Single and Multiple Blending Problems

Pharr Owt: a simple minimization problem

Pharr Owt, an old friend of yours, has just been made Warden/Headmaster of the South Florida Prison and Institute of Chemical Technology. She is concerned about the nutritional state of her new charges, and would like to start by improving their breakfasts. She must buy food from the state prison commissary system. The only breakfast foods they offer are oatmeal and eggs. Pharr is on a tight budget, and would like to meet the PICT inmates' nutritional needs as cheaply as possible.

She has 1,000 inmates to feed. Even though on a given day not everyone will eat whatever combination she buys, she is willing to assume that over the long haul it will even out. She has worked out this information:

	One Egg	One Cup Oatmeal	Minimum per Inmate
Fat	5	2	8 fat units
Fiber	0	6	8 fiber units
Protein	5	4	12 protein units
Cost	$0.10	$0.05	

With 1000 inmates, her question is simply this: what combination of oatmeal and eggs will give these inmates at least the required nutrition at least cost?

Defining the activity variables is a critical issue in formulating LP problems. Fortunately it is pretty easy in this case. Let OAT be the number of cups of oatmeal to provide 1000 prisoners each day, and EGG be the number of eggs to provide those 1000 unfortunates. She'll let the kitchen staff worry about whether inmates choose between eggs and oatmeal or just have a porridge.

To meet the nutritional requirements, she must provide no less than 8000 (8×1000) units each of fat and fiber, and at least 12,000 units of protein. These are minimum performance requirements, so they must be $\geq$ constraints. The fact that Pharr wants to meet these requirements at *least cost* implies that this is a cost minimization problem. The purchase prices are fixed and stable, so there is nothing tricky about the objective function. Working together, you agree that this is the problem:

```
Minimize Z =   .10  EGG  + .05 OAT
subject to:   5 EGG      +   2 OAT   >=   8,000    Fat
                             6 OAT   >=   8,000    Fiber
              5 EGG      +  4 OAT    >=  12,000    Protein
```

Setting up to graph the constraints yielded this:

Constraint	Connect these points to draw the boundary line (EGG, OAT)	Test these points to find the feasible side of the line (EGG, OAT)	RHS	Result
Fat	(1600. 0) (0, 4000)	(2000, 2000)	14,000	feasible
Fiber	$OAT = 1333\frac{1}{3}$ (horizontal)	(2000, 2000)	12,000	feasible
Protein	(2400, 0) (0, 3000)	(2000, 2000)	18,000	feasible

You and Pharr used this information to draw the graph. As a first try, you looked, optimistically (you thought) at a $200/day isocost line.

$$.10\ EGG + .05\ OAT = 200$$

$$EGG = 0,\ OAT = 4000$$

$$OAT = 0,\ EGG = 2000$$

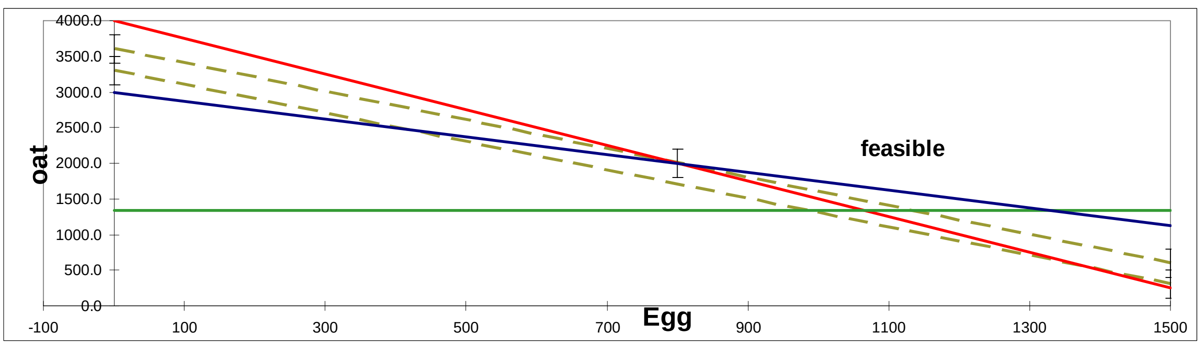
Amazingly, there was room to drop it more. Your next guess was $180/day.

$$.10\ EGG + .05\ OAT = 180$$

$$EGG = 0,\ OAT = 3600$$

$$OAT = 0,\ EGG = 1800$$

To your amazement and amusement, you hit it dead on the money on your second try. $180 per day buys the inmates all the nutritional breakfast they can stand. Pharr Owt needs to order up 2000 cups of oatmeal and 800 eggs for her flock each day. That will exactly meet the requirements for fat and protein, and more than meet the fiber requirement. You're getting pretty slick at this LP stuff!



Comment

Having put all this effort into graphical solutions of LP problems, it would be unfair not to acknowledge we *never* solve real problems graphically. The graphical solution is a very useful pedagogical device, and an instrument for understanding both the basics and some of the nuances of LP. From here on out (after we master this chapter), we'll be solving problems using the Simplex Method on the computer. Even so, when you encounter something you don't quite understand, you might benefit from coming back to the graphical solution to see if, just maybe, it can help you to close the gap. And if the problem concerning you has only 2 activity variables, of course you can graph the particular problem.

Pharr Owt's Problem by Computer

Let's see if you can come up with the same answers to my questions that I do, using the EXCEL output. Some of the answers, of course, we already know from the graphical solution.

1. What should Pharr give the inmates for breakfast?
2. What will that cost per day?
3. Are they getting more than the minimum of any nutrient? If so, which one(s)?
4. Other than saving some money, would there be any effect on the solution if Pharr found she could get oatmeal for 3¢ per cup? What would happen?
5. Other than costing some money, would there be any effect if Pharr found out that the price of eggs had gone up to 11¢ each? What would happen?
6. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Fiber standard for 1000 prisoners is a minimum of 12,000 units per day. How much more (or less) will this cost daily?
7. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Protein standard for 1000 prisoners is a minimum of 15,000 units per day. How much more (or less) will this cost daily?
8. Prices haven't changed. Suppose the Florida Bureau of Prisons tells her that the new Fat standard for 1000 prisoners is a minimum of 7,000 units per day. How much more (or less) will this cost daily?

Pharr Owt's Summary Output

```
MIN      .05 OAT + .1 EGG
 SUBJECT TO
           2 OAT + 5 EGG >=   8000
           6 OAT         >=   8000
           4 OAT + 5 EGG >=   12000
   OBJECTIVE FUNCTION VALUE      180.000000
                           RANGES IN WHICH THE BASIS IS UNCHANGED:
                                                      OBJ COEFFICIENT RANGES
VARIABLE VALUE    REDUCED   VARIABLE    CURRENT    ALLOWABLE     ALLOWABLE
                   COST                 COEFF      INCREASE      DECREASE
OAT      2000    .000000   OAT        .050000     .030000       .010000
EGG       800    .000000   EGG        .100000     .025000       .037500

                                                     RIGHTHAND SIDE RANGES
    SLACK OR     DUAL             CURREN     ALLOWABLE     ALLOWABLE
     SURPLUS     PRICES            RHS        INCREASE      DECREASE
       .0      -.0150            8000.0     1333.333      2000.000
   4000.0       .0000            8000.0     4000.000       INFINITY
       .0      -.0050           12000.0     4000.000      1333.333
```

Pharr Owt's EXCEL Output

Eggs	Oatmeal			
800	2,000			
0.1	0.05	180	Cost	
0	2	8,000	Fat	8,000
0	6	12,000	Fiber	8,000
5	4	12,000	Protein	12,000
1	0	800	NNEgg	0
0	1	2,000	NNOat	0

Answer Report						
Target Cell (Min)						
	Cell	Name	Original Value	Final Value		
	$C$3	Cost	0	180		
Adjustable Cells						
	Cell	Name	Original Value	Final Value		
	$A$2	Eggs	0	800		
	$B$2	Oatmeal	0	2,000		
Constraints						
	Cell	Name	Cell Value	Formula	Status	Slack
	$C$5	Fat	8,000	$C$5>=$E$5	Binding	0
	$C$6	Fiber	12,000	$C$6>=$E$6	Not Binding	4,000
	$C$7	Protein	12,000	$C$7>=$E$7	Binding	0
	$C$8		800	$C$8>=$E$8	Not Binding	800
	$C$9		2,000	$C$9>=$E$9	Not Binding	2,000

Sensitivity Report							
Changing Cells							
	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	$A$2	Eggs	800	0	0.1	0.03	0.04
	$B$2	Oatmeal	2,000	0	0.05	0.03	0.01
Constraints							
	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	$C$5	Fat	8,000	0.02	8,000	1,333.33	2,000
	$C$6	Fiber	12,000	0	8,000	4,000	1E+30
	$C$7	Protein	12,000	0.01	12,000	4,000	1,333.33
	$C$8		800	0	0	800	1E+30
	$C$9		2,000	0	0	2,000	1E+30

Answers:

1. Feed them 2000 cups of oatmeal and 800 eggs per day
2. It will cost $180 per day
3. Yes, they are getting 4000 units more fiber than required.
4. 3¢ per cup is below the 4¢ lower limit for oatmeal in the objective function ranging. That tells us that a different intersection of constraints would become optimal, providing a new and different solution (nothing but oatmeal - yum!)
5. If the price of eggs goes up 1¢ per egg, that remains within the objective function coefficient range for eggs. She pays more, but gives the same breakfast.
6. Pharr is one step ahead of the Bureaucrats. The 4000 unit surplus of fiber means she is already meeting this standard.
7. This *tightens* the Protein standard by 3000 units. The shadow price for the Protein constraint is $0.005, and a 3000 unit increase is within the RHS range for this constraint. The daily cost of breakfast increases by 3000 × $0.005 = $15.00. She must rerun the problem to find out what $195 breakfast will meet this standard.
8. This change *relaxes* the fat constraint by 1000 units. The shadow price for the Fat constraint is $0.015, and a 1000 unit decrease is within the RHS range for this constraint. The daily cost of breakfast decreases by 1000 × $0.015 = $15.00. She must rerun the problem to find out what $165 breakfast will meet this standard.

Formulation hints

We have already had a look at some really important formulation issues. These include

- Limit your activity variables to things that the decision maker can change directly and needs to make a decision on
- *Define* your activity variables, don't just describe them
- Use incremental future cash flows in the objective function
- If any doubt exists *at all* about your formulation, conduct dimensional analysis

The first of these deserves to be expanded on. Many beginners at LP would take a problem such as Beau Jarble's and do things that, once they are experienced LP modelers, would amaze them as being incredibly naive. Like what? The classic mistake is to make things like lathe time, hole drilling capacity, and so forth into activity variables.

But they are already set. Beau is <u>not</u> looking to make a decision about whether to ask the drill press operator to work extra hours. After the problem is solved, Beau might see a threat or an opportunity there, and then raise the issue. But for now, he's assuming the operator will work the 10 hours that we began by assuming. Beau's focus is directly on the things that make revenue. Wheels.

There are some other situations that commonly trip up beginners. Let's look at a few of the most common ones.

PROPORTIONALITY CONSTRAINTS

Suppose that Beau had said that he wanted to make at least twice as many of the ITS model as of the ITA. A common (wrong) way to formulate that would be to say

$$\text{ITA} \leq 50$$

What's wrong with that? Several things. First, it combines two restrictions (no more that 100 ITS wheels and at least twice as many ITS wheels as ITA's). This can, and usually will, blur both restrictions. Second, it <u>assumes</u>, before solving the problem, that the answer will include 100 ITS wheels. Until we solve, we do *not* know that. Third, it can easily fail to enforce this new restriction. We could easily end up with the solution ITA = 50, ITS = 80. LP can't know that this is wrong, but you can bet that Beau knows.

The right way to develop the constraint is to write an algebraic expression that captures exactly what Beau said, then rearrange it if necessary. Beau said

$$\text{ITS} \geq 2\ \text{ITA}$$

This doesn't look like a regular LP constraint (it has a variable, but no number, in the RHS). But look what happens if we subtract 2 ITA from both sides.

$$\text{ITS} - 2\ \text{ITA} \geq 0$$

This may be less intuitively appealing to look at, but it will enforce exactly the restriction Beau wanted.

Let's consider another case. Suppose Pharr had said that she feared the effects of cholesterol in eggs, and as a result wanted to spend no more than half her budget on eggs. Again, remember that at the time we get this information, we do not know how the solution will come out. Even if we did know, most beginners would get stuck trying to figure out how to limit expenditures on eggs to $90 or less (which is not, anyway, what Pharr requested!)

To do it right, we do something truly amazing. We write down what she said, algebraically. That isn't as hard as you might think. We just do it in thoughtful stages. She said

$$\text{Amount spent on eggs} \leq .5 \times \text{Total amount spent}$$

The amount spent on eggs must be .10 EGG. The total amount spent is going to be .05 OAT + .10 EGG. So that must mean that

$$.10 \text{ EGG} \leq .5 (.05 \text{ OAT} + .10 \text{ EGG})$$

Just to make this a little easier to work with (we don't really need to do this), let's multiply *both* sides by 2. That gives us

$$.20 \text{ EGG} \leq .05 \text{ OAT} + .10 \text{ EGG}$$

We still need to put it into standard LP format. As with the Beau example, we manage that by subtracting the RHS from both sides, which gives us

$$- .05 \text{ OAT} + .10 \text{ EGG} \leq 0$$

Most of us, now that it is in front of our faces, can see that as making sense. Many fewer of us could have arrived at that intuitively. I know I couldn't.

EQUALITY CONSTRAINTS

We have said very little about constraints that are equations instead of inequalities. That is because although LP has little trouble digesting them, they usually are ***misteaks***. For example, Beau said that he didn't think the market would take more than 100 ITS wheels. A rookie might easily have written that constraint as ITS = 100. In this instance, we would get the same answer; the only damage would have been to the sensitivity analysis. But it still isn't what Beau told us.

The more "equal to" constraints you put into a problem, the more closely the problem represents simultaneous equations. Simultaneous equations don't always have solutions! An equality constraint risks making the feasible solution space nonexistent. When an LP program tells you "NO FEASIBLE SOLUTION EXISTS", the first place to look for the source of the trouble is equality constraints.

I have never seen an equality constraint that was truly needed, with the exception of one kind. That kind is when you want to define one variable in terms of others. That can express a physical law (2 HYDROGEN + OXYGEN = WATER) or it can define a supplementary variable (ITA + ITS = WHEELS) if Beau wanted a separate variable to count wheels made.

Multiple Blending Problems

In most problems, including resources among the activity variables is a serious error, as we just noted in discussing Beau's hole drilling. But there is one special category of problems that breaks that rule. To properly set up a **multiple blending problem**, you absolutely must use that sort of mixed variable. Be cautious - many problems are called blending problems that are not full-fledged multiple blending problems.

A blending problem is one in which 2 or more ingredients are blended in different ways to form 2 or more end products. If you have a common list of ingredients used in different proportions to make 3 brands of steak sauce, you probably have a blending problem. If you have a list of petrochemicals and additives that are blended together to make NoxaLot, GutLess, and MeDiocre grades of gasoline, you almost surely have a blending problem. *If the exact proportions of the ingredients required to make each product are fixed and known, it is* ***not*** *a* ***true*** *blending problem!*

If a bottle of DeadCow barbecue sauce calls for 4 ounces of ingredient A, 2 ounces of B, and 2 ounces of C, then it is not a true blending problem. If a bottle of DeadCow contains at least 50% A, no more than 30% B, and enough C to complete filling an 8 ounce bottle, then we are looking at a true blending problem.

Suppose the percentages do describe the formula for DeadCow. The other product made by the good folks who bring you DeadCow is called PigCarass. PigCarcass is at least 40% C, at least 10% B, and no more than 10% A. A bottle of either contains 8 ounces. They have 1000 ounces of each ingredient on hand. There is a price for each product, say P_{DC} and P_{PC} per bottle. Replacement costs for each ingredient are C_A, C_B, and C_C.

Wrestle with this a while, and you will soon see that variables like DC and PC just won't work. Since you don't know how much of each ingredient is in each bottle, you can't figure the margin. Since you don't know how much of each ingredient is in each bottle, you don't know when you will run out of an ingredient. Since you don't know when you'll run out, you can't figure how much DC and PC you can make. It is a nightmare.

The way out is to define the activity variables as the total number of ounces of each ingredient that ends up in each product. Thus one variable would be defined as

DCA = number of ounces of A used to make DeadCow

DCA's contribution margin would be $P_{DC} - C_A$. This margin could even be negative - maybe we make it up on DCB and/or DCC. The total DeadCow made is DCA + DCB + DCC. The total amount of A used up is DCA + PCA. This way, we can keep everything under control. It isn't intuitively obvious, but it works well.

Full details of this delicious problem can be found in Exercise 9. After you've studied the discussion of Snako Oil on the following pages, you should be able to do Exercise 9 on your own.

Snako Oil Company: A Multiple Blend Problem

The Snako Oil Company blends and markets 2 grades of gasoline, Zoom and Slug. Zoom is a Superduper Premium fuel that has a very high octane rating and is packed with lots of thermal energy, and sells for $2.00 per gallon. Slug is a barely adequate fuel well suited to thoroughly worn older cars. Despite its $1.45 per gallon price, Slug is popular because it can help an old worn car pass emissions inspection. Both fuels are made by mixing 3 petro-chemical ingredients. DiethylMulehide is a low octane, low energy, very clean burning component costing Snako only $0.50 per gallon. Gruntane is packed densely with energy, has a moderate octane rating, and costs Snako $1.50 per gallon. Hoctane contributes a very high octane rating, moderate energy density, and burns very cleanly. At a cost to Snako of $2.10 per gallon it certainly should.

The specifications for Zoom require that Zoom contain no more than 20% diethylMulehide, while Slug may contain up to 70% diethylMulehide. Zoom must be at least 40% Gruntane, while Slug does not need to be more than 5% Gruntane. Zoom always contains at least 30% Hoctane, but Slug only requires 20% or more. This week, Snako has access to 30,000 gallons of diethylMulehide, 8,000 gallons of Gruntane, and 10,000 gallons of Hoctane.

Snako has promised their distributors that they will deliver 10,000 or more gallons of Zoom and 25,000 or more gallons of Slug this week. They would like to make as much money as possible using the currently available resources without breaking their promises and without violating the integrity of their specifications for Zoom and Slug. We will use LP to help them.

For an LP beginner, this is a ***hard*** problem. I would not think of making you do it on your own. It combines 2 aspects that beginners often have trouble with. In addition, we will use some extra variables that we have hardly mentioned.

1. It is a *true* **blending problem**. That means that the proportions of ingredients in the blend are <u>not</u> <u>fixed</u>. This has a profound effect on the way we must define our activity variables.
2. Because it is true blending problem, it has **proportionality constraints**. People frequently are troubled, at first, by proportionality constraints.
3. The "extra variables" are called ***supplementary variables***. We use them in some problems to make the output easier to read. First we will set up the problem without them, then we will reformulate with them.

Because this is a real blending problem, we cannot depend on variables called ZOOM and SLUG as we would in, for example, a product mix problem. Since we do not know exactly what resources Zoom and Slug will require this week, we would be unable to figure out our resource constraints. And in setting up our objective function coefficients, we would get stuck because we don't know what Zoom and Slug are going to cost to make. As briefly mentioned in the last chapter, we need a different approach for a blending problem.

In this problem, we need variables that mean "the amount of *this* ingredient we will use in *this* product". Then we can sum variables using the same resource to represent the amount of that resource that will be used. We can, similarly, sum variables that are parts of the same product to represent the amount of that product we will blend. We can figure the profit contributions of the variables by subtracting the cost of the ingredient from the price of the product. But this is hardly something that is obvious to the average bear. Let's do it.

ZM = Gallons of diethylMulehide used to make Zoom. The contribution/gallon is the $2.00 selling price of Zoom less the $0.50 cost of a gallon of diethylMulehide, or $1.50.

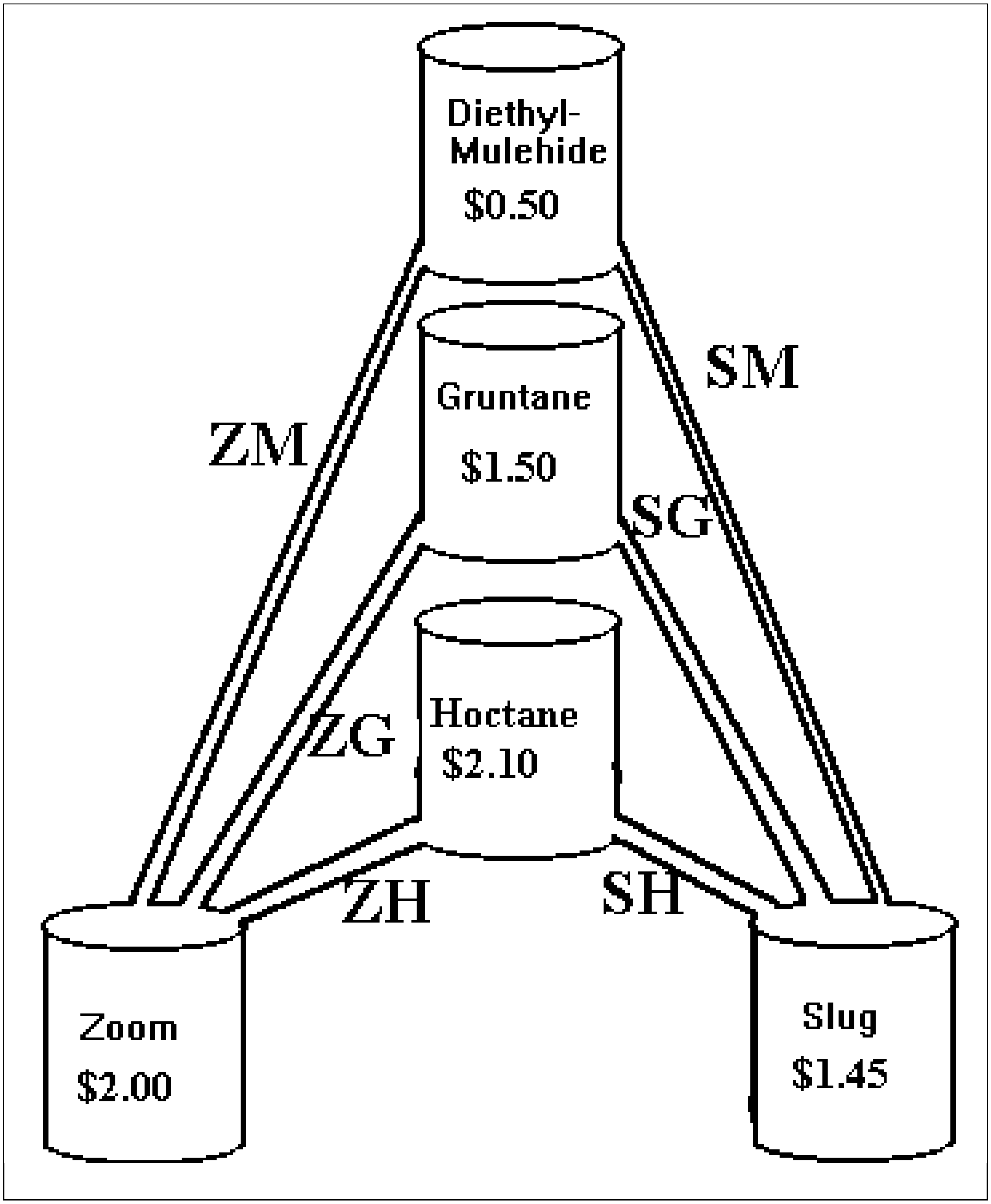
SM = Gallons of diethylMulehide used to make Slug. The contribution/gallon is the $1.45 selling price of Slug less the $0.50 cost of a gallon of diethylMulehide, or $0.95.

ZG = Gallons of Gruntane used to make Zoom. The contribution is $2.00 - $1.50 = $0.50

SG = Gallons of Gruntane used to make Slug. The contribution is $1.45 - $1.50 = ($0.05)

ZH = Gallons of Hoctane used to make Zoom. The contribution is $2.00 - $2.10 = ($0.10)

SH = Gallons of Hoctane used to make Slug. The contribution is $1.45 - $2.10 = ($0.65)



Flow Diagram for Snako Oil

Do not be put off by the negative contribution margins on some of the variables. That is very common in blending problems, and LP does take them into account appropriately. In this case, Snako is in effect dressing up the very cheap diethylMulehide with high grade components so that they can sell it at a high price. If that isn't profitable, LP will let us know.

So now we have an objective function:

$$\text{Maximize } Z = 1.5\ ZM + .95\ SM + .5\ ZG - .05\ SG - .1\ ZH - .65\ SH$$

The first 3 constraints are not too difficult. We cannot use more of the 3 components of these fuels than we have available:

$$ZM + SM \leq 30000$$

$$ZG + SG \leq 8000$$

$$ZH + SH \leq 10000$$

Delivery promises mean that they must deliver at least 10,000 gallons of Zoom and 25,000 gallons of Slug, which requires:

$$ZM + ZG + ZH \geq 10000$$

$$SM + SG + SH \geq 25000$$

Now comes the hard part: those pesky proportionality constraints

Zoom cannot contain more than 20% diethylMulehide and Slug is limited to no more than 70% diethylMulehide, or

$$ZM \leq .2\ (ZM + ZG + ZH)$$

$$SM \leq .7\ (SM + SG + SH)$$

which becomes

$$.8\ ZM - .2\ ZG - .2\ ZH \leq 0$$

$$.3\ SM - .7\ SG - .7\ SH \leq 0$$

Zoom needs at least 40% Gruntane and Slug needs at least 5% Gruntane:

$$ZG \geq .4\ (ZM + ZG + ZH)$$

$$SG \geq .05\ (SM + SG + SH)$$

or

$$-\ .4\ ZM + .6\ ZG - .4\ ZH \geq 0$$

$$-\ .05\ SM + .95\ SG - .05\ SH \geq 0$$

Zoom must contain at least 30% Hoctane and Slug needs at least 20% Hoctane:

$$ZH \geq .3\ (ZM + ZG + ZH)$$

$$SH \geq .2\ (SM + SG + SH)$$

or

$$-\ .3\ ZM - .3\ ZG + .7\ ZH \geq 0$$

$$-\ .2\ SM - .2\ SG + .8\ SH \geq 0$$

So the total formulation, when you bring this all together, is:

Maximize Z =	1.5 ZM	+.95 SM	+.5 ZG	-.05 SG	-.1 ZH	-.65 SH		
Subject to	ZM	+SM					≤ 30,000	DiethylMulehide
			ZG	+SG			≤ 8,000	Gruntane
					ZH	+SH	≤ 10,000	Hoctane
	ZM		+ZG		+ZH		≥ 10,000	Zoom
		SM		+SG		+SH	≥ 25,000	Slug
	.8 ZM		-.2 ZG		-2. ZH		≥ 0	≤ 20% M in Zoom
		.3 SM		-.7 SG		-.7 SH	≥ 0	≤ 70% M in Slug
	-.4 ZM		+.6 ZG		-.4 ZM		≥ 0	≥40%G in Zoom
		-.05 SM		+.95 SG		-.05 SH	≥ 0	≥5% G in Slug
	-.3 ZM		-.3 ZG		+.7 ZH		≥ 0	≥30% H in Zoom
		-.2 SH		-.2 SG		+.8 SH	≥ 0	≥20 H in Slug

Excel Analysis for Snako Oil: No Supplementary Variables

Snako Oil: Traditional (flat) LP formulation

	Slug Diethyl-Mulehide	Slug Gruntane	Slug Hoctane	Zoom Diethyl-Mulehide	Zoom Gruntane	Zoom Hoctane			
	23333.33	3000	7000	2000	5000	3000			
Contribution	0.95	-0.05	-0.65	1.5	0.5	-0.1	$22,666.67		
Diethyl-Mulehide	1			1			25333.3333	<=	30000
Gruntane		1			1		8000	<=	8000
Hoctane			1			1	10000	<=	10000
Slug	1	1	1				33333.3333	>=	25000
Zoom				1	1	1	10000	>=	10000
M in Zoom				-0.8	0.2	0.2	0	>=	0
M in Slug	-0.3	0.7	0.7				0	>=	0
G in Zoom				-0.4	0.6	-0.4	1000	>=	0
G in Slug	-0.05	0.95	-0.05				1333	>=	0
H in Zoom				-0.3	-0.3	0.7	0	>=	0
H in Slug	-0.2	-0.2	0.8				333	>=	0

Microsoft Excel 10.0 Sensitivity Report
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Slug Diethyl- Mulehide	23333.33286	0	0.950000049	1E+30	0.553571476
$C$3	Slug Gruntane	2999.999939	0	-0.050000017	0	2.066666842
$D$3	Slug Hoctane	6999.999859	0	-0.649999947	2.32157E+12	0
$E$3	Zoom Diethyl- Mulehide	2000	0	1.5	5.166667002	3.16666672
$F$3	Zoom Gruntane	5000	0	0.5	2.066666801	0
$G$3	Zoom Hoctane	3000	0	-0.1	0	2.32103E+12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$8	Slug	33333.33266	0	25000	8333.33266	1E+30
$H$9	Zoom	10000	-1.0333334	10000	3124.999811	1428.571429
$H$10	M in Zoom	0	-3	0	1600	500
$H$11	M in Slug	0	-3	0	2499.999798	499.9999899
$H$12	G in Zoom	1000	0	0	1000	1E+30
$H$13	G in Slug	1333	0	0	1333.333306	1E+30
$H$14	H in Zoom	0	0	0	333.3333333	1333.333333
$H$15	H in Slug	333	0	0	333.3333266	1E+30
$H$5	Diethyl- Mulehide	25333.33286	0	30000	1E+30	4666.667197
$H$6	Gruntane	7999.999939	2.16666672	8000	500	1600
$H$7	Hoctane	9999.999859	1.566666802	10000	2000.000273	1000

Snako Oil: No supplementary Variables

	Diethyl-Mulehide	Grun-tane	Hoc-tane	Total Made	Revenue	Min Slug	Min Zoom
Slug	**23333.33**	**3000**	**7000**	**33333.33**	**$1.45**	**1**	
Zoom	**2000**	**5000**	**3000**	**10000**	**$2.00**		**1**
Ttoal	**25333.33**	**8000**	**10000**		**$68,333.33**	**33333.33**	**10000**

Contribution						>=	>=
$ 22,666.67						25000	10000
					Total		
Cost	$0.50	$1.50	$2.10		$45,666.67		
Max M	1				25333.3333	<=	30000
Max G		1			8000	<=	8000
Max H			1		10000	<=	10000
Excess M in S	1			-70%	0.00	<=	0
Short G in S		-1		5%	-1333.33	<=	0
Short H in S			-1	20%	-333.33	<=	0
Excess M in Z	1			-20%	0.00	<=	0
Short G in Z		-1		40%	-1000.00	<=	0
Short H in Z			-1	30%	0.00	<=	0

Microsoft Excel 10.0 Answer Report
Target Cell (Max)

Cell	Name	Original Value	Final Value
$A$7	Contribution	$ 22,666.67	$ 22,666.67

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Slug Diethyl- Mulehide	23333.33333	23333.33333
$C$3	Slug Gruntane	3000	3000
$D$3	Slug Hoctane	7000	7000
$B$4	Zoom Diethyl- Mulehide	2000	2000
$C$4	Zoom Gruntane	5000	5000
$D$4	Zoom Hoctane	3000	3000

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$10	Max M Total	25333.33333	$F$10<=$H$10	Not Binding	4666.666667
$F$11	Max G Total	8000	$F$11<=$H$11	Binding	0
$F$12	Max H Total	10000	$F$12<=$H$12	Binding	0
$F$13	Excess M in S Total	0.00	$F$13<=$H$13	Binding	0
$F$14	Short G in S Total	-1333.33	$F$14<=$H$14	Not Binding	1333.333333
$F$15	Short H in S Total	-333.33	$F$15<=$H$15	Not Binding	333.3333333
$F$16	Excess M in Z Total	0.00	$F$16<=$H$16	Binding	0
$F$17	Short G in Z Total	-1000.00	$F$17<=$H$17	Not Binding	1000
$F$18	Short H in Z Total	0.00	$F$18<=$H$18	Binding	0
$G$5	Ttoal Min Slug	33333.33333	$G$5>=$G$7	Not Binding	8333.333333
$H$5	Ttoal Min Zoom	10000	$H$5>=$H$7	Binding	0

Microsoft Excel 10.0 Sensitivity Report
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Slug Diethyl- Mulehide	23333.33333	0	0.95	1E+30	0.553571429
$C$3	Slug Gruntane	3000	0	-0.05	0	2.066666667
$D$3	Slug Hoctane	7000	0	-0.65	1E+30	0
$B$4	Zoom Diethyl- Mulehide	2000	0	1.5	5.166666667	3.166666667
$C$4	Zoom Gruntane	5000	0	0.5	2.066666667	0
$D$4	Zoom Hoctane	3000	0	-0.1	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$10	Max M Total	25333.33333	0	30000	1E+30	4666.666667
$F$11	Max G Total	8000	2.166666667	8000	500	1600
$F$12	Max H Total	10000	1.566666667	10000	2000	1000
$F$13	Excess M in S Total	0.00	3.17	0	500	2500
$F$14	Short G in S Total	-1333.33	0.00	0	1E+30	1333.333333
$F$15	Short H in S Total	-333.33	0.00	0	1E+30	333.3333333
$F$16	Excess M in Z Total	0.00	3.17	0	500	1600
$F$17	Short G in Z Total	-1000.00	0.00	0	1E+30	1000
$F$18	Short H in Z Total	0.00	0.00	0	1333.333333	333.3333333
$G$5	Ttoal Min Slug	33333.33333	0	25000	8333.333333	1E+30
$H$5	Ttoal Min Zoom	10000	-1.033333333	10000	3125	1428.571429

Alternate Solution

Several of the objective coefficients in the sensitivity analysis on the previous page have zero allowable increase or allocable decrease. This is a sign that, while there can be no solution better than the one above, there may be solutions just as good. Tricking Excel into giving this to you is not easy, but here's an example of another solutio that satisfies all the constraints and is just as good as the one above. Note that the total Slug and Zoom produced and the total of each ingredient consumed are the same; only the recipes differ.

Snako Oil: No supplementary Variables

	Diethyl-Mulehide	Gruntane	Hoctane	Total Made	Revenue	Min Slug		Min Zoom
Slug	**23333.33**	**3333.33**	**6666.67**	**33333.33**	**$1.45**	**1**		
Zoom	**2000**	**4666.67**	**3333.33**	**10000**	**$2.00**			**1**
Ttoal	**25333.33**	**8000**	**10000**		**$68,333.33**	**33333.33**		**10000**
Contribution						>=		>=
$ 22,666.67						25000		10000
					Total			
Cost	$0.50	$1.50	$2.10		$45,666.67			
Max M	1				25333.3333	<=		30000
Max G		1			8000	<=		8000
Max H			1		10000	<=		10000
Excess M in S	1			-70%	0.00	<=		0
Short G in S		-1		5%	-1666.67	<=		0
Short H in S			-1	20%	0.00	<=		0
Excess M in Z	1			-20%	0.00	<=		0
Short G in Z		-1		40%	-666.67	<=		0
Short H in Z			-1	30%	-333.33	<=		0

Supplementary Variables

The previous formulation will give you the values of decision variables that optimize the model, The problem is in interpreting the results. The sensitivity analysis on objective coefficients for the no-supplementary variables version is almost totally useless, since there is no plausible way that just one of the six "pipe" variables could change without one or more others changing to, since what would really be changing is the cost of an ingredient or the price of a product.

The solution is to use supplementary variables. We create five more decision variables, one for each tank. These variables are constrained to e equal to the calculated consumption of the corresponding ingredient or the calculated production of each grade of gasoline.

Zoom = ZM + ZG + ZH
Slug = SM + SG + SH

Mule = ZM + SM
Grunt = ZG + SG
Hoct = ZH + SH

To make the magic work, we also need to restate the cost and revenue in terms of the supplementary variables instead of the corresponding calculated amounts.

A good rule of thumb is to solve the problem both with and without supplementary variables if you have any doubt that you have "tied down" the supplementary variables properly.

There's another bonus when we re-formulate the problem using supplementary variables: the proportionality constraints can be expressed in a way that is much clearer to the human mind, and therefore less prone to formulation error. To express the constraint that the DiethylMulehide in Zoom cannot be more than 20% of the total composition of Zoom, we only need to write ZM $\leq$ .2 ZOOM, and move all variables to the left for input to the computer:

$$-.2\ ZOOM + ZM \leq 0$$

Similarly, the requirement that the DiethylMulehide in Slug cannot be more than 70% of the total composition of Slug is written SM $\leq$ .7 SLUG or

$$-.7\ SLUG \leq SM$$

The other four proportionality constraints, in logical form and computer-input form are

```
ZG ≥  .4 ZOOM          - .4 ZOOM + ZG ≥   0
SG ≥ .05 SLUG          -.05 SLUG + SG ≥   0
ZH ≥  .3 ZOOM          - .3 ZOOM + ZH ≥   0
SH ≥  .2 SLUG          - .2 SLUG + SH ≥   0
```

Supplementary variables often simplify the objective function as well. In the case of Snako, we can tie the revenues to the products ZOOM and SLUG, and tie the costs to the resource variables MULE, GRUNT, and HOCT. The summary formulation of the extended problem appears on the next page.

The supplementary variable ""

Analysis for Snako Oil Using Supplementary Variables

Snako Oil: Supplementary Variables

	Diethyl-Mulehide	Gruntane	Hoctane	SuppTotal Made	CalcTotal Made	Revenue	Min Slug	Min Zoom
Slug	**23333.33**	**3000**	**7000**	**33333.333**	**33333.3333**	**$1.45**	**1**	
Zoom	**2000**	**5000**	**3000**	**10000**	**10000**	**$2.00**		**1**
Supp Total	**25333.33**	**8000**	**10000**	**0**				
Calc Total	**25333.33**	**8000**	**10000**			**$68,333.33**	**33333.3**	**10000**
Contribution							>=	>=
$ 22,666.67							25000	10000
					Total			
Cost	$0.50	$1.50	$2.10		$45,666.67			
Max M	1				25333.3333	<=	30000	
Max G		1			8000	<=	8000	
Max H			1		10000	<=	10000	
Excess M in S	1			-70%	-1.819E-11	<=	0	
Short G in S		-1		5%	-1333.3333	<=	0	
Short H in S			-1	20%	-333.33333	<=	0	
Excess M in Z	1			-20%	0	<=	0	
Short G in Z		-1		40%	-1000	<=	0	
Short H in Z			-1	30%	0	<=	0	

Microsoft Excel 10.0 Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$A$8	Contribution	$ 22,666.67	$ 22,666.67

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Slug Diethyl- Mulehide	23333.3338	23333.33333
$C$3	Slug Gruntane	3000.000126	3000
$D$3	Slug Hoctane	7000.000076	7000
$E$3	Slug SuppTotal Made	33333.33401	33333.33333
$B$4	Zoom Diethyl- Mulehide	2000	2000
$C$4	Zoom Gruntane	5000	5000
$D$4	Zoom Hoctane	3000	3000
$E$4	Zoom SuppTotal Made	10000	10000
$B$5	Supp Total Diethyl- Mulehide	25333.33385	25333.33333
$C$5	Supp Total Gruntane	8000	8000
$D$5	Supp Total Hoctane	10000	10000
$E$5	Supp Total SuppTotal Made	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$11	Max M Total	25333.33333	$F$11<=$H$11	Not Binding	4666.666667
$F$12	Max G Total	8000	$F$12<=$H$12	Binding	0
$F$13	Max H Total	10000	$F$13<=$H$13	Binding	0
$F$14	Excess M in S Total	-1.81899E-11	$F$14<=$H$14	Binding	0
$F$15	Short G in S Total	-1333.333333	$F$15<=$H$15	Not Binding	1333.333333
$F$16	Short H in S Total	-333.3333333	$F$16<=$H$16	Not Binding	333.3333333
$F$17	Excess M in Z Total	0	$F$17<=$H$17	Binding	0
$F$18	Short G in Z Total	-1000	$F$18<=$H$18	Not Binding	1000
$F$19	Short H in Z Total	0	$F$19<=$H$19	Binding	0
$H$6	Calc Total Min Slug	33333.33333	$H$6>=$H$8	Not Binding	8333.333333
$I$6	Calc Total Min Zoom	10000	$I$6>=$I$8	Binding	0
$B$5	Supp Total Diethyl- Mulehide	25333.33333	$B$5=$B$6	Not Binding	0
$C$5	Supp Total Gruntane	8000	$C$5=$C$6	Not Binding	0
$D$5	Supp Total Hoctane	10000	$D$5=$D$6	Not Binding	0
$E$3	Slug SuppTotal Made	33333.33333	$E$3=$F$3	Not Binding	0
$E$4	Zoom SuppTotal Made	10000	$E$4=$F$4	Not Binding	0

Microsoft Excel 10.0 Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Slug Diethyl- Mulehide	23333.33333	0	$ -	1E+30	0.553571429
$C$3	Slug Gruntane	3000	0	$ -	0	2.066666667
$D$3	Slug Hoctane	7000	0	$ -	1E+30	0
$E$3	Slug SuppTotal Made	33333.33333	0	$ 1.45	1E+30	0.3875
$B$4	Zoom Diethyl- Mulehide	2000	0	$ -	5.166666666	3.166666667
$C$4	Zoom Gruntane	5000	0	$ -	2.066666667	0
$D$4	Zoom Hoctane	3000	0	$ -	0	1E+30
$E$4	Zoom SuppTotal Made	10000	0	$ 2.00	1.033333333	1E+30
$B$5	Supp Total Diethyl- Mulehide	25333.33333	0	$ (0.50)	1E+30	0.62
$C$5	Supp Total Gruntane	8000	0	$ (1.50)	1E+30	2.166666667
$D$5	Supp Total Hoctane	10000	0	$ (2.10)	1E+30	1.566666667
$E$5	Supp Total SuppTotal Made	0	0	$ -	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Max M Total	25333.33333	0	30000	1E+30	4666.666667
$F$12	Max G Total	8000	2.166666667	8000	500	1600
$F$13	Max H Total	10000	1.566666667	10000	2000	1000
$F$14	Excess M in S Total	-1.81899E-11	3.166666667	0	500	2500
$F$15	Short G in S Total	-1333.333333	0	0	1E+30	1333.333333
$F$16	Short H in S Total	-333.3333333	0	0	1E+30	333.3333333
$F$17	Excess M in Z Total	0	3.166666667	0	500	1600
$F$18	Short G in Z Total	-1000	0	0	1E+30	1000
$F$19	Short H in Z Total	0	0	0	1333.333333	333.3333333
$H$6	Calc Total Min Slug	33333.33333	0	25000	8333.333333	1E+30
$I$6	Calc Total Min Zoom	10000	-1.033333333	10000	3125	1428.571429
$B$5	Supp Total Diethyl- Mulehide	25333.33333	-0.5	0	4666.666667	25333.33333
$C$5	Supp Total Gruntane	8000	-3.666666667	0	1600	500
$D$5	Supp Total Hoctane	10000	-3.666666667	0	1000	2000
$E$3	Slug SuppTotal Made	33333.33333	3.666666667	0	500	2500
$E$4	Zoom SuppTotal Made	10000	3.666666667	0	500	1600

THE SNAKO RESULTS

Now let's see if we can answer a few questions that LeRoy, the high muckety muck at the Snake Oil Company might want answers to. Leroy's questions are:

1. How much of each product should we make this week? How much money will we earn from doing that?
2. One of our dealers wants an extra 1000 gallons of Zoom this week. Purely on a short term profit basis, should we oblige the dealer? Can we? How will that hit the bottom line?
3. Our petrochemical supplier is overloaded with Gruntane today, and has offered to bring us an extra 2000 gallons at our usual cost with no special-delivery charge. Should we take it? Would we gain any profit by doing so? How much?
4. The same petrochemical supplier is also overloaded with diethylMulehide today, and has offered to bring us an extra 5000 gallons at our usual cost instead of Gruntane with no special-delivery charge. Should we take it? Would we gain any profit by doing so? How much?
5. I took a little LP when I was majoring in Party Dynamics at UGA. Looking at the printout with supplementary variables, I see that constraint 11 has a shadow price of $3.17. Doesn't that present some kind of opportunity for us?
6. And while we're at it, constraint 1 has a $3.67 shadow price. Shouldn't we do something about that?

Try to answer these questions using the outputs. No peeking allowed, but after you have written your answers, you'll find mine on the next page.

Here are my answers. If you really answered the questions, you probably found that the extra effort to include the supplementary variables was worth it in this case.

1. You should blend 10,000 gallons of Zoom and 33,333 $\frac{1}{3}$ gallons of Slug. I really can't tell you what your ***profit*** will be without getting all of the details of your accounting system and running a *pro forma* income statement, but this course of action should contribute $22666.67 toward overhead and profit.
2. The RHS ranging for the contract constraint tells me that you can blend up to 3125 more gallons of Zoom this week, but the shadow price tells me that each extra gallon of Zoom you force into your mix will cut total contribution by $1.033. Dealer relations are a lot more important in the long run than this week's bottom line. Is it worth $1033 to you to keep the dealer happy? If so, do it. I'll be happy to rerun the LP program so you'll know your new mix.
3. The marginal value to you of an extra gallon of Gruntane is $2.167, but the range on the Gruntane constraint only goes up to 8,500 gallons. If you don't mind storing an excess of 1500 gallons 'till next week, go for it. If you plan to both buy the extra Gruntane *and* please your dealer, I'd have to rerun the problem to tell you the effect. My guess is that the 2 changes would about counterbalance each other in their profit contribution effect.
4. You are already going to have 4,666 $\frac{2}{3}$ gallons of diethylMulehide left over. I don't see much point in getting more this week.
5. Constraint 11 is a proportionality constraint, LeRoy. All that shadow price tells you is that it costs something to meet quality standards for Zoom, which you probably already guessed.
6. Constraint 1 defines the supplementary variable ZOOM in terms of its components ZM, ZG, and ZH. Trying to work with a shadow price on that is like asking "If a horse had wings, would that make it a horsefly?".

Multiperiod Problems

Some problems involve decisions that are made repeatedly over multiple time periods. Given projected enrollments and counselor attrition, how many Incept Counselors must we train each Quarter over the next year? Given a very seasonal sales forecast with prices and costs varying month by month, how many Widgets must we make each month for the next year? These are multiperiod problems.

A multiperiod problem requires a large number of activity variables, and there is no short cut for that. Let's consider Widgets, assuming the problem starts with January. The greatest number of Widgets we can sell in March will be our March manufacturing capacity, plus the Widgets made in January for sale in March, plus the Widgets made in February for sale in March, less the number of Widgets we make in March for April sale, the number of Widgets we make in March for May sale, and so forth.

Every multiperiod problem is a little different, but they all require you to ask what is or is not carried forward from period to period. Depending on your conclusion, you may need to define a few or a great many variables.

ECOWAGEN
EgoWagen Gmbh has asked for your help in planning production of their two models for the next two quarters. The Jetson, is a four door car whose market segment is Grumpies (Grownup Upwardly Mobile Pretenders) mit kinder. It wholesales to their US distributor for $15,000. The GIT is a shorter version of the same car with a better suspension, more powerful motor, and even more pretensions, aimed at those who cannot simultaneously afford gold chains and Porsches. Its wholesale price is $18,000. Both prices

are F.O.B. Nuttgart.

Their sales forecast for the next two quarters shows the following demand levels, given their current marketing plans:

	GIT	Jetson
Fall	2,000	5,000
Winter	2,500	8,000

Unfortunately, their demand will exceed their manufacturing capacity of 10000 units/Quarter in Winter quarter. One partial solution may be to build cars in advance. Storage and interest charges amount to $1000/quarter for each quarter a car is stored. Thus a car made in the fall would cost them an extra $3000 if held for summer sale. They have a two-quarter planning horizon; for simplicity, we do not want any cars left over at the end of the horizon, so we have no interest in planning cars for sale beyond whiter quarter. In a going concern, we would re-run the model at the end of falll and think about spring then.

A further restriction arises from their labor contract. The GIT is more time consuming and difficult to make than the Jetson. As a result, they have agreed that in any period, GITs will constitute no more than 30% of their production. Incremental production cost for a GIT is $9000, while that cost for a Jetson is $8000.

Incremental production cost for a GIT is $9000, while that cost for a Jetson is $8000.

What they want from you, at the very least, is a production schedule for the year. Further insights would be nice.

Solution:

The key to solving this problem is in rcognizing that the decision what to produce and the decision what to release to the market are two different decisions that have to be made for each model each quarter, so there are eight decision variables in all.

There are three groups of constraints (plus non-negativity):

1. Cells B11 and B12 calculate the total number of cars made each quarter, each of these has to bel les than or equal to the production capacity for that quarter, which is specified at 10,000 cars.
2. The decision variables in cells D2:E3 correspond to cars sold. They need to be less than or equal to demand (cells F15:G15).
3. The numbers of cars sold also need to be less than equal to supply (Cells F2:G3). The supply of a particular model i a articlar quarter is the number of that model car made that quarter plus the number left in inventory at the end of the previous quarter.

The following pages give the Excel solution to this problem. For a more interesting version, see Exercise 6.

	A	B	C	D	E	F	G	H	I
1	EcoWagen	MakeGit	MakeJet	SellGit	SellJet	AvailGit	AvailJet	StoreGit	StoreJet
2	Fall	2000	5500	2000	5000	=B2	=C2	=B2-D2	=C2-E2
3	Winter	2500	7500	2500	8000	=B3+H2	=C3+I2	=H2+B3-D3	=I2+C3-E3
4	Total	=SUM(B2:B3)	=SUM(C2:C3)	=SUM(D2:D3)	=SUM(E2:E3)			=SUM(H2:H3)	=SUM(I2:I3)
5	Dollars	-9000	-8000	18000	15000			-1000	-1000
6	Total Contribution								
7	=SUMPRODUCT(B4:I4,B5:I5)								
8									
9									
10		cars	capacity						
11	Fall	=B2+C2	10000						
12	Winter	=B3+C3	10000						
13									
14				SellGit	SellJet	DemGit	DemJet		
15	Fall Demand			=D2	=E2	2000	5000		
16	Winter Demand			=D3	=E3	2500	8000		

EcoWagen	MakeGit	MakeJet	SellGit	SellJet	AvailGit	AvailJet	StoreGit	StoreJet
Fall	2000	5500	2000	5000	2000	5500	0	500
Winter	2500	7500	2500	8000	2500	8000	0	0
Total	4500	13000	4500	13000			0	500
Dollars	-$9,000	-$8,000	$18,000	$15,000			-$1,000	-$1,000
Total Contribution								
$131,000,000								

	cars	capacity
Fall	7500	10000
Winter	10000	10000

	SellGit	SellJet	DemGit	DemJet
Fall Demand	2000	5000	2000	5000
Winter Demand	2500	8000	2500	8000

Answer Report

$A$7	Total Contribution	0	$131,000,000

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	Fall MakeGit	0	2500
$C$2	Fall MakeJet	0	5000
$D$2	Fall SellGit	0	2000
$E$2	Fall SellJet	0	5000
$B$3	Winter MakeGit	0	2000
$C$3	Winter MakeJet	0	8000
$D$3	Winter SellGit	0	2500
$E$3	Winter SellJet	0	8000

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$2	Fall AvailGit	2500	$F$2>=$D$2	Not Binding	500
$G$2	Fall AvailJet	5000	$G$2>=$E$2	Binding	0
$F$3	Winter AvailGit	2500	$F$3>=$D$3	Binding	0
$G$3	Winter AvailJet	8000	$G$3>=$E$3	Binding	0
$D$15	Fall Demand SellGit	2000	$D$15<=$F$15	Binding	0
$E$15	Fall Demand SellJet	5000	$E$15<=$G$15	Binding	0
$D$16	Winter Demand SellGit	2500	$D$16<=$F$16	Binding	0
$E$16	Winter Demand SellJet	8000	$E$16<=$G$16	Binding	0
$B$11	Fall cars	7500	$B$11<=$C$11	Not Binding	2500
$B$12	Winter cars	10000	$B$12<=$C$12	Binding	0

Sensitivity Report

Report Created: 1/6/2005 8:31:11 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Fall MakeGit	2500	0	-11000	1000	0
$C$2	Fall MakeJet	5000	0	-10000	0	7000
$D$2	Fall SellGit	2000	0	20000	1E+30	9000
$E$2	Fall SellJet	5000	0	17000	1E+30	7000
$B$3	Winter MakeGit	2000	0	-10000	0	1000
$C$3	Winter MakeJet	8000	0	-9000	10000	0
$D$3	Winter SellGit	2500	0	19000	1E+30	8000
$E$3	Winter SellJet	8000	0	16000	1E+30	6000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$2	Fall AvailGit	2500	0	0	500	1E+30
$G$2	Fall AvailJet	5000	0	0	500	2000
$F$3	Winter AvailGit	2500	-11000	0	2500	500
$G$3	Winter AvailJet	8000	-10000	0	2000	500
$D$15	Fall Demand SellGit	2000	9000	2000	2500	2000
$E$15	Fall Demand SellJet	5000	7000	5000	2500	5000
$D$16	Winter Demand SellGit	2500	8000	2500	2500	500
$E$16	Winter Demand SellJet	8000	6000	8000	2000	500
$B$11	Fall cars	7500	0	10000	1E+30	2500
$B$12	Winter cars	10000	1000	10000	500	2000

Transportation and Trans-shipment

Minimizing Transportation Cost

The Georgia Cracker Company manufactures crackers at two factories in Albany and Atlanta and ships them around the state. The capacity of the Albany factory is 500 cases per day and the capacity of the Atlanta factory is 1000 cases per day. The maximum number of cases that can be sold per day in each city is:

Albany	Atlanta	Augusta	Cass	Colum-bus	Dalton	Harlem	Lexing-ton	Macon	Mc-Intyre	Savan-nah	Senoia
100	200	75	50	100	90	50	60	200	40	180	75

Here is the distance from the factories to each market area:

Miles	Albany	Atlanta	Augusta	Cass	Colum-bus	Dalton	Harlem	Lexing-ton	Macon	Mc-Intyre	Savan-nah	Senoia
Albany	0	165	207	212	89	254	192	195	102	124	210	142
Atlanta	165	0	148	43	102	85	128	83	77	104	243	33

It costs 5 cents per case per mile to ship the crackers by truck

This is a simple example of a very important class of linear programming applications referred to as "The Transportation Problem." The linear programming solution is as follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2	Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3	Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4	Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5															
6	Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
7	Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
8	Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
9															
10															
11		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
12	From Albany	100	0	0	0	100	0	0	0	0	0	180	0	380	500
13	From Atlanta	0	200	75	50	0	90	50	60	200	40	0	75	840	1000
14	Total to	100	200	75	50	100	90	50	60	200	40	180	75		
15	Demand	100	200	75	50	100	90	50	60	200	40	180	75		
16															
17	101,015	Total miles													
18	$ 5,050.75	Transportation cost													

The decision variables in cells B12 through N13 specify how many cases of crackers should be shipped from each factory to each market area. The constraints are that the total from each factory has to be less than or equal to its capacity, and the total to each market area has to be equal to its demand. The objective is to minimize the total transportation cost found by multiplying the number of cases in each shipment times the mileage times the cost per mile and adding the result for all shipments.

Minimizing Total Variable Cost

However, transportation is not the only variable cost associated with our crackers. There is also the variable cost of manufacturing them, which costs $22 per case in Albany and $24 per case in Atlanta. We get a different result if we include the manufacturing cost (the product of the cost per case at each factory times the "total from" that factory, added together for the two factories.)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5																
6		Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
7		Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
8		Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
9																
10																
11	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
12	$ 22	From Albany	100	0	0	0	100	0	0	0	80	40	180	0	500	500
13	$ 24	From Atlanta	0	200	75	50	0	90	50	60	120	0	0	75	720	1000
14		Total to	100	200	75	50	100	90	50	60	200	40	180	75		
15		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
16																
17		103,815	Total miles													
18		$ 5,190.75	Tramsportation Cost													
19		$ 28,280.00	Manufacturing Variable Cost													
20		$ 33,470.75	Total VariableCost													

Notice that we have used fewer cases from Atlanta and more from Albany even though we have to ship more miles, since we more than make up for it by saving money on manufacturing. (There is excess manufacturing capacity, and we move the excess from Albany to Atlanta.

Maximizing Contribution to Profit and Overhead

Instead of insisting that the maximum possible demand at each location must be met, it makes more sense to look at what's profitable. It turns out that we take a loss supplying some of the cities.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5																
6		Cost Per Mile	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
7		Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
8		Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
9																
10																
11	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
12	$ 22	From Albany	100	0	0	0	100	0	0	0	200	0	0	0	400	500
13	$ 24	From Atlanta	0	200	0	50	0	0	0	0	0	0	0	75	325	1000
14		Total to	100	200	0	50	100	0	0	0	200	0	0	75		
15		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
16		Price	$30	$32	$ 30	$27	$28	$28	$26	$26	$30	$26	$30	$27		
17																
18		33,925	Total miles													
19		$ 1,696.25	Tramsportation Cost													
20		$ 16,600.00	Manufacturing Variable Cost													
21		$ 18,296.25	Total Cost													
22		$ 21,575.00	Total Revenue													
23		$ 3,278.75	Contribution to Profit and Overhead													

The Trans-shipment Problem

Up to now we have just been looking at elaborations of the transportation problem, The next step up in sophistication comes when Georgia Cracker Company opens a warehouse in Macon with a capacity to unload 400 cases per day from rail to trucks. Crackers can be shipped by train from either factory for just one cent per case per mile, then transferred (or "trans-shipped") to trucks to go on to the final destination at the usual cost of five cents per case per mile. In a full-fledged trans-shipment problem all locations are potential trans-shipment points, but for simplicity here only Macon is.

Row 16 of the spreadsheet gives the total number of cases shipped to each market, whether directly from Albany or Atlanta, or from Macon after being shipped to Macon by rail. Row 17, which in this simplified model pertains only to Macon, is the number of cases trans-shipped out. Row 18 calculated the "Net to" as the "Total To" in row 16 take away the "Total From" in row 17. This is constrained to be less or equal to the maximum demand for each city, and multiplied times the selling price per case in each city to calculate the revenue as in the third transportation spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2		Miles	Albany	Atlanta	Augusta	Cass	Columbus	Dalton	Harlem	Lexington	Macon	McIntyre	Savannah	Senoia		
3		Albany	0	165	207	212	89	254	192	195	102	124	210	142		
4		Atlanta	165	0	148	43	102	85	128	83	77	104	243	33		
5		Macon	102	77	121	128	95	170	100	93	0	32	170	70		
6																
7		Cost Per Mile	to Albany	to Atlanta	to August	to Cass	to Colum	to Dalton	to Harlem	to Lexingt	to Macon	to McIntyr	to Savann	to Senoia		
8		From Albany	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.01	0.05	0.05	0.05		
9		From Atlanta	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.01	0.05	0.05	0.05		
10		From Macon	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
11																
12	Var. Cost per Case		to Albany	to Atlanta	to Augusta	to Cass	to Columbus	to Dalton	to Harlem	to Lexington	to Macon	to McIntyre	to Savannah	to Senoia	Total From	Capacity
13	$ 22	From Albany	100	0	0	0	100	0	0	0	300	0	0	0	500	500
14	$ 24	From Atlanta	0	200	0	50	0	0	0	0	0	0	0	75	325	1000
15		From Macon	0	0	60	0	0	0	0	0	0	40	0	0	100	400
16		Total To	100	200	60	50	100	0	0	0	300	40	0	75		
17		Total From									100					
18		Net To	100	200	60	50	100	0	0	0	200	40	0	75		
19		Demand	100	200	75	50	100	90	50	60	200	40	180	75		
20		Price	$30	$32	$30	$27	$28	$28	$26	$26	$30	$26	$30	$27		
21																
22		52,665	Total miles													
23		$ 1,409.25	Tramsportation Cost													
24		$ 18,800.00	Manufacturing Variable Cost													
25		$ 20,209.25	Total Cost													
26		$ 24,415.00	Total Revenue													
27		$ 4,205.75	Contribution to Profit and Overhead													