

## Some Key Terms

**Activity Variable** Also called "Decision Variable;" something the decision maker actually needs to make a decision about and carry out that decision in the real world.

**Solution Value** The quantity of the variable in the optimal solution.

**Dual Price** (also called Shadow Price) The rate at which the objective function changes as we change the value of the Right Hand Side of a constraint. It is meaningful only if it is meaningful to change the RHS value.

**Reduced Cost** For an activity variable with a nonzero value, the reduced cost is zero. For an activity variable with a value of zero, it is what it would cost to *force* a marginal unit of this activity variable into the solution.

**RHS Range** Right Hand Side Ranging tells you the range of values for the RHS of a constraint to which a shadow price applies. Outside the range, *improvements* to the objective function get smaller or nonexistent. Outside the range, *damage* to the objective function is done at a higher rate, or the problem becomes infeasible.

**O.F. Range** The Objective Function Ranging permits you to examine the effect of changing one objective function coefficient. Within the Objective Function Coefficient Range, a revised coefficient for a basic activity variable affects only the total value of the objective function. If a coefficient is revised to a value outside the Objective Function Coefficient Range, a different basic feasible solution (constraint intersection) becomes optimal.

**Unbounded Solution** The constraint set was not sufficient to keep the solution from going to infinity in the optimizing direction. This can't happen in reality, so it means you overlooked something.

**No Feasible Solution** Whoops! You overconstrained that sucker big time. This usually indicates an error in formulating the problem, but it occasionally *can* mean that the decision maker is between a rock and a hard place.

## Problems

When you submit computer output for a problem, it should be a cleanly printed output, beginning at the top of a page. Show the formulation properly printed (not a PrtSc of an input screen). Include solution values, marginal values, dimensional analysis, and ranging information.

1) The bedframes product group of the *Seasick WaterBeds Company* makes and sells two models of waterbed frame. The Mediterranean yields a contribution margin of \$85/unit, while the Caribbean is good for \$125. While other products are also made, the bed frame product group has the use of the cutting shop for 300 man-hours, the assembly shop for 210 man-hours, and the final finishing shop for 336 man-hours next week. A Mediterranean frame takes 3, 1.5, and 3 hours in those shops respectively, while a Caribbean frame requires 2, 3, and 4 hours. Marketing has requested that no more than 70 Mediterranean frames be made available next week, but they need at least 20 Caribbean frames. **Formulate the problem.**

1<sup>1/2</sup>) a. Solve the *Seasick WaterBeds Company* problem using EXCEL.

b. If you could get overtime for an *extra* \$12.00/hour in one of the 3 shops, which one would you choose? How many hours would you buy? Ignoring the overtime premium, what is the effect on the objective function?

2) *Crashcar Enterprises* makes Trailing Arms and Panhard Bars. A pair of Trailing Arms requires 3 feet of steel rod stock, 4 rod ends, and 4 hours of labor to machine and assemble. A Panhard Bar is made using 3 feet of steel rod, 2 rod ends, and 1 hour of labor. These resources must be arranged for in advance of actual production. For next week, they have available 180 feet of steel rod, 160 rod ends, and 140 labor hours.

College students working part time provide the labor, as needed, for \$5/hour. The correct type of steel rod is available to Widget at \$1/foot. Crashcar has recently converted their plant from military aerospace work. Their present stock of rod ends are FAA approved MilSpec unobtainium, and were purchased for \$1,475.80 each. The same rod ends are readily available through surplus sources for \$5.00 apiece. On the advice of their chief accountant, overhead is charged to all work at \$5.00 per direct labor hour.

A pair of Trailing Arms sells for \$73.00 FOB Crashcar, while a Panhard Bar sells for \$38.00. At these prices, they have a stable market slightly exceeding their capacity. In fact, their pricing is so attractive that Blind Hog Motorsports has just signed a contract to purchase 25 pairs of Trailing Arms per week.

As a profit-oriented enterprise, how many of each product ought they make and sell next week? Formulate this as an LP problem, and solve graphically.

2 1/2) Solve Crashcar Enterprises using EXCEL

3) *Microhard Inc.* produces hinges and latches for Gates. The unit contribution margin for a latch is \$7 and for a hinge it is \$6. Each latch requires 6 square inches of steel and 4 minutes of labor. A hinge requires 12 square inches of steel and 3 minutes of labor. For the period of interest, they have 17 square feet of steel and 20 man-hours of labor available. Demand for both products is far beyond capacity, but they only have packaging materials for 150 hinges. **Formulate and solve graphically.**

3<sup>1/2</sup>) Solve the *Microhard Inc.* problem using EXCEL.

4) *Hammersham Furniture Company* makes both Dining Room Suites and Bedroom Suites. A Dining Suite requires 36 square yards of hardwood, 15 hours of skilled labor, and 6 yards of imported brocade fabric. A Bedroom Suite uses 27 square yards of hardwood lumber and 30 hours of skilled labor. Both are quite profitable. The contribution margin on a Dining set is \$1,000 and that on a Bedroom set is \$1,200.

This week, they have available for use 432 square yards of hardwood, 330 person-hours of skilled labor, and 60 yards of fine imported brocade. What should they make this week? **Formulate and solve graphically.**

4<sup>1/2</sup>) Solve the *Hammersham Furniture Company* problem using EXCEL

5) The manager of the *FollyGlass Tire Company* wishes to determine the production schedule for the next two months. FollyGlass produces both glass belted and steel belted radial tires, selling to both auto manufacturers and the aftermarket. Four machine hours are required for each glass belted tire, and five per steel belted tire. For Month 1, 15,000 machine hours are available. For Month 2, only 10,000 machine hours can be used. Adequate labor is available for both periods.

Variable production costs incurred each month must be paid in cash the same month, before the proceeds of that month's sales have been collected. The variable production cost for a glass-belted tire is \$25; for a steel belted tire the cost is \$33. At the start of Month 1, the cash account holds \$70,000.

FollyGlass has contracts with two auto manufacturers during this planning period. They have agreed to sell 650 glass belted tires to Yugo in Month 1, and to sell 1000 steel belted tires to Trabant in Month 2. All sales are at their normal low wholesale prices, which are attractive enough that they can sell all that they can make of either tire, each month. The price of glass belted tires is stable at \$40 each. Steel belted tires will sell for \$50 each in Month 1. In Month 2, that price rises to \$55.

**Formulate this as a linear programming problem.**

Note: Since it is possible for tires made in Month 1 to be sold in Month 2, and for sales in Month 1 to finance production in Month 2, this is a single multiperiod problem, not two single period problems.

5<sup>1/2</sup>) Solve the *FollyGlass Tire Company* problem. Would FollyGlass like to borrow money in either month (or both)? How much? What is the absolute highest monthly rate of interest they can pay on the money without making themselves worse off?

6) *EcoWagen GmbH*, the company that makes cars with *Fahrtfi Newton*, has asked for your help in planning next year's production of their two models. The Jetson, also known as the Aryan's Choice, is a four door car whose market segment is Grumpies (Grownup Upwardly Mobile Pretenders) mit kinder. It wholesales to their US distributor for \$15,000. The GIT is a shorter version of the same car with a better suspension, more powerful motor, and even more pretensions, aimed at those who cannot simultaneously afford gold chains and Porsches. Its wholesale price is \$18,000. Both prices are F.O.B. Nuttgart.

Their sales forecast for next year shows the following demand levels, given their current marketing plans:

|        | <b>GIT</b> | <b>Jetson</b> |
|--------|------------|---------------|
| Fall   | 2000       | 5000          |
| Winter | 2500       | 8000          |
| Spring | 3000       | 5500          |
| Summer | 3000       | 7000          |

Unfortunately, their demand will exceed their manufacturing capacity of 10000 units/Quarter some quarters. One partial solution may be to build cars in advance. Storage and interest charges amount to \$1000/quarter for each quarter a car is stored. Thus a car made in the fall would cost them an extra \$3000 if held for summer sale. They have no interest in planning cars for sale beyond next summer.

A further restriction arises from their labor contract. The GIT is more time consuming and difficult to make than the Jetson. As a result, they have agreed that in any period, GITs will constitute no more than 30% of their production. Incremental production cost for a GIT is \$9000, while that cost for a Jetson is \$8000.

What they want from you, at the very least, is a production schedule for the year. Further insights would be nice. For now, just **formulate the problem**.

6<sup>1/2</sup>) Solve the EcoWagen problem.

7) *Firestone Cement Company* makes various grades of portland cement by firing a mix of limestone and clay. The prices of these commodities vary. Presently, limestone can be had for \$16/ton, and clay can be had for \$22/ton. Limestone is needed for adhesion, and clay for hardness. They presently have 5 tons of each which must be used next week; they can easily buy more. They must make a batch of a grade which *must* contain at least 60% clay. They have promised to deliver at least 20 tons of that grade. How should they make that batch? **Formulate the problem. Solve it graphically.**

7<sup>1/2</sup>) Solve the *Firestone Cement Company* problem.

**8)** *YUPS R' US* is planning a big nationwide sale of the latest Yuppie toy, Cellular Car Fax machines. They have two suppliers who build and label them to YUPS R' US specifications. Tijuana Telefax builds them for only \$50 each, but their shipping costs are somewhat higher than Boston Technofax, who charges the company \$60 for each machine. Both companies sell F.O.B. their own loading dock. Tijuana can produce 16,500 machines in time for the sale, while Boston can make 21,000 available. The L.A. warehouse will need 8,000 machines, Kansas City needs 4,500, Houston needs 6,500, Washington D.C. needs 6,000, and Atlanta needs 5,000.

Shipping costs from Tijuana to those warehouses are (in the same order) \$15, \$20, \$25, \$30, and \$30 per fax. From Boston, the shipping charges are \$25, \$15, \$15, \$5, and \$10 per fax. Both companies can ship in time for distribution from your warehouses to your retail outlets in time for the Sale if you act quickly. Place your orders. For now, just **formulate the problem**.

**8<sup>1/2</sup>)** Solve the *YUPS R' US* problem. Tell what they should order from each supplier to send to each regional warehouse. Is there another order pattern that would work as well? If they just remembered that the Miami warehouse needs 4,500 machines at any cost, can they meet that need without shorting another warehouse? How?

**9)** *Carniverous Comestibles* produces 2 brands of barbecue sauce, DeadCow and PigCarcass, using ingredients A, B, and C. Their flavors are distinctive, and quite different from each other. DeadCow is composed of at least 50% A, no more than 30% B, and the remainder C. PigCarcass must be no more than 10% ingredient A, at least 10% B, and at least 40% C. They have on hand and available for today's production 1,000 ounces of each ingredient.

Ingredient A costs them \$0.10/ounce, ingredient B costs \$0.05/ounce, and ingredient C costs \$0.15/ounce. An 8 ounce bottle of DeadCow sells to their distributor for \$1.00, and an 8 ounce bottle of PigCarcass sells for \$1.28. **Formulate the linear programming problem** that will let them make the optimal amount of DeadCow and PigCarcass.

**9<sup>1/2</sup>)** Solve the *Carniverous Comestibles* problem without using supplementary variables. Now reformulate it to include supplementary variables representing A, B, C, DeadCow, and PigCarcass and run it again. Comment on the difference you see in the runs.

**10) Purdy Paint Company's** manufacturing process requires that each batch be exactly 1000 gallons. The procedure they use involves combining the base ingredients in a closed vat, and then homogenizing the mixture ultrasonically. Once the paint base is ready, they add pigment and put the paint in cans. The paint is unsalable without pigment. Today they are going to make Blue, Green, and Yellow paints. If you have been in a paint store, you know that these colors will have much fancier names, but we'll call them what they are.

A gallon of Blue paint is made with a gallon of base plus 2 ounces of blue pigment. A gallon of Yellow paint contains the base plus 3 ounces of yellow pigment. A gallon of Green paint has  $\frac{1}{2}$  ounce of blue pigment and 2 ounces of yellow pigment added to the base. This sounds like they get more than a gallon per gallon! Sometimes they do; their cans have room for a little extra. There are some losses in the process, and they want to be sure that every can contains at least the advertised quantity.

Today they have 1000 ounces of blue pigment and 2000 ounces of yellow pigment on hand. In addition, they have a prepaid order for 350 gallons of Green paint. Their Blue and Yellow paints are also popular, so any paint they make today will sell soon at a normal price. The contribution margins are \$5/gallon of Blue, \$6/gallon of Green, and \$7/gallon of Yellow. What combination of Blue, Green, and Yellow paint should they make today to maximize their total contribution to profit and overhead?

Formulate the linear programming problem that will let them make the optimal amount of Blue, Green, and Yellow paint.

**10<sup>1/2</sup>)** Solve the *Purdy Paint Company* problem using Excel

**Project:** Epidemiologists at the Centers for Disease Corraling (CFDC) have identified a frightening new disease which they have named *Virulence*. Scattered cases are already occurring in this country. Indications point to massive new outbreaks within the next 6 months. CFDC has been motoring the situation since this new disease first came to light, and at this point several important facts are now well established:

- Virulence is highly contagious.
- Immunity to virulence is virtually nonexistent in this country.
- No effective treatment specific to the disease has yet been developed, but conventional treatments for the symptoms yield 91% complete remission of symptoms and 4% partial recovery with some significant disablement. The other 5% die.
- Survivors seem to be totally immune to getting the disease again.
- All types of vaccine under test have shown rather good effectiveness and almost complete absence of side effects for most of the population. Oral vaccines seem to be necessary to the safe vaccination of diabetics and certain others.

*You* have been appointed by the President of the United States to take immediate charge of the immunization program in this country. Four vaccines have been given emergency type approval by the FDA as being safe and effective. However, no vaccine is perfect. In this disease, between 1% and 10% of those vaccinated do not develop immunity, but the more people are immunized the less chance those not immunized, either from not being vaccinated or from an ineffective vaccination, will be exposed and get sick. The approved vaccines, and their effective immunization rates, are:

| <u>Vaccine</u> | <u>Type</u>                 | <u>% of those vaccinated who become immune</u> |
|----------------|-----------------------------|--|
| I              | Oral, live virus            | 90%  |
| II             | Oral, killed virus          | 96%  |
| III            | Intramuscular, live virus   | 95%  |
| IV             | Intramuscular, killed virus | 99%  |

Under the President's recent Executive Order, you are being asked to effectively immunize as many people as possible. In any event, there are approximately 48 million individuals who *must* be vaccinated to prevent destruction of the fabric of our society. This includes individuals who are at high risk of exposure, such as police, teachers, and medical professionals. It also includes individuals otherwise critical to the continued functioning of society, such as the judiciary and virtually all politicians, major political contributors, and lawyers. Of these 48 million, at least 10 million are subject to conditions that make an oral vaccine mandatory.

Many materials and processes are involved in the production of any safe and effective vaccine. Not all of these are scarce enough to limit the Virulence Program, but a few are. The process of attenuating a live virus for vaccine will require .3 units of the attenuating medium per dose of the oral vaccine and 1.7 units per dose of the intramuscular vaccine. 20 million units of attenuating medium can be made available in time to meet your deadline. Killed virus vaccines require a virus killing medium. .25 (i.e.,  $\frac{1}{4}$ ) units of killing medium are needed per oral dose and 1.5 units are required for each dose of intramuscular killed virus vaccine. The industry can provide up to 30 million units of virus killing medium.

The processes for converting a killed or attenuated virus culture into a vaccine are complex. Due to recent industry over-expansion, those processes that are in common between oral and intramuscular vaccines are available in such generous supply that there is no possibility that they might limit the Virulence Program.

Unfortunately, this is not true for the processes that are specific to either oral or intramuscular vaccines. Using a composite unit of measure of manufacturing capacity, 80 million such units can be made available for intramuscular vaccine production, of which a dose of the killed virus vaccine requires 3 and the live (attenuated) virus vaccine requires 2. Only 6 million composite oral vaccine manufacturing units are available to the program, but oral vaccines only require .3 units per dose of live virus vaccine and only .32 units per dose of the killed virus vaccine.

The industry is cooperating fully, but they require direction and coordination. Specifically, they need you to tell them how many doses of each vaccine they should produce in order to meet the country's needs. Once they know that, they can proceed to implement the vaccine manufacturing program.

The President is also interested. Based on available resources, what will you and the industry be able to accomplish?

Formulate this situation as an LP problem and solve it using EXCEL.

Answer each of the following with a complete, clearly labeled paragraph. Justify your answer using only the printout for the original situation; you answer must not rely on rerunning the model. (You may rerun the model as a check, but do not include the printout or any references to it in your report!)

Do not carry over the changes from one question to another; for example answer part c assuming you do have to provide 10 million oral vaccinations, answer part d assuming you do have to provide 10 million oral vaccinations and you get no help from Canada.

- a) How many immunities could be produced? How many units of each kind of vaccine need to be made to achieve this?
- b) How many immunities could be produced if you didn't have to meet the constraint of 10 million oral vaccinations? How many units of each kind of vaccine need to be made to achieve this?
- c) Canada has offered to supply the US with 100,000 units of any of the four key raw materials (Attenuating medium, killing medium, oral composite, intramuscular composite). However, because of the very long and expensive process of certifying pharmaceuticals coming from foreign countries, we can only accept one of the four materials. Which offer of 100,000 units should CFDC accept? How many total immunities will the new situation enable the U.S. to produce?
- d) How many immunities could be produced if you didn't have to meet the constraint of 48 million vaccinations?
- e) Give a brief description of what would happen if the efficiency of the Oral Live vaccine could be raised from 90% to 91%. Give an even briefer description of what would happen if, instead, the efficiency of the Oral Live vaccine could be raised from 90% to 92%.
- f) Treat 48 million vaccinations as one goal and the number of immunities from (d) as another goal. Give shortfalls from either goal the same weight of 1. What solution minimizes the weighted sum of shortfalls?
- g) Membership in the fuzzy set of politically bad outcomes = shortfall from goal 1 divided by 48 million. Membership in the fuzzy set of medically bad outcomes = shortfall from goal 2 divided by the answer to (d). Membership in the fuzzy set of bad outcomes = membership in the set of politically bad outcomes or membership in the set of medically bad outcomes, whichever is more. What solution minimizes membership in the fuzzy set of bad outcomes?