually appealing, but it <u>is</u> in the "standard" form for a linear programming constraint and <u>will</u> enforce the intended restriction.

Notice that we did **not** try to express the ratio of Cabinets to the sum of all four products and constrain that ratio to be less than or equal to .10. That would have blown the whole thing, because the resulting inequality would have been a non - linear form: $\frac{\text{CABNT}}{\text{DESK+CHAIR+TABLE+CABNT}} \le .10$

The only restrictions we have not dealt with come from the requirement that the Plant Manager must plan to meet all firm orders. That provides us with the four easiest constraints to write:

```
DESK  >= 75
CHAIR >= 120
TABLE >= 100
CABNT >= 50
```

Before we go to the computer, we should write out the whole problem in one place

```
MAX  Z = 150 DESK + 45 CHAIR + 100 TABLE + 40 CABNT  Our objective
     Subject to:
     -.10 DESK - .10 CHAIR - .10 TABLE + .90 CABNT <= 0     10% Cabinets
                  3 CHAIR +   5 TABLE              <= 2000  wood
        6 DESK +   1 CHAIR +   2 TABLE              <= 2400  plastic
        9 DESK +   1 CHAIR +   2 TABLE +  15 CABNT <= 2000  steel
      2.5 DESK + 1.2 CHAIR + 2.2 TABLE + 1.9 CABNT <= 1000  labor hours
          DESK                                     >= 75    orders desks
                   CHAIR                           >= 120   orders chairs
                               TABLE               >= 100   orders tables
                                           CABNT   >= 50    orders cabinets
```

Now we won't fumble around with a dozen scraps of paper when we sit down at the computer. By the way. If you hadn't guessed, coefficients of one are implicit on the left hand side of the last four constraints.

Formulation and Solution to Hardcase (LINDO Format)

```
MAX     150 DESK + 45 CHAIR + 100 TABLE + 40 CABNT
SUBJECT TO
       2) - .1 DESK - .1 CHAIR - .1 TABLE + .9 CABNT <= 0
       3)  3 CHAIR + 5 TABLE <=   2000
       4)  6 DESK + CHAIR + 2 TABLE <=   2400
       5)  9 DESK + CHAIR + 2 TABLE + 15 CABNT <= 2000
       6)  2.5 DESK + 1.2 CHAIR + 2.2 TABLE + 1.9 CABNT <= 1000
       7)   DESK >=   75
       8)   CHAIR >= 120
       9)   TABLE >= 100
      10)   CABNT >=  50

         OBJECTIVE FUNCTION VALUE   1)      41125.0000

 VARIABLE        VALUE          REDUCED COST
     DESK        75.000000           .000000
    CHAIR       175.000000           .000000
    TABLE       200.000000           .000000
    CABNT        50.000000           .000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000       100.000000
       3)       475.000000          .000000
       4)      1375.000000          .000000
       5)          .000000        55.000000
       6)        67.499980          .000000
       7)          .000000      -335.000000
       8)        55.000000          .000000
       9)       100.000000          .000000
      10)          .000000      -875.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:
                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
    DESK        150.000000      335.000000        INFINITY
   CHAIR         45.000000        5.000000        INFINITY
   TABLE        100.000000      INFINITY          10.000000
   CABNT         40.000000      875.000000        INFINITY

                          RIGHTHAND SIDE RANGES
    ROW          CURRENT        ALLOWABLE        ALLOWABLE
                   RHS          INCREASE         DECREASE
       2           .000000        2.750000        10.000000
       3       2000.000000      INFINITY         475.000000
       4       2400.000000      INFINITY        1375.000000
       5       2000.000000       55.000000       100.000000
       6       1000.000000      INFINITY          67.499980
       7         75.000000       12.500000         7.857142
       8        120.000000       55.000000        INFINITY
       9        100.000000      100.000000        INFINITY
      10         50.000000        4.166667         1.666667
```

Interpreting the Solution

```
OBJECTIVE FUNCTION VALUE   1) 41125.0000
```

The total contribution to profit and overhead is $41,125

VARIABLE	VALUE	REDUCED COST	
DESK	75.000000	.000000	***Make seventy-five desks next week,***
CHAIR	175.000000	.000000	***One hundred seventy-five chairs***
TABLE	200.000000	.000000	***Two hundred Tables***
CABNT	50.000000	.000000	***And fifty cabinets***

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	100.000000

If the required fifty file cabinets could be more than 10% of the "things" produced, we could rearrange the production plan to make fewer "things" but more profit. [1]

3)	475.000000	.000000

We'll have 475 board feet of wood left over for the week after next

4)	1375.000000	.000000

We'll have 1,375 square feet of plastic left over.

5)	.000000	55.000000

Steel is a seriously constraining resource; the marginal value of a pound of steel for next week's production is $55, <u>over and above the cost of steel allowed for in the objective function</u>. As always, tightening a constraint (less steel available) hurts the objective function. Loosening the constraint (more than 2000 pounds available) helps the objective function.

6)	67.499980	.000000

Sixty-seven and a half hour of labor can be reassigned

7)	.000000	-335.000000

Desks take resources from more profitable items; if any of those firm orders are canceled, she could rearrange production so that profits go up by $335 for each canceled desk (but only down to a point that we'll discover later). Discovering that <u>more</u> orders exist would have the reverse effect ... profit goes <u>down</u> by $335 for each extra desk ordered. Another way to look at this is that Hardcase has set the price of desks $335 too low. If the contribution margin were $335 or more higher than it is now, the resource tradeoffs with the other products would favor making more, not fewer, desks.

8)	55.000000	.000000

We're making fifty-five more chairs for the open market, over and above the seventy-five that we have firm orders for.

9)	100.000000	.000000

We're making 100 tables for the open market, over and above the one hundred that we have firm orders for.

10)	.000000	-875.000000

File cabinets, like desks, suck up resources that could be used more profitably elsewhere. Every cabinet we can weasel out of making frees up enough steel to shift to a production schedule worth $875 more than this one. (Some of this gain also comes from that pesky 10% requirement; working out the precise details would give you a headache so we'll take the computer's word for it just this once.)

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[1] This type of constraint can yield shadow prices that are very tricky to interpret; as you see, it can be done, but students in beginning courses are not usually asked to do so.

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
DESK	150.000000	335.000000	INFINITY

We must make 75 desks regardless of profitability. If the price went up by $335.01, we would want to make desks

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
CHAIR	45.000000	5.000000	INFINITY

If the contribution from chairs went up by $5.01, we'd make more of them.

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
TABLE	100.000000	INFINITY	10.000000

It looks like tables give good resource tradeoffs down to a contribution of $90. We are probably making all of them we can. The "no upper limit" just means that we'd love to make more money off them, but we can't make any more of them. If they gained us $10.01 less than they do, we'd cut back.

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
CABNT	40.000000	875.000000	INFINITY

We gotta make file cabinets. Boy, do we wish we didn't. No way are we going to get the price high enough to want to make them.

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	2.750000	10.000000

This was the "10%" constraint. Interpreting it is tortuous at best. We usually don't make the attempt for a proportionality constraint, since the intended restriction is a proportion, but the range information is a quantity.

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
3	2000.000000	INFINITY	475.000000

Four hundred seventy-five board feet of wood we could use for something else

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
4	2400.000000	INFINITY	1375.000000

1,375 square feet of spare plastic available for other uses

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
5	2000.000000	55.000000	100.000000

The $55/lb marginal value for steel applies from 1900 to 2055 pounds of steel available.

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
6	1000.000000	INFINITY	67.499980

Sixty-seven and a half labor hours are available for special projects without affecting contribution.

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
7	75.000000	12.500000	7.857142

Total contribution goes down by $335 per desk for increases in pre-ordered desk up to a total of 87 desks; total contribution goes up by $335 per desk for dscreases in pre-ordered desk down to a total of 68 desks.

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
8	120.000000	55.000000	INFINITY

Total contribution remains the same as long as no more than 175 chairs are pre-ordered .

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
9	100.000000	100.000000	INFINITY

Total contribution remains the same as long as no more than 200 tables are pre-ordered .

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
10	50.000000	4.166667	1.6666

Hardcase is between a rock and a hard place on file cabinets. If orders go down to 48 or up to 55, they won't have any feasible production schedule a-tall! In between, the $875 shadow price applies

A Report to the Plant Manager

Boss, our best production schedule for next week calls for 75 desks, 175 chairs, 200 tables, and 50 file cabinets. Somebody should let Purchasing know that we'll have 475 board feet of wood and 1375 square feet of plastic left over. We'll use every last scrap of our steel supply; if they could rush us some more (I know we could use another 55 pounds), I'll give you an even better schedule. I've scheduled the assembly force at 93% utilization (67.5 idle worker hours). That should be acceptable both to you and to the union.

The 10% restriction on file cabinets is a real pain. If we could talk Mr. Hardcase into lifting it, and marketing into kinda forgetting that we make those, it would really help. Let's do lunch later this week and discuss it.

Melvin

The Excel printout for Hardcase looks like this:

	Desks	Chairs	Tables	Cabinets			
Quantity	75	175	200	50	LHS	RHS	
Contribution	$150	$45	$100	$40	$41,125		
Wood	0	3	5	0	1525	2000 board feet	
Plastic	6	1	2	0	1025	2400 sq. ft.	
Steel	9	1	2	15	2000	2000 lb.	
Assembly	2.5	1.2	2.2	1.9	932.5	1000 hours	
Firm Order Desks	1				75	75 Desks	
Firm Order Chairs		1			175	120 Chairs	
Firm Order Tables			1		200	100 Tables	
Firm Order Cabinets				1	50	50 File Cabinets	
max cabinets				1	50	50 Desks <=	10%

Microsoft Excel 11.0 Sensitivity Report
Worksheet: [Hardcase.xls]No supplementaries
Report Created: 12/22/2005 3:38:46 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Quantity Desks	75	$ -	$ 150.00	$ 335.00	1E+30
$C$2	Quantity Chairs	175	$ -	$ 45.00	$ 5.00	1E+30
$D$2	Quantity Tables	200	$ -	$ 100.00	1E+30	$ 10.00
$E$2	Quantity Cabinets	50	$ -	$ 40.00	$ 875.00	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$4	Wood LHS	1525	$ -	2000	1E+30	475
$F$5	Plastic LHS	1025	$ -	2400	1E+30	1375
$F$6	Steel LHS	2000	$ 55.00	2000	55	100
$F$7	Assembly LHS	932.5	$ -	1000	1E+30	67.5
$F$8	Firm Order Desks LHS	75	$(335.00)	75	12.5	7.857
$F$9	Firm Order Chairs LHS	175	$ -	120	55	1E+30
$F$10	Firm Order Tables LHS	200	$ -	100	100	1E+30
$F$11	Firm Order Cabinets LHS	50	$(875.00)	50	4.167	1.667
$F$12	max cabinets LHS	50	$ 100.00	0	2.75	10

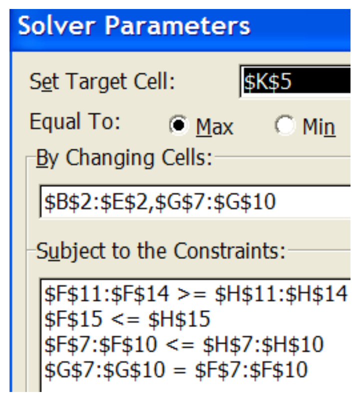
Cost Accounting at Hardcase Office Equipment

Suppose Hardcase's current cost estimates are $1.00 per board foot of wood, $1.00 per square foot of steel, $0.40 per pound of plastic, and $12.00 per hour of labor, along with miscellaneous expenses of $110.00 per desk, $26.20 per chair, $65.80 per table, and $1.20 per cabinet. The selling price our customers pay us is $300 per desk, $90 per chair, $200 per table, and $50 per cabinet. We need to know how to respond if market forces cause any one of these to change.

The sensitivity analysis we've seen so far can account nicely for a change in any one of the constraints. but if the cost of wood, plastic, steel, or assembly labor were to change this would change the objective coefficient of several variables, which would invalidate the sensitivity analysis, which is only valid if one and only one parameter changes.

If we want to see what happens if there is a change in the per-unit costs of one of the inputs to the manufacturing process, we need to move the cost line items into the objective coefficients section of the sensitivity report. To do this, we hoodwink the Simplex method used by Solver into thinking that the amount of each resource consumed is a separate decision, but add constraints that actually force these decision variables to be equal to the actual resource consumption based on the real decision variables (the quantity of each of the four products produced).

The Solver setup at right, along with the formulas view of the spreadsheet on the next page, show how this trick is pulled off. The target cell K5 is equal to total revenue (K3) minus total variable cost (K4). The "Changing Cells" are in two blocks, separated by a comma. The four cells in the block $B$2:$E$2 are the real decision variables, specifying how much of each product to make. The four cells in the block $G$7:$G$10 are required by the last constraint to be equal to the consumption of wood, plastic, steel, and assembly labor determined by the quantities of each product made.



The first three blocks of constraints specify the firm orders, the limitation on cabinets, and the limited availability of wood, plastic, steel, and assembly labor.

The key to the magic trick is in cell K4, total variable cost. This amount is calculated using the supplementary variables in column G (via the formulas in cells K7 through K110, NOT the calculated amounts in column F. While these are numerically the same, if you don't use the supplementary variables to calculate the cost, the sensitivity analysis will give them incorrect objective coefficients of zero.

Because the variables in the block $G$7:$G$10 are not really needed to guide the production decision with the initial cost estimates, but they are needed to move these estimates into the sensitivity analysis, we call them **supplementary variables**.

	A	B	C	D	E	F	G	H	I	J	K
1		Desks	Chairs	Tables	Cabinets						
2	Quantity	75	175.000000000002	199.999999999998	50.0000000000001	LHS		RHS			
3	Unit Price	300	90	200	70				Total Revenue		=SUMPRODUCT($B$2:$E$2,B3:E3)
4	Unit Cost	=SUMPRODUCT(B7:B10,$J$7:$J$10)+B16	=SUMPRODUCT(C7:C10,$J$7:$J$10)+C16	=SUMPRODUCT(D7:D10,$J$7:$J$10)+D16	=SUMPRODUCT(E7:E10,$J$7:$J$10)+E16				Total Variable Cost		=SUM(K7:K16)
5									Contribution		=K3-K4
6	Contribution	=B3-B4	=C3-C4	=D3-D4	=E3-E4	=SUMPRODUCT($B$2:$E$2,B6:E6)	Supplementary			Unit Cost	Total cost
7	Wood	0	3	5	0	=SUMPRODUCT($B$2:$E$2,B7:E7)	1525	2000	board feet	1	=G7*J7
8	Plastic	6	1	2	0	=SUMPRODUCT($B$2:$E$2,B8:E8)	1025	2400	sq. ft.	1	=G8*J8
9	Steel	9	1	2	15	=SUMPRODUCT($B$2:$E$2,B9:E9)	2000	2000	lb.	0.4	=G9*J9
10	Assembly	2.5	1.2	2.2	1.9	=SUMPRODUCT($B$2:$E$2,B10:E10)	932.50	1000	hours	12	=G10*J10
11	Firm Order Desks	1				=SUMPRODUCT($B$2:$E$2,B11:E11)		75	Desks		
12	Firm Order Vhairs		1			=SUMPRODUCT($B$2:$E$2,B12:E12)		120	Chairs		
13	Firm Order Tables			1		=SUMPRODUCT($B$2:$E$2,B13:E13)		100	Tables		
14	Firm Order Cabinets				1	=SUMPRODUCT($B$2:$E$2, B14:E14)		50	File Cabinets		
15	max cabinets				1	=SUMPRODUCT($B$2:$E$2,B15:E15)		=SUM(B2:E2)*J15	Desks <=	0.1	
16	Misc. Expens.	110.4	26.2	65.8	1.2						=SUMPRODUCT(B2:E2,B16:E16)

Here is the sensitivity analysis for Hardcase using supplementary variables:

Microsoft Excel 10.0 Sensitivity Report
Worksheet: [Hardcase-4.xls]Supplementaries
Report Created: 2/1/2008 2:22:30 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Quantity Desks	75	$ -	$ 189.60	$ 335.00	1E+30
$C$2	Quantity Chairs	175	$ -	$ 63.80	$ 5.00	1E+30
$D$2	Quantity Tables	200	$ -	$ 134.20	1E+30	$ 10.00
$E$2	Quantity Cabinets	50	$ -	$ 68.80	$ 875.00	1E+30
$G$7	Wood Supplementary	1525	$ -	$ (1.00)	$ 10.00	$ 17.63
$G$8	Plastic Supplementary	1025	$ -	$ (1.00)	1E+30	$ 55.00
$G$9	Steel Supplementary	2000	$ -	$ (0.40)	1E+30	$ 55.00
$G$10	Assembly Supplementary	932.5	$ -	$ (12.00)	$ 50.00	$ 50.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$7	Wood LHS	1525	$ -	2000	1E+30	475
$F$8	Plastic LHS	1025	$ -	2400	1E+30	1375
$F$9	Steel LHS	2000	$ 55.00	2000	55	100
$F$10	Assembly LHS	932.5	$ -	1000	1E+30	67.5
$F$11	Firm Order Desks LHS	75	$(335.00)	75	12.5	7.86
$F$12	Firm Order Chairs LHS	175	$ -	120	55	1E+30
$F$13	Firm Order Tables LHS	200	$ -	100	100	1E+30
$F$14	Firm Order Cabinets LHS	50	$(875.00)	50	4.17	1.67
$F$15	max cabinets LHS	50	$ 100.00	0	2.75	10
$G$7	Wood Supplementary	1525	$ (1.00)	0	1E+30	1525
$G$8	Plastic Supplementary	1025	$ (1.00)	0	4E+17	1025
$G$9	Steel Supplementary	2000	$ (0.40)	0	1E+30	2000
$G$10	Assembly Supplementary	932.5	$ (12.00)	0	8E+16	932.5

The objective coefficients for the four products are now equal to their selling price minus their unique miscellaneous costs (verify this). They can be merged like this because neither a change in the selling price of a product, nor a change in its unique miscellaneous costs, affects any of the other products directly. If the selling price of a desk increases by an amount less than $335 or the miscellaneous costs for a desk decrease by an amount less than $335, or some combination adding up to less than $335, and nothing else changes, then we can keep the same production schedule and just happily pocket the extra money. If the combined beneficial changes to selling price and miscellaneous costs come to more than $335 per desk, then we should rerun the model to find a better production schedule than the current one to take advantage of this good fortune. On the other hand, if luck turns against us and we have to lower the price of a desk. or our miscellaneous expenses for a desk go up, we have to just grin and bear it; we can't soften the blow by changing the production schedule. This is the same result as for the unit contribution of desks in the original version.

The interpretation of the sensitivity analysis for chairs and cabinets is similar in principle to that for desks.

If the selling price increases and/or the miscellaneous costs for tables decrease by any amount, we keep the same production schedule and happily pocket the extra money. If, unluckily, we have to sell tables more cheaply or we have to incur more miscellaneous costs per table, we can reduce the negative consequences with a different production schedule if the change is more than $10; if it's less than that, we keep the same production schedule and just suffer the consequences.

The objective coefficients for wood, plastic, steel, and assembly labor are negative because they are costs, not revenues. It is important to keep in mind that "increase" and "decrease" in the sensitivity analysis are defined in terms of pure mathematics, not in terms of ordinary business English. Mathematically, an "increase" in a negative number moves it closer to zero, which in business English we call a "decrease" in a cost. A mathematical "decrease" in a negative number moves it farther down away from zero, which in business English we call an "increase" in cost.

The fact that the numbers in the "Allowable Increase" column for wood, plastic, steel, and assembly labor are larger in magnitude than the unit costs means, in practical terms, that if one of these things becomes cheaper for us, we happily pocket the extra money without changing the production schedule. Mathematically, we would not even change if somebody paid US to consume these resources, but that's a mathematical fairy tale. The large numbers in the allowable decrease column mean that any likely increase in the cost of one input would leave us sad, but still committed to the same production schedule. Only if we had to pay more than $18.68 per board foot of wood, or (remember. "or," not "and") we had to pay more than $56 per square foot of steel, or more than $55.40 per pound of plastic, or more than $62 per hour of assembly labor, would we change the production schedule to soften the blow.

The extra constraints in the Constraints section have RHS of zero, so we will ignore their shadow prices.

Funds Allocation

Case 4: Neighborly Loan:
Jacob Marley, recently promoted to manager of the newest branch of the Neighborly Loan Company, is eager to please his new boss by loaning his annual $15 million budget profitably. Each local branch office at Neighborly generates profit by the interest income from three types of loans: first Mortgage loans on real estate at 7% annual interest, Automobile loans collateralized by liens on automobiles at 12% annual interest, and Signature loans with no collateral at 15% annual interest. Having the highest risk, Signature loans carry the highest loan interest rate. Neighborly's home office has set loan limits to guide branch managers and to protect the company from excessive amounts of high-risk loans. Neighborly requires each branch manager to place at least 60% of its loans into First Mortgages and no more than 10% of its loans in risky Signature loans.

If you make the mistake of expressing the mortgage and signature constraints as percentage of budget rather than percentage of amount loaned, you get the right solution but an incorrect sensitivity analysis The sensitivity analysis gives a dual price of 12 cents per dollar change in the amount budgeted, but actually changing the amount yields only an extra 9.3 cents on the dollar!.

The sensitivity analysis says each extra dollar of budget increases total interest earned by 12 cents, but in fact each extra dollar in the budget only increases total interest by 9.3 cents!

The reason is that changing the amount budgeted changes the maximum dollar amount for signature loans and the minimum dollar amount for first mortgages, which violates the "change just one constraint" assumption of the sensitivity analysis.

If you express the constraints as percentages of the amount loaned, you get the correct solution and also the correct sensitivity analysis with the correct dual price of 9.3 cents per dollar change in amount budgeted.

Note that the sensitivity report for the correct formulation shows the RHS of the mortgage and signature loans equal to zero. The constraints on signature loans and first mortgages are examples of a very important class of "proportionality constraints" in which a decision variable is constrained to be at least or at most some proportion of a total of which the variable itself is a part. These will play a big role in "blending problems." In fact, Neighborly Loan can be viewed as a "blending problem" in which financial assets are blended into a portfolio.

First Formulation: Numeric View

	A	B	C	D	E	F	G	H
1		**Mortgage**	**Auto**	**Siganture**				
2	**Amount Loaned**	**$9,000,000.00**	**$4,500,000.00**	**$1,500,000.00**	**LHS**		**RHS**	
3	**Interest rate**	**0.07**	**0.12**	**0.15**	**$ 1,395,000.00**			
4	**Total Budget**	**1**	**1**	**1**	**$15,000,000.00**	**≤**	**$15,000,000.00**	
5	**Mortgage minumum**	**1**			**$ 9,000,000.00**	**≥**	**$ 9,000,000.00**	**60%**
6	**Signature maximum**			**1**	**$ 1,500,000.00**	**≤**	**$ 1,500,000.00**	**10%**

First Formulation: Formulas View

	A	B	C	D	E	F	G	H
1		**Mortgage**	**Auto**	**Siganture**				
2	**Amount Loaned**	**9000000**	**4500000**	**1500000**	**LHS**		**RHS**	
3	**Interest rate**	**0.07**	**0.12**	**0.15**	**=SUMPRODUCT($B$2:$D$2,B3:D3)**			
4	**Total Budget**	**1**	**1**	**1**	**=SUMPRODUCT($B$2:$D$2,B4:D4)**	**≤**	**15000000**	
5	**Mortgage minumum**	**1**			**=SUMPRODUCT($B$2:$D$2,B5:D5)**	**≥**	**=$G$4*H5**	**0.6**
6	**Signature maximum**			**1**	**=SUMPRODUCT($B$2:$D$2,B6:D6)**	**≤**	**=$G$4*H6**	**0.1**

First Formulation: Sensitivity Analysis

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Amount Loaned Mortgage	$ 9,000,000.00	$ -	0.07	0.05	1E+30
$C$2	Amount Loaned Auto	$ 4,500,000.00	$ -	0.12	0.03	0.05
$D$2	Amount Loaned Siganture	$ 1,500,000.00	$ -	0.15	1E+30	0.03

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$4	Total Budget LHS	$ 15,000,000.00	**$ 0.12**	15000000	1E+30	4500000
$E$5	Mortgage minumum LHS	$ 9,000,000.00	$ (0.05)	9000000	4500000	9000000
$E$6	Signature maximum LHS	$ 1,500,000.00	$ 0.03	1500000	4500000	1500000

First Formulation: Effect of One Additional Dollar

	A	B	C	D	E	F	G	H
1		**Mortgage**	**Auto**	**Siganture**				
2	**Amount Loaned**	**$9,000,000.60**	**$4,500,000.30**	**$1,500,000.10**	**LHS**		**RHS**	
3	**Interest rate**	**0.07**	**0.12**	**0.15**	**$1,395,000.093**			
4	**Total Budget**	**1**	**1**	**1**	**$15,000,001.00**	**≤**	**$15,000,001.00**	
5	**Mortgage minumum**	**1**			**$ 9,000,000.60**	**≥**	**$ 9,000,000.60**	**60%**
6	**Signature maximum**			**1**	**$ 1,500,000.10**	**≤**	**$ 1,500,000.10**	**10%**

Second Formulation: Formulas View

	A	B	C	D	E	F	G	H
1		Mortgage	Auto	Siganture				
2	Amount Loaned	9000000	4500000	1500000	LHS		RHS	
3	Interest rate	0.07	0.12	0.15	=SUMPRODUCT($B$2:$D$2,B3:D3)			
4	Total Budget	1	1	1	=SUMPRODUCT($B$2:$D$2,B4:D4)	≤	15000000	
5	Mortgage minumum	1			=SUMPRODUCT($B$2:$D$2,B5:D5)	≥	=$E$4*H5	0.6
6	Signature maximum			1	=SUMPRODUCT($B$2:$D$2,B6:D6)	≤	=$E$4*H6	0.1

Second Formulation: Sensitivity Analysis

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Amount Loaned Mortgage	$ 9,000,000.00	$ -	0.07	0.05	0.155
$C$2	Amount Loaned Auto	$ 4,500,000.00	$ -	0.12	0.03	0.05
$D$2	Amount Loaned Siganture	$ 1,500,000.00	$ -	0.15	1E+30	0.03

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$4	Total Budget LHS	$ 15,000,000.00	$0.093	15000000	1E+30	15000000
$E$5	Mortgage minumum LHS	$ 9,000,000.00	$ (0.05)	0	4500000	9000000
$E$6	Signature maximum LHS	$ 1,500,000.00	$ 0.03	0	4500000	1500000

These constraints are not expressed in standard form since there's a function on the right hand side.
Note how mush less human-friendly it is than the form on this and the previous page.
To Excel, the two are exactly the same.

Standard Form

	A	B	C	D	E	F	G	H
1		Mortgage	Auto	Siganture				
2	Amount Loaned	$9,000,000.00	$4,500,000.00	$1,500,000.00	LHS		RHS	
3	Interest rate	0.07	0.12	0.15	$1,395,000.000			
4	Total Budget	1	1	1	$15,000,000.00	≤	$15,000,000.00	
5	Mortgage minumum	40%	-60%	-60%	$ -	≥	$ -	60%
6	Signature maximum	-10%	-10%	90%	$ -	≤	$ -	10%

Case 5: Yödelheim Ministry of Transport

The country of Yödelheim has three tunnels and two bridges that need to be repaired:
the Niffelheim tunnel, the Moria tunnel, the Orpheus tunnel, the Bifrost bridge, and the Gjallarbrú bridge.

The Yödelheim Ministry of Transport has only enough equipment and labor force in-house to repair any one of these. Because of the nature of government accounting, it will cost ten million Rheingolds (®10,000,000) whichever project they do in-house.

Whichever project they do themselves, they will outsource each of the other four to a private company. The Prime Minister has decreed that they cannot give more than one of these large projects to any one company, even if it is the lowest bidder on more than one.

Four private companies have each bid on some of the projects: Trolls R Us, NoTrump Bridge & Tunnel, Dante's Underworld Enterprises, and the Tallahatchee Bridge Company.

Trolls R Us has bid ®12,000,000 to repair the Niffelheim tunnel, ®7,500,000 to repair the Moria tunnel, ®18,000,000 to repair the Orpheus tunnel, ®5,000,000 to repair the Bifrost bridge, and ®11,000,000 to repair the Gjallarbrú bridge.

NoTrump Bridge & Tunnel has bid ®11,000,000 to repair the Niffelheim tunnel, ®16,000,000 to repair the Moria tunnel, ®15,000,000 to repair the Orpheus tunnel, ®5,000,000 to repair the Bifrost bridge, and ®15,000,000 to repair the Gjallarbrú bridge.

Dante's Underworld Enterprises has bid ®8,000,000 to repair the Niffelheim tunnel, ®15,000,000 to repair the Moria tunnel, and ®6,000,000 to repair the Orpheus tunnel. As an underground specialist, they did not submit any bids to repair the Bifrost bridge or the Gjallarbrú bridge.

The Tallahatchee Bridge Company has bid ®7,000,000 to repair the Bifrost bridge, and ®11,000,000 to repair the Gjallarbrú bridge. They did not submit bids on any of the tunnels.

The Ministry of Transport needs to decide which project to do in-house, and which project to award to each of the four companies.

One way to do this would be to start with the lowest bid, then the lowest bid among the remaining companies and projects, and so on. Expressing costs in millions of Rheingolds, the bids are as follows:

	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge
YMOT	10	10	10	10	10
Trolls	12	7.5	18	5	11
NoTrump	11	16	15	10	15
Dante	8	15	6	99	99
Tall	99	99	99	7	11

The lowest bid is Trolls R Us's ®5,000,000 to repair the Bifrost bridge. If we make this award, the remaining decision looks like this:

	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Gjallarbrú bridge
YMOT	10	10	10	10
NoTrump	11	16	15	15
Dante	8	15	6	99
Tall	99	99	99	11

The best opportunity in this table is to award the Orpheus Tunnel project to Dante's Underworld Enterprises for ®6,000,000. After that, we are down to

	Niffelheim tunnel	Moria tunnel	Gjallarbrú bridge
YMOT	10	10	10
NoTrump	11	16	15
Tall	99	99	11

The Yödelheim Ministry of Transport itself is the lowest cost option for each of the three remaining projects, at ®10,000,000. It should be clear that they should take on the Moria tunnel repairs since the only other option is the highest of the remaining bids. (99 is used to indicate no bid.) That brings us to the last two:

	Niffelheim tunnel	Gjallarbrú bridge
NoTrump	11	15
Tall	99	11

NoTrump Bridge & Tunnel is the only option left for the Niffelheim tunnel at ®11,000,000, leaving Tallahatchee Bridge Company to repair the Gjallarbrú bridge also for ®11,000,000.

The total cost of this selection is ®5,000,000 + ®6,000,000+ ®10,000,000+ ®11,000,000+ ®11,000,000, or 43 million Rheingolds

However, this method (known as the "***greedy algorithm***") actually wastes 1.5 million Rheingolds.
The best solution can be found using Linear Programming; problems of this type are collectively known as Assignment problems.

We can do substantially better using linear programming to minimize total costs without violating any of the rules.

The matrix B3:F7 contains the costs (in millions of Rheingolds) for each potential assignment of a project to an organization. These are the coefficients of the objective function. They are called "Objective Coefficients" for short in the sensitivity report.

The numbers in the matrix B12:F15 are the assignments: 1 means the organization specified in the row does the project specified in the column. These assignments are the decision variables in an assignment problem, so they are specified in the "By Changing Cells" box in the Solver setup. They're called "Adjustable Cells" in the Sensitivity Report.

There are two blocks of constraints in addition to the usual non-negativity constraints. The first block requires each organization to do one and only one project; the total number of projects assigned to each organization can be found in cells G 12:G15, and there is a constraint line that says each of these cells must be EQUAL (not less than or equal to) the RHS constant 1. Similarly, each project must be assigned to one and only one organization; the total number of organizations assigned to each project can be found in cells B 16:F15, and there is a constraint line that says each of these cells must be EQUAL (not less than or EQUAL to) the RHS constant 1.

	A	B	C	D	E	F	G
1	**Yödelheim Ministry of Transport**						
2							
3	Costs	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	
4	YMOT	10	10	10	10	10	
5	Trolls	12	7.5	18	5	11	
6	NoTrump	11	16	15	10	15	
7	Dante	8	15	6	99	99	
8	Tall	99	99	99	7	11	
9							
10							
11	Assign ments	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	Total
12	YMOT	0	0	0	0	1	1
13	Trolls	0	1	0	0	0	1
14	NoTrump	1	0	0	0	0	1
15	Dante	0	0	1	0	0	1
16	Tall	0	0	0	1	0	1
17	Total	1	1	1	1	1	
18							
19	Cost	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	
20	YMOT	0	0	0	0	10	
21	Trolls	0	7.5	0	0	0	
22	NoTrump	11	0	0	0	0	
23	DWM	0	0	6	0	0	
24	Tall	0	0	0	7	0	
25	Total	11	7.5	6	7	10	
26							
27	Total Cost	41.5					

	A	B	C	D	E	F	G
1	**Yödelheim Ministry of Transport**						
2							
3	Costs	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	
4	YMOT	10	10	10	10	10	
5	Trolls	12	7.5	18	5	11	
6	NoTrum	11	16	15	10	15	
7	Dante	8	15	6	99	99	
8	Tall	99	99	99	7	11	
9							
10							
11	Assign ments	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	Total
12	YMOT	0	0	0	0	1	=SUM(B12:F12)
13	Trolls	0	1	0	0	0	=SUM(B13:F13)
14	NoTrum	1	0	0	0	0	=SUM(B14:F14)
15	Dante	0	0	1	0	0	=SUM(B15:F15)
16	Tall	0	0	0	1	0	=SUM(B16:F16)
17	Total	=SUM(B12:B16)	=SUM(C12:C16)	=SUM(D12:D16)	=SUM(E12:E16)	=SUM(F12:F16)	
18							
19	Cost	Niffelheim tunnel	Moria tunnel	Orpheus tunnel	Bifrost bridge	Gjallarbrú bridge	
20	YMOT	=B4*B12	=C4*C12	=D4*D12	=E4*E12	=F4*F12	
21	Trolls	=B5*B13	=C5*C13	=D5*D13	=E5*E13	=F5*F13	
22	NoTrum	=B6*B14	=C6*C14	=D6*D14	=E6*E14	=F6*F14	
23	DWM	=B7*B15	=C7*C15	=D7*D15	=E7*E15	=F7*F15	
24	Tall	=B8*B16	=C8*C16	=D8*D16	=E8*E16	=F8*F16	
25	Total	=SUM(B20:B24)	=SUM(C20:C24)	=SUM(D20:D24)	=SUM(E20:E24)	=SUM(F20:F24)	
26							
27	Total Cost	=SUM(B20:F24)					

The matrix B20:F24 shows the assignments in terms of the cost; it is found by multiplying corresponding numbers in the cost matrix and the assignment matrix. Row 25 summarizes the costs for each project. The objective function, total cost, is the sum of row 25, found in cell B27. So, in the Solver setup, the Target Cell is B27. Of course, min is checked instead of max; the Yödelheim taxpayers would not appreciate a Ministry of Transport that used the maximum cost option!

	A	B	C	D	E	F	G	H
6	Adjustable Cells							
7				**Final**	**Reduced**	**Objective**	**Allowable**	**Allowable**
8		**Cell**	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
9		$B$12	YMOT Niffelheim tunnel	0	0	10	2	1.5
10		$C$12	YMOT Moria tunnel	0	0	10	1.5	1.5
11		$D$12	YMOT Orpheus tunnel	0	0	10	4	2
12		$E$12	YMOT Bifrost bridge	0	2.5	10	1E+30	2.5
13		$F$12	YMOT Gjallarbrú bridge	1	0	10	1.5	1E+30
14		$B$13	Trolls Niffelheim tunnel	0	4.5	12	1E+30	4.5
15		$C$13	Trolls Moria tunnel	1	0	7.5	1.5	1.5
16		$D$13	Trolls Orpheus tunnel	0	10.5	18	1E+30	10.5
17		$E$13	Trolls Bifrost bridge	0	0	5	1.5	1.5
18		$F$13	Trolls Gjallarbrú bridge	0	3.5	11	1E+30	3.5
19		$B$14	NoTrump Niffelheim tunnel	1	0	11	1.5	1E+30
20		$C$14	NoTrump Moria tunnel	0	5	16	1E+30	5
21		$D$14	NoTrump Orpheus tunnel	0	4	15	1E+30	4
22		$E$14	NoTrump Bifrost bridge	0	1.5	10	1E+30	1.5
23		$F$14	NoTrump Gjallarbrú bridge	0	4.000000001	15	1E+30	4.000000001
24		$B$15	Dante Niffelheim tunnel	0	2	8	1E+30	2
25		$C$15	Dante Moria tunnel	0	9	15	1E+30	9
26		$D$15	Dante Orpheus tunnel	1	0	6	2	1E+30
27		$E$15	Dante Bifrost bridge	0	95.5	99	1E+30	95.5
28		$F$15	Dante Gjallarbrú bridge	0	93	99	1E+30	93
29		$B$16	Tall Niffelheim tunnel	0	89.5	99	1E+30	89.5
30		$C$16	Tall Moria tunnel	0	89.5	99	1E+30	89.5
31		$D$16	Tall Orpheus tunnel	0	89.5	99	1E+30	89.5
32		$E$16	Tall Bifrost bridge	1	0	7	1.5	1E+30
33		$F$16	Tall Gjallarbrú bridge	0	1.5	11	1E+30	1.5
34								
35	Constraints							
36				**Final**	**Shadow**	**Constraint**	**Allowable**	**Allowable**
37		**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
38		$B$17	Total Niffelheim tunnel	1	6	1	0	0
39		$C$17	Total Moria tunnel	1	6	1	0	0
40		$D$17	Total Orpheus tunnel	1	6	1	0	1
41		$E$17	Total Bifrost bridge	1	3.5	1	0	0
42		$F$17	Total Gjallarbrú bridge	1	6	1	0	1
43		$G$12	YMOT Total	1	4	1	1	0
44		$G$13	Trolls Total	1	1.5	1	0	0
45		$G$14	NoTrump Total	1	5	1	0	0
46		$G$15	Dante Total	1	0	1	0	1E+30
47		$G$16	Tall Total	1	3.5	1	0	0

Technical notes:
This is NOT an "integer programming" or "binary programming" problem, The mathematics of the problem automatically give all decision variables an value of zero or one without having to force this directly.
Also, assignment problems are always highly degenerate, but that's OK since changing the RHS of a constraint would violate the basic assumptions of the assignment problem anyway.

Finally, if there are more rows than columns and one project had to be carried over until next year, we would create another row labeled "next year" and fill in the cost matrix for this row with the cost of delaying each of the six projects. If, instead, we had five projects and six organizations, we could create another column called "no project." The Ministry's costs in this column of the cost matrix would ordinarily be zero.

Case 6: Georgia Cracker Company

Minimizing Transportation Cost

The Georgia Cracker Company manufactures crackers at two factories in Albany and Atlanta and ships them around the state. The capacity of the Albany factory is 500 cases per day and the capacity of the Atlanta factory is 1000 cases per day. The maximum number of cases that can be sold per day in each city is:

Albany	Atlanta	Augusta	Cass	Colum-bus	Dalton	Harlem	Lexing-ton	Macon	Mc-Intyre	Savan-nah	Senoia
100	200	75	50	100	90	50	60	200	40	180	75

Here is the distance from the factories to each market area:

Miles	Albany	Atlanta	Augusta	Cass	Colum-bus	Dalton	Harlem	Lexing-ton	Macon	Mc-Intyre	Savan-nah	Senoia
Albany	0	165	207	212	89	254	192	195	102	124	210	142
Atlanta	165	0	148	43	102	85	128	83	77	104	243	33

It costs 5 cents per case per mile to ship the crackers by truck

This is a simple example of a very important class of linear programming applications referred to as "The Transportation Problem." The linear programming solution is as follows.

Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
Albany	0	165	207	212	89	254	192	195	102	124	210	142		
Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
From Albany	100	0	0	0	100	0	0	0	0	0	180	0	380	500
From Atlanta	0	200	75	50	0	90	50	60	200	40	0	75	840	1000
Total to	100	200	75	50	100	90	50	60	200	40	180	75		
Demand	100	200	75	50	100	90	50	60	200	40	180	75		
101,015	Total miles													
$ 5,050.75	Transportation cost													

The decision variables in cells B12 through N13 specify how many cases of crackers should be shipped from each factory to each market area. The constraints are that the total from each factory has to be less than or equal to its capacity, and the total to each market area has to be equal to its demand. The objective is to minimize the total transportation cost found by multiplying the number of cases in each shipment times the mileage times the cost per mile and adding the result for all shipments.

Minimizing Total Variable Cost

However, transportation is not the only variable cost associated with our crackers. There is also the variable cost of manufacturing them, which costs $22 per case in Albany and $24 per case in Atlanta. We get a different result if we include the manufacturing cost (the product of the cost per case at each factory times the "total from" that factory, added together for the two factories.)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5																
6		Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
7		Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
8		Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
9																
10																
11	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
12	$ 22	From Albany	100	0	0	0	100	0	0	0	80	40	180	0	500	500
13	$ 24	From Atlanta	0	200	75	50	0	90	50	60	120	0	0	75	720	1000
14		Total to	100	200	75	50	100	90	50	60	200	40	180	75		
15		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
16																
17		103,815	Total miles													
18		$ 5,190.75	Tramsportation Cost													
19		$ 28,280.00	Manufacturing Variable Cost													
20		$ 33,470.75	Total VariableCost													

Notice that we have used fewer cases from Atlanta and more from Albany even though we have to ship more miles, since we more than make up for it by saving money on manufacturing. (There is excess manufacturing capacity, and we move the excess from Albany to Atlanta.

Maximizing Contribution to Profit and Overhead

Instead of insisting that the maximum possible demand at each location must be met, it makes more sense to look at what's profitable. It turns out that we take a loss supplying some of the cities.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5																
6		Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
7		Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
8		Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
9																
10																
11	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
12	$ 22	From Albany	100	0	0	0	100	0	0	0	200	0	0	0	400	500
13	$ 24	From Atlanta	0	200	0	50	0	0	0	0	0	0	0	75	325	1000
14		Total to	100	200	0	50	100	0	0	0	200	0	0	75		
15		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
16		Price	$30	$32	$ 30	$27	$28	$28	$26	$26	$30	$26	$30	$27		
17																
18		33,925	Total miles													
19		$ 1,696.25	Tramsportation Cost													
20		$ 16,600.00	Manufacturing Variable Cost													
21		$ 18,296.25	Total Cost													
22		$ 21,575.00	Total Revenue													
23		$ 3,278.75	Contribution to Profit and Overhead													

The Trans-shipment Problem

Up to now we have just been looking at elaborations of the transportation problem, The next step up in sophistication comes when Georgia Cracker Company opens a warehouse in Macon with a capacity to unload 400 cases per day from rail to trucks. Crackers can be shipped by train from either factory for just one cent per case per mile, then transferred (or "trans-shipped") to trucks to go on to the final destination at the usual cost of five cents per case per mile. In a full-fledged trans-shipment problem all locations are potential trans-shipment points, but for simplicity here only Macon is.

Row 16 of the spreadsheet gives the total number of cases shipped to each market, whether directly from Albany or Atlanta, or from Macon after being shipped to Macon by rail. Row 17, which in this simplified model pertains only to Macon, is the number of cases trans-shipped out. Row 18 calculated the "Net to" as the "Total To" in row 16 take away the "Total From" in row 17. This is constrained to be less or equal to the maximum demand for each city, and multiplied times the selling price per case in each city to calculate the revenue as in the third transportation spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5		Macon	102	77	121	128	95	170	100	93	0	32	170	70		
6																
7		Cost Per Mile	to Albany	to Atlanta	to August	to Cass	to Colum	to Dalton	to Harlem	to Lexingt	to Macon	to McIntyr	to Savann	to Senoia		
8		From Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.01	0.05	0.05	0.05		
9		From Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.01	0.05	0.05	0.05		
10		From Macon	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
11																
12	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
13	$ 22	From Albany	100	0	0	0	100	0	0	0	300	0	0	0	500	500
14	$ 24	From Atlanta	0	200	0	50	0	0	0	0	0	0	0	75	325	1000
15		From Macon	0	0	60	0	0	0	0	0	0	40	0	0	100	400
16		Total To	100	200	60	50	100	0	0	0	300	40	0	75		
17		Total From									100					
18		Net To	100	200	60	50	100	0	0	0	200	40	0	75		
19		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
20		Price	$30	$32	$ 30	$27	$28	$28	$26	$26	$30	$26	$30	$27		
21																
22		52,665	Total miles													
23		$ 1,409.25	Tramsportation Cost													
24		$ 18,800.00	Manufacturing Variable Cost													
25		$ 20,209.25	Total Cost													
26		$ 24,415.00	Total Revenue													
27		$ 4,205.75	Contribution to Profit and Overhead													

As long as the warehouse rental and operating costs are less that ($4,205.73-$3,278.75 =) $927.00 per week, Georgia Cracker Company is better off renting the warehouse and using it for transshipment.