Case 7: Pharr Owt, a simple minimization problem

Pharr Owt, an old friend of yours, has just been made Warden/Headmaster of the South Florida Prison and Institute of Chemical Technology. She is concerned about the nutritional state of her new charges, and would like to start by improving their breakfasts. She must buy food from the state prison commissary system. The only breakfast foods they offer are oatmeal and eggs. Pharr is on a tight budget, and would like to meet the PICT inmates' nutritional needs as cheaply as possible.

She has 1,000 inmates to feed. Even though on a given day not everyone will eat whatever combination she buys, she is willing to assume that over the long haul it will even out. She has worked out this information:

	One Egg	**One Cup Oatmeal**	**Minimum per Inmate**
Fat	5	2	8 fat units
Fiber	0	6	8 fiber units
Protein	5	4	12 protein units
Cost	$0.10	$0.05	

With 1000 inmates, her question is simply this: what combination of oatmeal and eggs will give these inmates at least the required nutrition at least cost?

Defining the activity variables is a critical issue in formulating LP problems. Fortunately it is pretty easy in this case. Let OAT be the number of cups of oatmeal to provide 1000 prisoners each day, and EGG be the number of eggs to provide those 1000 unfortunates. She'll let the kitchen staff worry about whether inmates choose between eggs and oatmeal or just have a porridge.

To meet the nutritional requirements, she must provide no less than 8000 (8×1000) units each of fat and fiber, and at least 12,000 units of protein. These are minimum performance requirements, so they must be ≥ constraints. The fact that Pharr wants to meet these requirements at *least cost* implies that this is a cost minimization problem. The purchase prices are fixed and stable, so there is nothing tricky about the objective function. Working together, you agree that this is the problem:

Minimize Z =	.10 EGG	+ .05 OAT			
subject to:	5 EGG	+ 2 OAT	≥	8,000	Fat
		6 OAT	≥	8,000	Fiber
	5 EGG	+ 4 OAT	≥	12,000	Protein

Setting up to graph the constraints yielded this:

Constraint	Connect these points to draw the boundary line **(EGG, OAT)**	Test these points to find the feasible side of the line **(EGG, OAT)**	**RHS**	**Result**
Fat	(1600. 0) (0, 4000)	(2000, 2000)	14,000	feasible
Fiber	Oar = 1333.33 (horizontal)			
Protein	(2400, 0) (0, 3000)	(2000, 2000)	18,000	feasible

You and Pharr used this information to draw the graph. As a first try, you looked, optimistically (you thought) at a $200/day isocost line.

$$.10\ EGG + .05\ OAT = 200$$

$$EGG = 0,\ OAT = 4000$$

$$OAT = 0,\ EGG = 2000$$

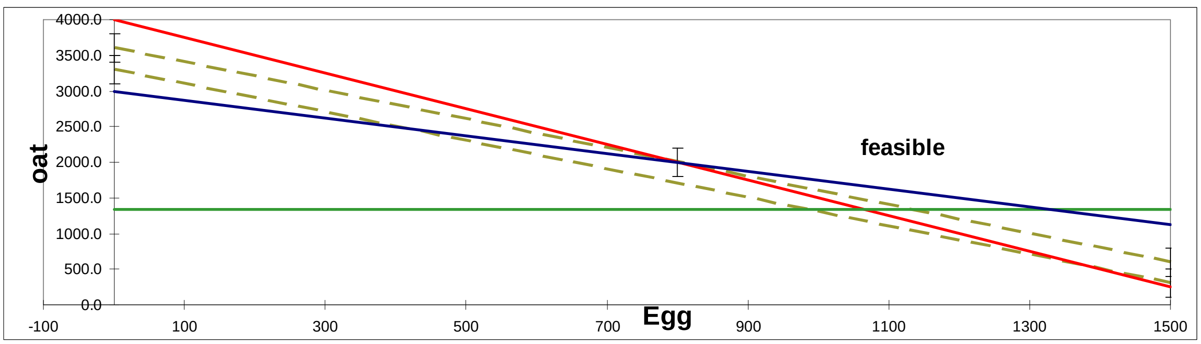
Amazingly, the $200/day isocost line passed through the interior of the feasible region. This indicated that there was room to drop it more. Your next guess was $180/day.

$$.10\ EGG + .05\ OAT = 180$$

$$EGG = 0,\ OAT = 3600$$

$$OAT = 0,\ EGG = 1800$$

To your amazement and amusement, you hit it dead on the money on your second try. The $180/day isocost just touched the feasible region at one of its corners, which means $180 per day buys the inmates all the nutritional breakfast they can stand. Pharr Owt needs to order up 2000 cups of oatmeal and 800 eggs for her flock each day. That will exactly meet the requirements for fat and protein, and more than meet the fiber requirement. You're



getting pretty slick at this LP stuff!

Comment

Having put all this effort into graphical solutions of LP problems, it would be unfair not to acknowledge we *never* solve real problems graphically. The graphical solution is a very useful pedagogical device, and an instrument for understanding both the basics and some of the nuances of LP. From here on out (<u>after</u> we master this chapter), we'll be solving problems using the Simplex Method on the computer. Even so, when you encounter something you don't quite understand, you might benefit from coming back to the graphical solution to see if, just maybe, it can help you to close the gap. And if the problem concerning you has only 2 activity variables, of course you can graph the particular problem.

PHARR OWT'S PROBLEM BY COMPUTER

Let's see if you can come up with the same answers to my questions that I do, using the EXCEL output. Some of the answers, of course, we already know from the graphical solution.

1. What should Pharr give the inmates for breakfast?
2. What will that cost per day?
3. Are they getting more than the minimum of any nutrient? If so, which one(s)?
4. Other than saving some money, would there be any effect on the solution if Pharr found she could get oatmeal for 3¢ per cup? What would happen?
5. Other than costing some money, would there be any effect if Pharr found out that the price of eggs had gone up to 11¢ each? What would happen?
6. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Fiber standard for 1000 prisoners is a minimum of 12,000 units per day. How much more (or less) will this cost daily?
7. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Protein standard for 1000 prisoners is a minimum of 15,000 units per day. How much more (or less) will this cost daily?
8. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Fat standard for 1000 prisoners is a minimum of 7,000 units per day. How much more (or less) will this cost daily?

Pharr Owt's LINDO Output

```
MIN     .05 OAT + .1 EGG
 SUBJECT TO
           2 OAT + 5 EGG >=   8000
           6 OAT         >=   8000
           4 OAT + 5 EGG >=   12000
   OBJECTIVE FUNCTION VALUE      180.000000
                           RANGES IN WHICH THE BASIS IS UNCHANGED:
                                                   OBJ COEFFICIENT RANGES
VARIABLE VALUE   REDUCED   VARIABLE    CURRENT    ALLOWABLE     ALLOWABLE
                  COST                 COEFF      INCREASE      DECREASE
OAT      2000    .000000   OAT        .050000     .030000       .010000
EGG       800    .000000   EGG        .100000     .025000       .037500

                                               RIGHTHAND SIDE RANGES
    SLACK OR    DUAL            CURREN     ALLOWABLE     ALLOWABLE
     SURPLUS    PRICES            RHS       INCREASE      DECREASE
        .0     -.0150           8000.0     1333.333      2000.000
   4000.0       .0000           8000.0     4000.000       INFINITY
        .0     -.0050          12000.0     4000.000      1333.333
```

Pharr Owt's EXCEL Output

Eggs	Oatmeal			
800	2,000			
0.1	0.05	180	Cost	
0	2	8,000	Fat	8,000
0	6	12,000	Fiber	8,000
5	4	12,000	Protein	12,000
1	0	800	NNEgg	0
0	1	2,000	NNOat	0

Answer Report						
Target Cell (Min)						
	Cell	Name	Original Value	Final Value		
	$C$3	Cost	0	180		
Adjustable Cells						
	Cell	Name	Original Value	Final Value		
	$A$2	Eggs	0	800		
	$B$2	Oatmeal	0	2,000		
Constraints						
	Cell	Name	Cell Value	Formula	Status	Slack
	$C$5	Fat	8,000	$C$5>=$E$5	Binding	0
	$C$6	Fiber	12,000	$C$6>=$E$6	Not Binding	4,000
	$C$7	Protein	12,000	$C$7>=$E$7	Binding	0
	$C$8		800	$C$8>=$E$8	Not Binding	800
	$C$9		2,000	$C$9>=$E$9	Not Binding	2,000

Sensitivity Report							
Changing Cells							
	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	$A$2	Eggs	800	0	0.1	0.03	0.04
	$B$2	Oatmeal	2,000	0	0.05	0.03	0.01
Constraints							
	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	$C$5	Fat	8,000	0.02	8,000	1,333.33	2,000
	$C$6	Fiber	12,000	0	8,000	4,000	1E+30
	$C$7	Protein	12,000	0.01	12,000	4,000	1,333.33
	$C$8		800	0	0	800	1E+30
	$C$9		2,000	0	0	2,000	1E+30

Answers:

1. Feed them 2000 cups of oatmeal and 800 eggs per day
2. It will cost $180 per day
3. Yes, they are getting 4000 units more fiber than required.
4. 3¢ per cup is below the 4¢ lower limit for oatmeal in the objective function ranging. That tells us that a different intersection of constraints would become optimal, providing a new and different solution (nothing but oatmeal - yum!)
5. If the price of eggs goes up 1¢ per egg, that remains within the objective function coefficient range for eggs. She pays more, but gives the same breakfast.
6. Pharr is one step ahead of the Bureaucrats. The 4000 unit surplus of fiber means she is already meeting this standard.
7. This *tightens* the Protein standard by 3000 units. The shadow price for the Protein constraint is $0.005, and a 3000 unit increase is within the RHS range for this constraint. The daily cost of breakfast increases by 3000 × $0.005 = $15.00. She must rerun the problem to find out what $195 breakfast will meet this standard.
8. This change *relaxes* the fat constraint by 1000 units. The shadow price for the Fat constraint is $0.015, and a 1000 unit decrease is within the RHS range for this constraint. The daily cost of breakfast decreases by 1000 × $0.015 = $15.00. She must rerun the problem to find out what $165 breakfast will meet this standard.

Formulation hints

We have already had a look at some really important formulation issues. These include

- ➢ Limit your activity variables to things that the decision maker can change directly and needs to make a decision on
- ➢ *Define* each one of your activity variables, including supplementary variables if any; don't just describe them with generalities
- ➢ Use incremental future cash flows in the objective function
- ➢ If any doubt exists *at all* about your formulation, conduct dimensional analysis

The first of these deserves to be expanded on. Many beginners at LP would take a problem such as Beau Jarble's and do things that, once they are experienced LP modelers, would amaze them as being incredibly naive. Like what? The classic mistake is to make things like lathe time, hole drilling capacity, and so forth into activity variables.

But they are already set. Beau is <u>not</u> looking to make a decision about whether to ask the drill press operator to work extra hours. After the problem is solved, Beau might see a threat or an opportunity there, and then raise the issue. But for now, he's assuming the operator will work the 10 hours that we began by assuming. Beau's focus is directly on the things that make revenue. Wheels.

There are some other situations that commonly trip up beginners. Let's look at a few of the most common ones.

PROPORTIONALITY CONSTRAINTS

Suppose that Beau had said that he wanted to make at least twice as many of the ITS model as of the ITA. A common (wrong) way to formulate that would be to say

$$ITA \leq 50$$

What's wrong with that? Several things. First, it combines two restrictions (no more that 100 ITS wheels and at least twice as many ITS wheels as ITA's). This can, and usually will, blur both restrictions. Second, it assumes, before solving the problem, that the answer will include 100 ITS wheels. Until we solve, we do *not* know that. Third, it can easily fail to enforce this new restriction. We could easily end up with the solution ITA = 50, ITS = 80. LP can't know that this is wrong, but you can bet that Beau knows.

The right way to develop the constraint is to write an algebraic expression that captures exactly what Beau said, then rearrange it if necessary. Beau said

$$ITS \geq 2\ ITA$$

This doesn't look like a regular LP constraint (it has a variable, but no number, in the RHS). Excel can handle this just fine, but it's worth a peek under the hood to see how it translates into a classic LP setup like the one on page 2. All we need to do is to subtract 2 ITA from both sides.

$$ITS - 2\ ITA \geq 0$$

This may be less intuitively appealing to look at, but it will enforce exactly the restriction Beau wanted.

Let's consider another case. Suppose Pharr had said that she feared the effects of cholesterol in eggs, and as a result wanted to spend no more than half her budget on eggs. Again, remember that at the time we get this information, we do not know how the solution will come out. Even if we did know, most beginners would get stuck trying to figure out how to limit expenditures on eggs to $90 or less (which is not, anyway, what Pharr requested!)

To do it right, we do something truly amazing. We write down what she said, algebraically. That isn't as hard as you might think. We just do it in thoughtful stages. She said

$$\text{Amount spent on eggs} \leq .5 \times \text{Total amount spent}$$

The amount spent on eggs must be .10 EGG. The total amount spent is going to be .05 OAT + .10 EGG. So that must mean that

$$.10\ EGG \leq .5\ (.05\ OAT + .10\ EGG)$$

This si probably the best way to put it into Exel since it makes the most sense to the human eye and brain. To put it into the classic LP formulation Excel uses behind the scenes, let's multiply *both* sides by 2. That gives us

$$.20\ EGG \leq .05\ OAT + .10\ EGG$$

now we can easily subtract the RHS .05 OAT + .10 EGG T from both sides, which gives us

$$-\ .05\ OAT + .10\ EGG \leq 0$$

Multiple Blending Problems

In most problems, including resources among the activity variables is a serious error, as we just noted in discussing Beau's hole drilling. But there is one special category of problems that breaks that rule. To properly set up a **multiple blending problem**, you absolutely must use that sort of mixed variable. Be cautious - many problems are called blending problems that are not full-fledged multiple blending problems.

A multiple blending problem is one in which 2 or more ingredients are blended in flexible ways to form 2 or more end products. If you have a common list of ingredients used in different proportions to make 3 brands of steak sauce, you probably have a blending problem. If you have a list of petrochemicals and additives that are blended together to make NoxaLot, GutLess, and MeDiocre grades of gasoline, you almost surely have a blending problem. *If the exact proportions of the ingredients required to make each product are fixed and known, it is **not** a **true** blending problem!*

If a bottle of DeadCow barbecue sauce calls for 4 ounces of ingredient A, 2 ounces of B, and 2 ounces of C, then it is not a true blending problem. If a bottle of DeadCow contains at least 50% A, no more than 30% B, and enough C to complete filling an 8 ounce bottle, then we are looking at a true blending problem.

Suppose the percentages do describe the formula for DeadCow. The other product made by the good folks who bring you DeadCow is called PigCarass. PigCarcass is at least 40% C, at least 10% B, and no more than 10% A. A bottle of either contains 8 ounces. They have 1000 ounces of each ingredient on hand. There is a price for each product, say P_{DC} and P_{PC} per bottle. Replacement costs for each ingredient are C_A , C_B , and C_C .

Wrestle with this a while, and you will soon see that variables like DC and PC just won't work. Since you don't know how much of each ingredient is in each bottle, you can't figure the margin. Since you don't know how much of each ingredient is in each bottle, you don't know when you will run out of an ingredient. Since you don't know when you'll run out, you can't figure how much DC and PC you can make. It is a nightmare.

The way out is to define the activity variables as the total number of ounces of each ingredient that ends up in each product. Thus one variable would be defined as

DCA = number of ounces of A used to make DeadCow

DCA's contribution margin would be $P_{DC} - C_A$. This margin could even be negative - maybe we make it up on DCB and/or DCC. The total DeadCow made is DCA + DCB + DCC. The total amount of A used up is DCA + PCA. This way, we can keep everything under control. It isn't intuitively obvious, but it works well.

Full details of this delicious problem can be found in Exercise 9. After you've studied the discussion of Snako Oil on the following pages, you should be able to do Exercise 9 on your own.

Case 8: Snako Oil Company, A Multiple Blend Problem

The Snako Oil Company blends and markets 2 grades of gasoline, Zoom and Slug. Zoom is a Superduper Premium fuel that has a very high octane rating and is packed with lots of thermal energy, and sells for $2.00 per gallon. Slug is a barely adequate fuel well suited to thoroughly worn older cars. Despite its $1.45 per gallon price, Slug is popular because it can help an old worn car pass emissions inspection. Both fuels are made by mixing 3 petro-chemical ingredients. DiethylMulehide is a low octane, low energy, very clean burning component costing Snako only $0.50 per gallon. Gruntane is packed densely with energy, has a moderate octane rating, and costs Snako $1.50 per gallon. Hoctane contributes a very high octane rating, moderate energy density, and burns very cleanly. At a cost to Snako of $2.10 per gallon it certainly should.

The specifications for Zoom require that Zoom contain no more than 20% diethylMulehide, while Slug may contain up to 70% diethylMulehide. Zoom must be at least 40% Gruntane, while Slug does not need to be more than 5% Gruntane. Zoom always contains at least 30% Hoctane, but Slug only requires 20% or more. This week, Snako has access to 30,000 gallons of diethylMulehide, 8,000 gallons of Gruntane, and 10,000 gallons of Hoctane.

Snako has promised their distributors that they will deliver 10,000 or more gallons of Zoom and 25,000 or more gallons of Slug this week. They would like to make as much money as possible using the currently available resources without breaking their promises and without violating the integrity of their specifications for Zoom and Slug. We will use LP to help them.

For an LP beginner, this is a ***hard*** problem. It combines 2 aspects that beginners often have trouble with. In addition, we will use some extra (supplementary) variables.

1. It is a *true* **blending problem**. That means that the proportions of ingredients in the blend are <u>not</u> <u>fixed</u>. This has a profound effect on the way we must define our activity variables.
2. Because it is true blending problem, it has **proportionality constraints**. People frequently are troubled, at first, by proportionality constraints.
3. The "extra variables" are called ***supplementary variables***. We use them in some problems to make the output easier to read. First we will set up the problem without them, then we will reformulate with them.

Because this is a real blending problem, we cannot depend on variables called ZOOM and SLUG as we would in, for example, a product mix problem. Since we do not know exactly what resources Zoom and Slug will require this week, we would be unable to figure out our resource constraints. And in setting up our objective function coefficients, we would get stuck because we don't know what Zoom and Slug are going to cost to make. So we need a different approach for a blending problem.

In this problem, we need variables that mean "the amount of *this* ingredient we will use in *this* product". Then we can sum variables using the same resource to represent the amount of that resource that will be used. We can, similarly, sum variables that are parts of the same product to represent the amount of that product we will blend. We can figure the profit contributions of the variables by subtracting the cost of the ingredient from the price of the product. But this is hardly something that is obvious to the average bear. Let's do it.

ZM = Gallons of diethylMulehide used to make Zoom. The contribution/gallon is the $2.00 selling price of Zoom less the $0.50 cost of a gallon of diethylMulehide, or $1.50.

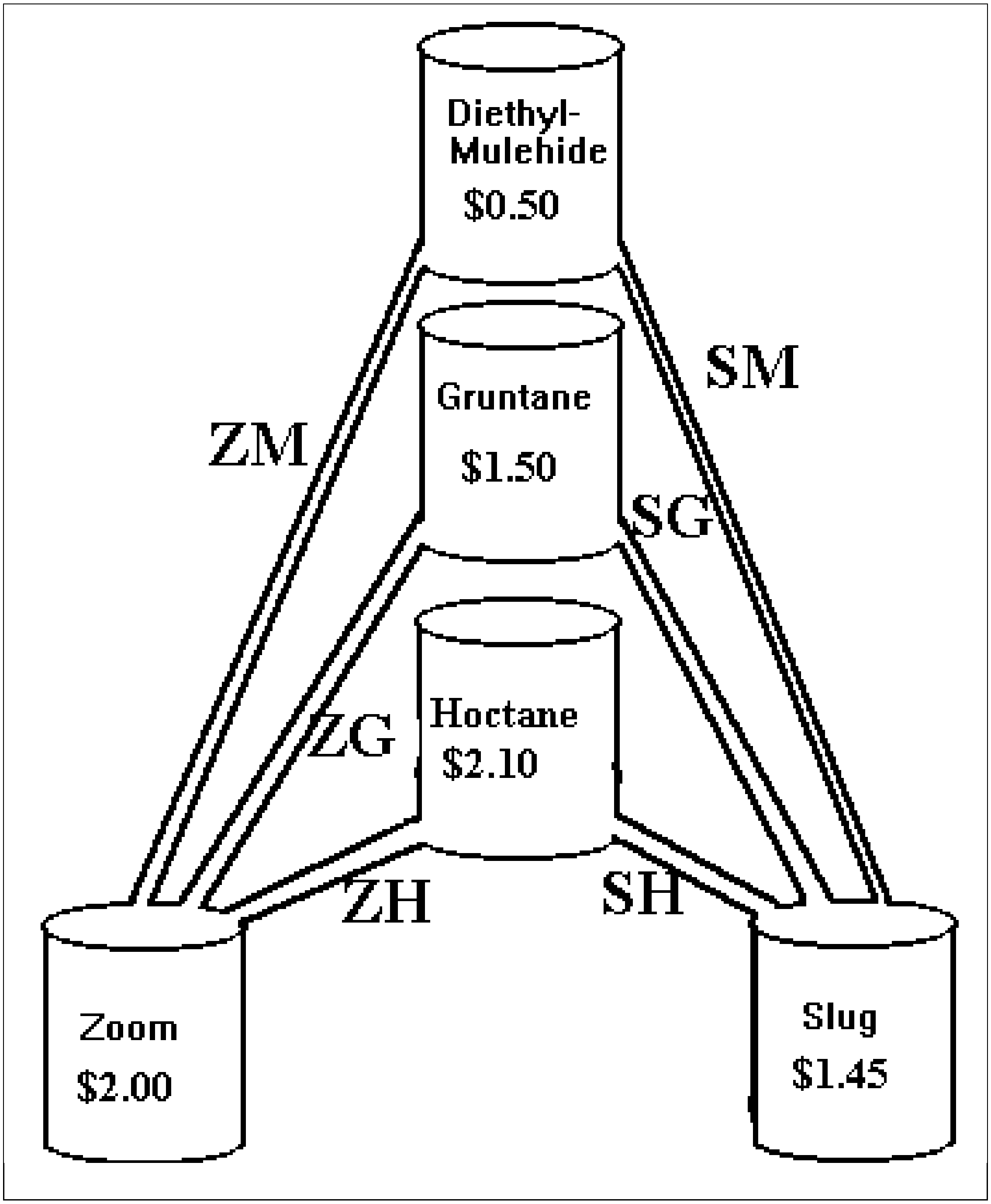
SM = Gallons of diethylMulehide used to make Slug. The contribution/gallon is the $1.45 selling price of Slug less the $0.50 cost of a gallon of diethylMulehide, or $0.95.

ZG = Gallons of Gruntane used to make Zoom. The contribution is $2.00 - $1.50 = $0.50

SG = Gallons of Gruntane used to make Slug. The contribution is $1.45 - $1.50 = ($0.05)

ZH = Gallons of Hoctane used to make Zoom. The contribution is $2.00 - $2.10 = ($0.10)

SH = Gallons of Hoctane used to make Slug. The contribution is $1.45 - $2.10 = ($0.65)



Flow Diagram for Snako Oil

Do not be put off by the negative contribution margins on some of the variables. That is very common in blending problems, and LP does take them into account appropriately. In this case, Snako is in effect dressing up the very cheap diethylMulehide with high grade components so that they can sell it at a high price. If that isn't profitable, LP will let us know.

So now we have an objective function:

$$\text{Maximize } Z = 1.5\ ZM + .95\ SM + .5\ ZG - .05\ SG - .1\ ZH - .65\ SH$$

The first 3 constraints are not too difficult. We cannot use more of the 3 components of these fuels than we have available:

$$ZM + SM \leq 30000$$

$$ZG + SG \leq 8000$$

$$ZH + SH \leq 10000$$

Delivery promises mean that they must deliver at least 10,000 gallons of Zoom and 25,000 gallons of Slug, which requires:

$$ZM + ZG + ZH \geq 10000$$

$$SM + SG + SH \geq 25000$$

Now comes the hard part: those pesky proportionality constraints

Zoom cannot contain more than 20% diethylMulehide and Slug is limited to no more than 70% diethylMulehide, or

$$ZM \leq .2\ (ZM + ZG + ZH)$$

$$SM \leq .7\ (SM + SG + SH)$$

Zoom needs at least 40% Gruntane and Slug needs at least 5% Gruntane:

$$ZG \geq .4\ (ZM + ZG + ZH)$$

$$SG \geq .05\ (SM + SG + SH)$$

Zoom must contain at least 30% Hoctane and Slug needs at least 20% Hoctane:

$$ZH \geq .3\ (ZM + ZG + ZH)$$

$$SH \geq .2\ (SM + SG + SH)$$

In standard LP formulation, the six proportionality constraints are tortured by algebra into the following equivalent forms:

$$.8\ ZM - .2\ ZG - .2\ ZH \leq 0$$

$$.3\ SM - .7\ SG - .7\ SH \leq 0$$

$$-\ .4\ ZM + .6\ ZG - .4\ ZH \geq 0$$

$$-\ .05\ SM + .95\ SG - .05\ SH \geq 0$$

$$-\ .3\ ZM - .3\ ZG + .7\ ZH \geq 0$$

$$-\ .2\ SM - .2\ SG + .8\ SH \geq 0$$

So the total formulation, when you bring this all together, using standard LP formulation, is:

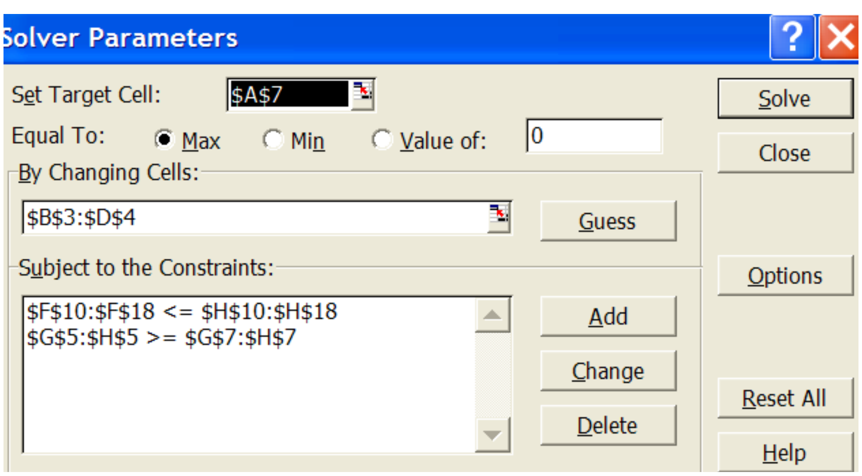
Maximize Z =	1.5 ZM	+.95 SM	+.5 ZG	-.05 SG	-.1 ZH	-.65 SH		
Subject to	ZM	+SM					≤ 30,000	DiethylMulehide
			ZG	+SG			≤ 8,000	Gruntane
					ZH	+SH	≤ 10,000	Hoctane
	ZM		+ZG		+ZH		≥ 10,000	Zoom
		SM		+SG		+SH	≥ 25,000	Slug
	.8 ZM		-.2 ZG		-.2 ZH		≥ 0	≤ 20% M in Zoom
		.3 SM		-.7 SG		-.7 SH	≥ 0	≤ 70% M in Slug
	-.4 ZM		+.6 ZG		-.4 ZM		≥ 0	≥ 40%G in Zoom
		-.05 SM		+.95 SG		-.05 SH	≥ 0	≥ 5% G in Slug
	-.3 ZM		-.3 ZG		+.7 ZH		≥ 0	≥ 30% H in Zoom
		-.2 SH		-.2 SG		+.8 SH	≥ 0	≥ 20 H in Slug

The Excel version on the next page uses a more human-friendly layout using subtraction to compare the each of the six "pipe" activity variables with the calculated total production of the appropriate product in Column E and requiring the excess diethylMulehide and the shortage of Hoctane or Gruntane to be ≥ zero.

Also note the revenue function and the min Slug and min Zoom constraints are at the upper right corner to take advantage of the matrix layout.

	A	B	C	D	E	F	G	H
1	**Snako Oil: No supplementary Variables**							
2		Diethyl-Mulehide	Gruntane	Hoctane	Total Made	Revenue	Min Slug	Min Zoom
3	Slug	23333.33	3000	7000	33333.33	$1.45	1	
4	Zoom	2000	5000	3000	10000	$2.00		1
5	Ttoal	25333.33	8000	10000		$68,333.33	33333.33	10000
6	**Contribution**						>=	>=
7	**$ 22,666.67**						25000	10000
8						Total		
9	Cost	$0.50	$1.50	$2.10		$45,666.67		
10	Max M	1				25333.3333	<=	30000
11	Max G		1			8000	<=	8000
12	Max H			1		10000	<=	10000
13	Excess M in S	1			-70%	0.00	<=	0
14	Short G in S		-1		5%	-1333.33	<=	0
15	Short H in S			-1	20%	-333.33	<=	0
16	Excess M in Z	1			-20%	0.00	<=	0
17	Short G in Z		-1		40%	-1000.00	<=	0
18	Short H in Z			-1	30%	0.00	<=	0

	A	B	C	D	E	F	G	H
1	**Snako Oil: No**							
2		Diethyl-Mulehide	Gruntane	Hoctane	Total Made	Revenue	Min Slug	Min Zoom
3	Slug	23333.333	3000	7000	=SUM(B3:D3)	1.45	1	
4	Zoom	2000	5000	3000	=SUM(B4:D4)	2		1
5	Ttoal	=+B3+B4	=+C3+C4	=+D3+D4		=SUMPRODUCT($E$3:$E$4,F3:F4)	=$E$3*G3+$E$4*G4	=$E$3*H3+$E$4*H4
6	**Contribution**						>=	>=
7	**=F5-F9**						25000	10000
8						Total		
9	Cost	0.5	1.5	2.1		=SUMPRODUCT($B$5:$D$5,B9:D9)		
10	Max M	1				=SUMPRODUCT($B$5:$D$5,B10:D10)	<=	30000
11	Max G		1			=SUMPRODUCT($B$5:$D$5,B11:D11)	<=	8000
12	Max H			1		=SUMPRODUCT($B$5:$D$5,B12:D12)	<=	10000
13	Excess M in S	1			-0.7	=SUMPRODUCT($B$3:$E$3,B13:E13)	<=	0
14	Short G in S		-1		0.05	=SUMPRODUCT($B$3:$E$3,B14:E14)	<=	0
15	Short H in S			-1	0.2	=SUMPRODUCT($B$3:$E$3,B15:E15)	<=	0
16	Excess M in Z	1			-0.2	=SUMPRODUCT($B$4:$E$4,B16:E16)	<=	0
17	Short G in Z		-1		0.4	=SUMPRODUCT($B$4:$E$4,B17:E17)	<=	0
18	Short H in Z			-1	0.3	=SUMPRODUCT($B$4:$E$4,B18:E18)	<=	0



Sensitivity Report:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Slug Diethyl- Mulehide	23333.33333	0	0.95	1E+30	0.553571429
$C$3	Slug Gruntane	3000	0	-0.05	0	2.066666667
$D$3	Slug Hoctane	7000	0	-0.65	1E+30	0
$B$4	Zoom Diethyl- Mulehide	2000	0	1.5	5.166666667	3.166666667
$C$4	Zoom Gruntane	5000	0	0.5	2.066666667	0
$D$4	Zoom Hoctane	3000	0	-0.1	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$10	Max M Total	25333.33333	0	30000	1E+30	4666.666667
$F$11	Max G Total	8000	2.166666667	8000	500	1600
$F$12	Max H Total	10000	1.566666667	10000	2000	1000
$F$13	Excess M in S Total	0.00	3.17	0	500	2500
$F$14	Short G in S Total	-1333.33	0.00	0	1E+30	1333.333333
$F$15	Short H in S Total	-333.33	0.00	0	1E+30	333.3333333
$F$16	Excess M in Z Total	0.00	3.17	0	500	1600
$F$17	Short G in Z Total	-1000.00	0.00	0	1E+30	1000
$F$18	Short H in Z Total	0.00	0.00	0	1333.333333	333.3333333
$G$5	Ttoal Min Slug	33333.33333	0	25000	8333.333333	1E+30
$H$5	Ttoal Min Zoom	10000	-1.033333333	10000	3125	1428.571429

See how the matrix makes the relationships among the six activity variables much clearer for humans. Excel doesn't care.

Microsoft Excel 10.0 Answer Report
Target Cell (Max)

Cell	Name	Original Value	Final Value
$A$7	Contribution	$ 22,666.67	$ 22,666.67

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Slug Diethyl- Mulehide	23333.33333	23333.33333
$C$3	Slug Gruntane	3000	3000
$D$3	Slug Hoctane	7000	7000
$B$4	Zoom Diethyl- Mulehide	2000	2000
$C$4	Zoom Gruntane	5000	5000
$D$4	Zoom Hoctane	3000	3000

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$10	Max M Total	25333.33333	$F$10<=$H$10	Not Binding	4666.666667
$F$11	Max G Total	8000	$F$11<=$H$11	Binding	0
$F$12	Max H Total	10000	$F$12<=$H$12	Binding	0
$F$13	Excess M in S Total	0.00	$F$13<=$H$13	Binding	0
$F$14	Short G in S Total	-1333.33	$F$14<=$H$14	Not Binding	1333.333333
$F$15	Short H in S Total	-333.33	$F$15<=$H$15	Not Binding	333.3333333
$F$16	Excess M in Z Total	0.00	$F$16<=$H$16	Binding	0
$F$17	Short G in Z Total	-1000.00	$F$17<=$H$17	Not Binding	1000
$F$18	Short H in Z Total	0.00	$F$18<=$H$18	Binding	0
$G$5	Ttoal Min Slug	33333.33333	$G$5>=$G$7	Not Binding	8333.333333
$H$5	Ttoal Min Zoom	10000	$H$5>=$H$7	Binding	0

Alternate Solution

Several of the objective coefficients in the sensitivity analysis on the previous page have zero allowable increase or allocable decrease. This is a sign that, while there can be no solution better than the one above, there may be solutions just as good. Tricking Excel into giving this to you is not easy, but here's an example of another solution that satisfies all the constraints and is just as good as the one above. Note that the total Slug and Zoom produced and the total of each ingredient consumed are the same; only the recipes differ.

	A	B	C	D	E	F	G	H
1	**Snako Oil: No supplementary Variables**							
2		Diethyl-Mulehide	Gruntane	Hoctane	Total Made	Revenue	Min Slug	Min Zoom
3	Slug	23333.33	3333.333	6666.67	33333.33	$1.45	1	
4	Zoom	2000	4666.667	3333.33	10000	$2.00		1
5	Ttoal	25333.33	8000	10000		$ 68,333.33	33333.33	10000
6	**Contribution**						>=	>=
7	**$ 22,666.67**						25000	10000
8						Total		
9	Cost	$0.50	$1.50	$2.10		$45,666.67		
10	Max M	1				25333.3333	<=	30000
11	Max G		1			8000	<=	8000
12	Max H			1		10000	<=	10000
13	Excess M in S	1			-70%	0.00	<=	0
14	Short G in S		-1		5%	-1666.67	<=	0
15	Short H in S			-1	20%	0.00	<=	0
16	Excess M in Z	1			-20%	0.00	<=	0
17	Short G in Z		-1		40%	-666.67	<=	0
18	Short H in Z			-1	30%	-333.33	<=	0

Supplementary Variables

The previous formulation will give you the values of decision variables that optimize the model, The problem is in interpreting the results. The sensitivity analysis on objective coefficients for the no-supplementary variables version is almost totally useless, because there is no plausible way that just one of the six "pipe" variables could change without one or more others changing too, since what would really be changing is the cost of an ingredient or the price of a product.

The solution is to use supplementary variables. We create five more decision variables, one for each tank. These variables are constrained to e equal to the calculated consumption of the corresponding ingredient or the calculated production of each grade of gasoline.

Zoom = ZM + ZG + ZH
Slug = SM + SG + SH

Mule = ZM + SM
Grunt = ZG + SG
Hoct = ZH + SH

To make the magic work, we also need to restate the cost and revenue in terms of the supplementary variables instead of the corresponding calculated amounts.

A good rule of thumb is to solve the problem both with and without supplementary variables if you have any doubt that you have "tied down" the supplementary variables properly.

There's another bonus when we re-formulate the problem using supplementary variables: the proportionality constraints can be expressed in a way that is much clearer to the human mind, and therefore less prone to formulation error. To express the constraint that the DiethylMulehide in Zoom cannot be more than 20% of the total composition of Zoom, we only need to write ZM $\leq$ .2 ZOOM

Similarly, the requirement that the DiethylMulehide in Slug cannot be more than 70% of the total composition of Slug is written SM $\leq$ .7 SLUG

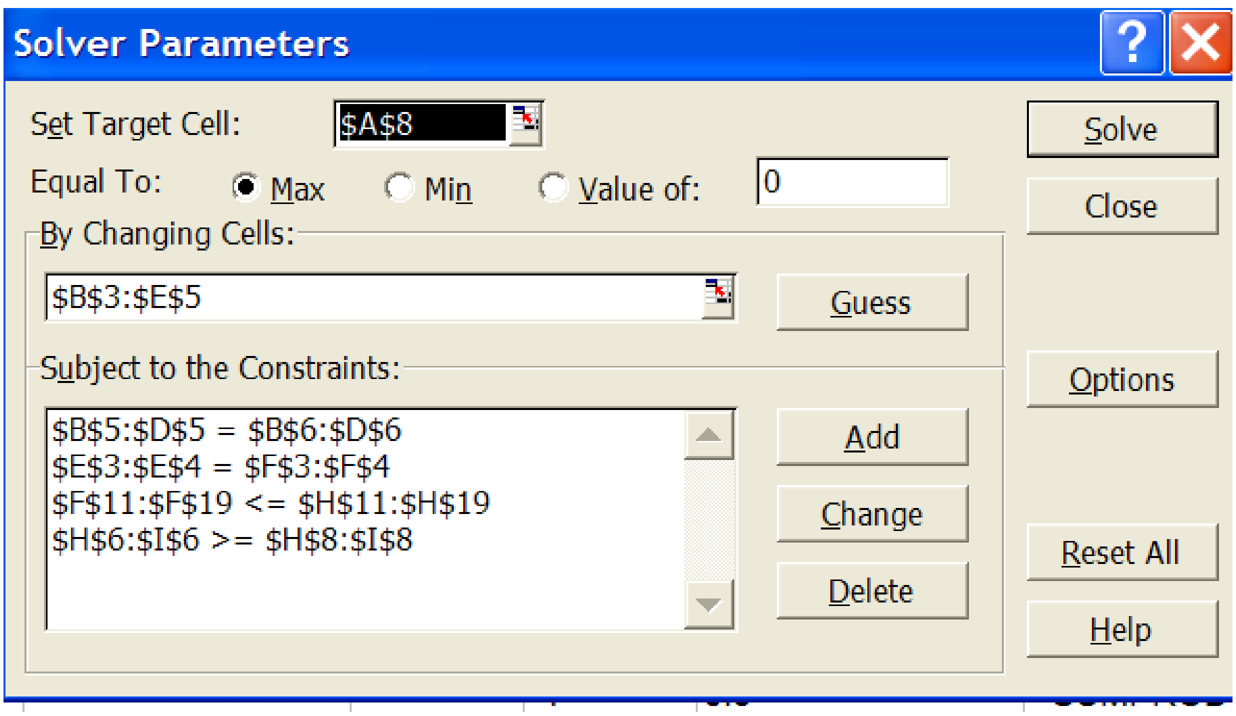
The other four proportionality constraints, in logical form and computer-input form are

```
ZG ≥  .4 ZOOM
SG ≥ .05 SLUG
ZH ≥  .3 ZOOM
SH ≥  .2 SLUG
```

Supplementary variables often simplify the objective function as well. In the case of Snako, we can tie the revenues to the products ZOOM and SLUG, and tie the costs to the resource variables MULE, GRUNT, and HOCT. The summary formulation of the extended problem appears on the next page.

The supplementary variable "SuppTotal Supp Total Made" is only there to square off the matrix; it has no other function so Excel just leaves it at zero.

	A	B	C	D	E	F	G	H	I
1	**Snako Oil: Supplementary Variables**								
2		Diethyl-Mulehide	Gruntane	Hoctane	SuppTotal Made	CalcTotal Made	Revenue	Min Slug	Min Zoom
3	Slug	23333.33	3000	7000	33333.333	33333.3333	$1.45	1	
4	Zoom	2000	5000	3000	10000	10000	$2.00		1
5	Supp Total	25333.33	8000	10000	0				
6	Calc Total	25333.33	8000	10000			$ 68,333.33	33333.3	10000
7	**Contribution**							>=	>=
8	**$ 22,666.67**							25000	10000
9						Total			
10	Cost	$0.50	$1.50	$2.10		$45,666.67			
11	Max M	1				25333.3333	<=	30000	
12	Max G		1			8000	<=	8000	
13	Max H			1		10000	<=	10000	
14	Excess M in S	1			-70%	-1.819E-11	<=	0	
15	Short G in S		-1		5%	-1333.3333	<=	0	
16	Short H in S			-1	20%	-333.33333	<=	0	
17	Excess M in Z	1			-20%	0	<=	0	
18	Short G in Z		-1		40%	-1000	<=	0	
19	Short H in Z			-1	30%	0	<=	0	



Microsoft Excel 10.0 Sensitivity Report
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Slug Diethyl- Mulehide	23333.33333	0	$ -	1E+30	0.553571429
$C$3	Slug Gruntane	3000	0	$ -	0	2.066666667
$D$3	Slug Hoctane	7000	0	$ -	1E+30	0
$E$3	Slug SuppTotal Made	33333.33333	0	$ 1.45	1E+30	0.3875
$B$4	Zoom Diethyl- Mulehide	2000	0	$ -	5.166666666	3.166666667
$C$4	Zoom Gruntane	5000	0	$ -	2.066666667	0
$D$4	Zoom Hoctane	3000	0	$ -	0	1E+30
$E$4	Zoom SuppTotal Made	10000	0	$ 2.00	1.033333333	1E+30
$B$5	Supp Total Diethyl- Mulehide	25333.33333	0	$ (0.50)	1E+30	0.62
$C$5	Supp Total Gruntane	8000	0	$ (1.50)	1E+30	2.166666667
$D$5	Supp Total Hoctane	10000	0	$ (2.10)	1E+30	1.566666667
$E$5	Supp Total SuppTotal Made	0	0	$ -	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Max M Total	25333.33333	0	30000	1E+30	4666.666667
$F$12	Max G Total	8000	2.166666667	8000	500	1600
$F$13	Max H Total	10000	1.566666667	10000	2000	1000
$F$14	Excess M in S Total	-1.81899E-11	3.166666667	0	500	2500
$F$15	Short G in S Total	-1333.333333	0	0	1E+30	1333.333333
$F$16	Short H in S Total	-333.3333333	0	0	1E+30	333.3333333
$F$17	Excess M in Z Total	0	3.166666667	0	500	1600
$F$18	Short G in Z Total	-1000	0	0	1E+30	1000
$F$19	Short H in Z Total	0	0	0	1333.333333	333.3333333
$H$6	Calc Total Min Slug	33333.33333	0	25000	8333.333333	1E+30
$I$6	Calc Total Min Zoom	10000	-1.033333333	10000	3125	1428.571429
$B$5	Supp Total Diethyl- Mulehide	25333.33333	-0.5	0	4666.666667	25333.33333
$C$5	Supp Total Gruntane	8000	-3.666666667	0	1600	500
$D$5	Supp Total Hoctane	10000	-3.666666667	0	1000	2000
$E$3	Slug SuppTotal Made	33333.33333	3.666666667	0	500	2500
$E$4	Zoom SuppTotal Made	10000	3.666666667	0	500	1600

Note how this analysis puts the nonzero objective coefficients on the tanks which Snako incurs cost to refill or gains revenue by selling from, rather than the pipes which are of purely internal concern.

THE SNAKO RESULTS

Now let's see if we can answer a few questions that LeRoy, the high muckety muck at the Snake Oil Company might want answers to. Leroy's questions are:

1. How much of each product should we make this week? How much money will we earn from doing that?
2. One of our dealers wants an extra 1000 gallons of Zoom this week. Purely on a short term profit basis, should we oblige the dealer? Can we? How will that hit the bottom line?
3. Our petrochemical supplier is overloaded with Gruntane today, and has offered to bring us an extra 2000 gallons at our usual cost with no special-delivery charge. Should we take it? Would we gain any profit by doing so? How much?
4. The same petrochemical supplier is also overloaded with diethylMulehide today, and has offered to bring us an extra 5000 gallons at our usual cost instead of Gruntane with no special-delivery charge. Should we take it? Would we gain any profit by doing so? How much?
5. I took a little LP when I was majoring in Party Dynamics at UGA. Looking at the printout with supplementary variables, I see that the constraint "Excess M in S Total" has a shadow price of $3.17. Doesn't that present some kind of opportunity for us?
6. And while we're at it, the constraint "Zoom Supp Total Made" has a $3.67 shadow price. Shouldn't we do something about that?

Try to answer these questions using the outputs. No peeking allowed, but after you have written your answers, you'll find mine on the next page.

Here are my answers. If you really answered the questions, you probably found that the extra effort to include the supplementary variables was worth it in this case.

1. You should blend 10,000 gallons of Zoom and $33{,}333\frac{1}{3}$ gallons of Slug. I really can't tell you what your ***profit*** will be without getting all of the details of your accounting system and running a *pro forma* income statement, but this course of action should contribute $22666.67 toward overhead and profit.
2. The RHS ranging for the contract constraint tells me that you <u>can</u> blend up to 3125 more gallons of Zoom this week, but the shadow price tells me that each extra gallon of Zoom you force into your mix will cut total contribution by $1.033. Dealer relations are a lot more important in the long run than this week's bottom line. Is it worth $1033 to you to keep the dealer happy? If so, do it. I'll be happy to rerun the LP program so you'll know your new mix.
3. The marginal value to you of an extra gallon of Gruntane is $2.167, but the range on the Gruntane constraint only goes up to 8,500 gallons. If you don't mind storing an excess of 1500 gallons 'till next week, go for it. If you plan to both buy the extra Gruntane *and* please your dealer, I'd have to rerun the problem to tell you the effect. My guess is that the 2 changes would about counterbalance each other in their profit contribution effect.
4. You are already going to have $4{,}666\frac{2}{3}$ gallons of diethylMulehide left over. I don't see much point in getting more this week.
5. The constraint "Excess M in S Total" is a proportionality constraint, LeRoy. All that shadow price tells you is that it costs something to meet quality standards for Zoom, which you probably already guessed.
6. The constraint "Zoom Supp Total Made" defines the supplementary variable ZOOM in terms of its components ZM, ZG, and ZH. Trying to work with a shadow price on that is like asking "If a horse had wings, would that make it a horsefly?".

Multiperiod Problems

Some problems involve decisions that are made repeatedly over multiple time periods. Given projected enrollments and counselor attrition, how many Incept Counselors must we train each Quarter over the next year? Given a very seasonal sales forecast with prices and costs varying month by month, how many Widgets must we make each month for the next year? These are multiperiod problems.

A multiperiod problem requires a large number of activity variables, and there is no short cut for that. Let's consider Widgets, assuming the problem starts with January. The greatest number of Widgets we can sell in March will be our March manufacturing capacity, plus the Widgets made in January for sale in March, plus the Widgets made in February for sale in March, less the number of Widgets we make in March for April sale, the number of Widgets we make in March for May sale, and so forth.

Every multiperiod problem is a little different, but they all require you to ask what is or is not carried forward from period to period. Depending on your conclusion, you may need to define a few or a great many variables.

Case 9: EcoWagen

EgoWagen Gmbh has asked for your help in planning production of their two models for the next two quarters. The Jetson, is a four door car whose market segment is Grumpies (Grownup Upwardly Mobile Pretenders) mit kinder. It wholesales to their US distributor for $15,000. The GIT is a shorter version of the same car with a better suspension, more powerful motor, and even more pretensions, aimed at those who cannot simultaneously afford gold chains and Porsches. Its wholesale price is $18,000. Both prices are F.O.B. Nuttgart.

Their sales forecast for the next two quarters shows the following demand levels, given their current marketing plans:

	GIT	Jetson
Fall	2,000	5,000
Winter	2,500	8,000

Unfortunately, their demand will exceed their manufacturing capacity of 10000 units/Quarter in Winter quarter. One partial solution may be to build cars in advance. Storage and interest charges amount to $1000/quarter for each quarter a car is stored. Thus a car made in the fall would cost them an extra $3000 if held for summer sale. They have a two-quarter planning horizon; for simplicity, we do not want any cars left over at the end of the horizon, so we have no interest in planning cars for sale beyond whiter quarter. In a going concern, we would re-run the model at the end of fall and think about spring then.

A further restriction arises from their labor contract. The GIT is more time consuming and difficult to make than the Jetson. As a result, they have agreed that in any period, GITs will constitute no more than 30% of their production. Incremental production cost for a GIT is $9000, while that cost for a Jetson is $8000. Incremental production cost for a GIT is $9000, while that cost for a Jetson is $8000. What they want from you, at the very least, is a production schedule for the year. Further insights would be nice.

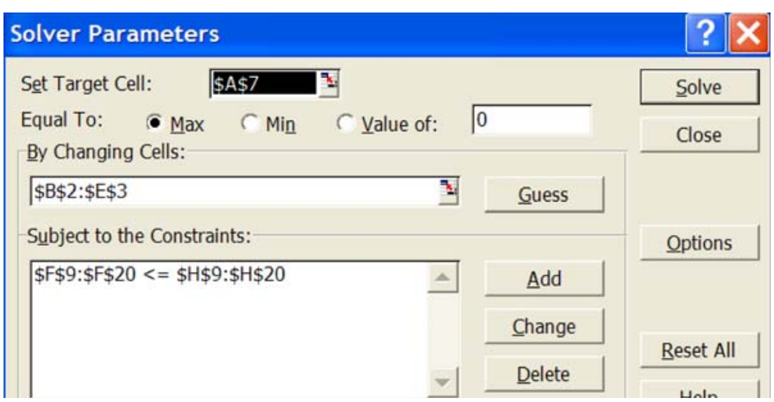
Solution:

The key to solving this problem is in recognizing that the decision what to produce and the decision what to release to the market are two different decisions that have to be made for each model each quarter, so there are eight decision variables in all.

There are four groups of constraints (plus non-negativity):

1. Cells F9 and F10 calculate the total number of cars made each quarter, each of these has to be less than or equal to the production capacity for that quarter, which is specified at 10,000 cars in cells H9 and H10.
2. Cells F11:F14 calculate the total number of cars sold by model and quarter. They need to be less than or equal to their respective maximum demand by model and quarter (cells H11:H14).
3. Cells F15:F18 are for human convenience, since they calculate the same numbers as cells F11:F14. What is necessary is that these numbers, i.e. the number of cars sold by model and quarter, can't be more than the supply of cars available for sale in that quarter. The supply of cars available for sale is found in cells F2:G3 based on the number of that model car made that quarter plus the number left in inventory at the end of the previous quarter.
4. Cells F19:F20 represent the proportionality constraints imposed by the labor union contract using the standard LP formulation. If MakeGit has to be less than or equal to 70% of (MakeGit + MakeJet), then -.3*MakeJet +.7*MakeGit has to be less than or equal to zero.

	A	B	C	D	E	F	G	H	I
1	EcoWagen	MakeGit	MakeJet	SellGit	SellJet	AvailGit	AvailJet	StoreGit	StoreJet
2	Fall	2000	5500	2000	5000	2000	5500	0	500
3	Winter	2500	7500	2500	8000	2500	8000	0	0
4	Total	4500	13000	4500	13000			0	500
5	Dollars	-$9,000	-$8,000	$18,000	$15,000			-$1,000	-$1,000
6	Total Contribution								
7	$131,000,000								
8						Total			
9	Fall capacity	1	1			7500	<=	10000	
10	Winter capacity	1	1			10000	<=	10000	
11	Fall Git Demand			1		2000	<=	2000	
12	Winter Git Demand			1		2500	<=	2500	
13	Fall Jet Demand				1	5000	<=	5000	
14	Winter Jet Demand				1	8000	<=	8000	
15	Fall Git Supply			1		2000	<=	2000	
16	Winter Git Supply			1		2500	<=	2500	
17	Fall Jet Supply				1	5000	<=	5500	
18	Winter Jet Supply				1	8000	<=	8000	
19	Fall excess Git	0.7	-0.3			-250	<=	0	
20	Winter excess Git	0.7	-0.3			-500	<=	0	



Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Fall MakeGit	2000	0	-11000	0	9000
$C$2	Fall MakeJet	5500	0	-10000	1000	0
$D$2	Fall SellGit	2000	0	20000	1E+30	9000
$E$2	Fall SellJet	5000	0	17000	1E+30	7000
$B$3	Winter MakeGit	2500	0	-10000	11000	0
$C$3	Winter MakeJet	7500	0	-9000	0	1000
$D$3	Winter SellGit	2500	0	19000	1E+30	8000
$E$3	Winter SellJet	8000	0	16000	1E+30	6000

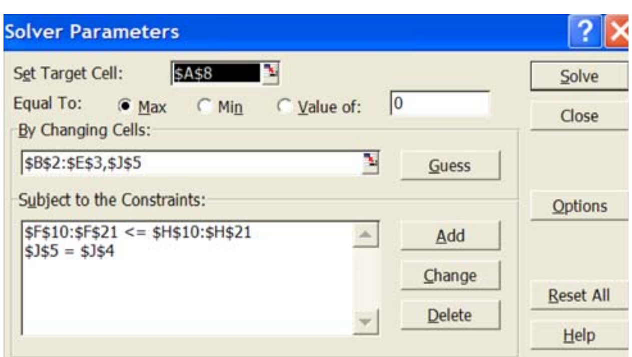
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$9	Fall capacity Total	7500	$0.00	10000	1E+30	2500
$F$10	Winter capacity Total	10000	$1,000.00	10000	500	1666.666667
$F$11	Fall Git Demand Total	2000	$9,000.00	2000	357.1428571	2000
$F$12	Winter Git Demand Total	2500	$8,000.00	2500	500	500
$F$13	Fall Jet Demand Total	5000	$7,000.00	5000	2500	833.3333333
$F$14	Winter Jet Demand Total	8000	$6,000.00	8000	2500	500
$F$15	Fall Git Supply Total	2000	$0.00	0	500	250
$F$16	Winter Git Supply Total	2500	$11,000.00	0	500	500
$F$17	Fall Jet Supply Total	5000	$0.00	0	1E+30	500
$F$18	Winter Jet Supply Total	8000	$10,000.00	0	500	2500
$F$19	Fall excess Git Total	-250	$0.00	0	1E+30	250
$F$20	Winter excess Git Total	-500	$0.00	0	1E+30	500

The objective coefficient column of the top part of the sensitivity analysis looks rather mysterious. This is because the cost of storing cars in inventory is not represented directly, so it ends up hidden in the other objective coefficients. The solution is to introduce one supplementary variable, called Supp Total Stored. When this is introduced, the objective coefficients are easily matched with the dollar amounts in the problem statement.

	A	B	C	D	E	F	G	H	I	J
1	EcoWagen	MakeGit	MakeJet	SellGit	SellJet	AvailGit	AvailJet	StoreGit	StoreJet	
2	Fall	2000	5500	2000	5000	2000	5500	0	500	
3	Winter	2500	7500	2500	8000	2500	8000	0	0	
4								Calc Total Stored		500
5	Total	4500	13000	4500	13000			Supp Total Stored		500
6	Dollars	-$9,000	-$8,000	$18,000	$15,000					-$1,000
7	Total Contribution									
8	$131,000,000									
9						Total				
10	Fall capacity	1	1			7500	<=	10000		
11	Winter capacity	1	1			10000	<=	10000		
12	Fall Git Demand			1		2000	<=	2000		
13	Winter Git Demand			1		2500	<=	2500		
14	Fall Jet Demand				1	5000	<=	5000		
15	Winter Jet Demand				1	8000	<=	8000		
16	Fall Git Supply			1		2000	<=	2000		
17	Winter Git Supply			1		2500	<=	2500		
18	Fall Jet Supply				1	5000	<=	5500		
19	Winter Jet Supply				1	8000	<=	8000		
20	Fall excess Git	0.7	-0.3			-250	<=	0		
21	Winter excess Git	0.7	-0.3			-500	<=	0		

	A	B	C	D	E	F	G	H	I	J
1	EcoWagen	MakeGit	MakeJet	SellGit	SellJet	AvailGit	AvailJet	StoreGit	StoreJet	
2	Fall	2000	5500	2000	5000	=B2	=C2	=B2-D2	=C2-E2	
3	Winter	2500	7500	2500	8000	=B3+H2	=C3+I2	=H2+B3-D3	=I2+C3-E3	
4									Calc Total Stored	=SUM(H2:I3)
5	Total	=SUM(B2:B3)	=SUM(C2:C3)	=SUM(D2:D3)	=SUM(E2:E3)				Supp Total Stored	499.9999999999
6	Dollars	-9000	-8000	18000	15000					-1000
7	Total Contribution									
8	=SUMPRODUCT(B5:J5,B6:J6)									
9						Total				
10	Fall capacity	1	1			=SUMPRODUCT(B2:E2,B10:E10)	<=	10000		
11	Winter capacity	1	1			=SUMPRODUCT(B3:E3,B11:E11)	<=	10000		
12	Fall Git Demand			1		=SUMPRODUCT(B2:E2,B12:E12)	<=	2000		
13	Winter Git Demand			1		=SUMPRODUCT(B3:E3,B13:E13)	<=	2500		
14	Fall Jet Demand				1	=SUMPRODUCT(B2:E2,B14:E14)	<=	5000		
15	Winter Jet Demand				1	=SUMPRODUCT(B3:E3,B15:E15)	<=	8000		
16	Fall Git Supply			1		=SUMPRODUCT(B2:E2,B16:E16)	<=	=F2		
17	Winter Git Supply			1		=SUMPRODUCT(B3:E3,B17:E17)	<=	=F3		
18	Fall Jet Supply				1	=SUMPRODUCT(B2:E2,B18:E18)	<=	=G2		
19	Winter Jet Supply				1	=SUMPRODUCT(B3:E3,B19:E19)	<=	=G3		
20	Fall excess Git	0.7	-0.3			=SUMPRODUCT(B2:E2,B20:E20)	<=	0		
21	Winter excess Git	0.7	-0.3			=SUMPRODUCT(B3:E3,B21:E21)	<=	0		



Sensitivity Report:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Fall MakeGit	2000	0	-9000	0	9000
$C$2	Fall MakeJet	5500	0	-8000	1000	0
$D$2	Fall SellGit	2000	0	18000	1E+30	9000
$E$2	Fall SellJet	5000	0	15000	1E+30	7000
$B$3	Winter MakeGit	2500	0	-9000	11000	0
$C$3	Winter MakeJet	7500	0	-8000	0	1000
$D$3	Winter SellGit	2500	0	18000	1E+30	8000
$E$3	Winter SellJet	8000	0	15000	1E+30	6000
$J$5	Supp Total Stored	500	0	-1000	1000	6000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$10	Fall capacity Total	7500	0	10000	1E+30	2500
$F$11	Winter capacity Total	10000	1000	10000	500	1666.666667
$F$12	Fall Git Demand Total	2000	9000	2000	357.1428571	2000
$F$13	Winter Git Demand Total	2500	8000	2500	500	500
$F$14	Fall Jet Demand Total	5000	7000	5000	2500	833.3333333
$F$15	Winter Jet Demand Total	8000	6000	8000	2500	500
$F$16	Fall Git Supply Total	2000	0	0	500	250
$F$17	Winter Git Supply Total	2500	11000	0	250	500
$F$18	Fall Jet Supply Total	5000	0	0	1E+30	500
$F$19	Winter Jet Supply Total	8000	10000	0	250	2500
$F$20	Fall excess Git Total	-250	0	0	1E+30	250
$F$21	Winter excess Git Total	-500	0	0	1E+30	500
$J$5	Supp Total Stored	500	-1000	0	1E+30	500