Widgets and Gadgets (Unpainted)

A widget takes 4 hours to assemble
A gadget takes 8 hours to assemble

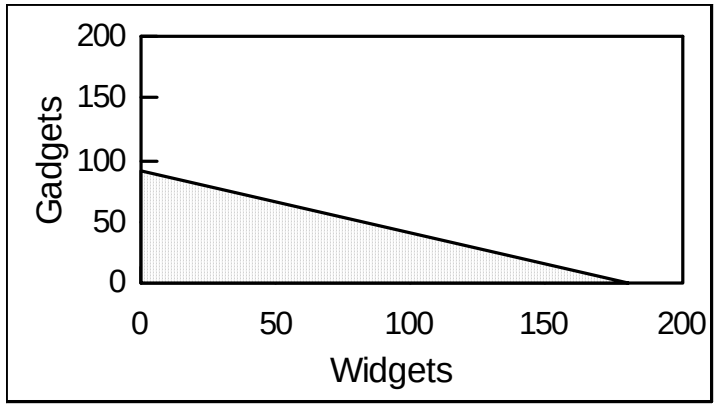
If W is the number of widgets produced and G is the number of gadgets produced, the number of hours of assembly time required is $4W + 8G$

A maximum of 720 assembly hours are available per eight hour day since your factory can only accommodate 90 workers in the assembly area.

$4W + 8G \leq 720$

Each widget earns $50 contribution to profit and overhead. The contribution of gadgets varies from month to month; some months it is as low as $20 per gadget while other months it is as high as $110.

All widgets and gadgets are sold immediately at the going rate; none are kept in inventory.



Every point in the graph represents a "production schedule" calling for producing a number of widgets equal to the horizontal coordinate of the point, and a number of gadgets equal to the vertical coordinate of the same point. Points outside the shaded triangular area are "infusible:" there are not enough assembly hours available to produce that many widgets and gadgets. The shaded area is called the "feasible region."

QUESTION: how many widgets and gadgets should you produce per day?

ANSWER: if the contribution per gadget is more than $100, produce 90 gadgets and no widgets. If the contribution per gadget is less than $100, produce 180 widgets and no gadgets.

(This problem is so simple that there's really no need for anything as powerful as linear programming, but it illustrates the barest essentials of an LP problem.)

Widgets and Gadgets (Painted)

You have acquired the facilities to paint your widgets and gadgets. (There is no longer any market for unpainted ones).

Your assembly constraints are as before: $4W + 8G \leq 720$ since each widget takes 4 hours to assemble, each gadget takes 8 hours to assemble, and the assembly department can accommodate up to 90 workers yielding 720 assembly hours per day.

Each widget takes 2 hours to paint, and each gadget takes 1 hour to paint. The paint department can accommodate up to 15 painters, yielding 120 painting hours per day. Thus, $2W + 1G \leq 120$

It is not necessary to use the full capacity if partial utilization is more profitable.

The net contribution for a painted widget is $100; the net contribution for a painted gadget varies, some months as low as $40 but other months as high as $210.

QUESTION:

How many widgets and gadgets should you produce per 8 hour day?

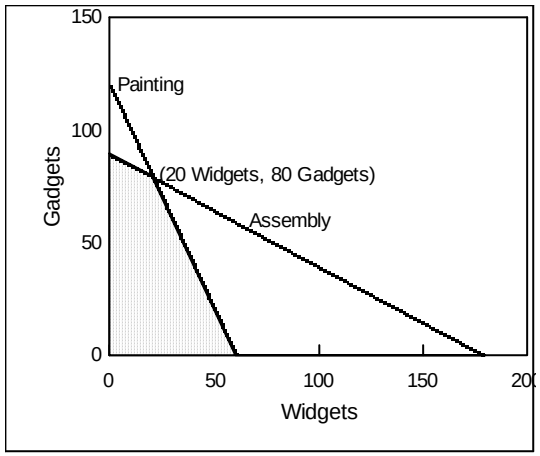
ANSWER:

If the contribution per gadget is less than $50, make 60 widgets and no gadgets. (60 is the most widgets you can paint; there is some idle assembly capacity)

If the contribution per gadget is between $50 and $200, make 20 widgets and 80 gadgets. (This is the only way to completely utilize both assembly and painting capacity)

If the contribution per gadget is over $200, make 90 gadgets and no widgets. (90 is the most gadgets you can assemble; there is some idle painting capacity.)

This problem is a bit more complicated, though still far simpler than the ones LP was invented for. The graph below and the Excel printouts on the following pages illustrate the solution when the going price for a gadget is $75. By the end of this course, you will be able to set up and interpret problems like this almost effortlessly



Base Case with contrib per Gadget = $75

	Widgets	Gadgets		
Production Schedule	20	80	Value	Constraint
Contribution	$100.00	$75.00	$8,000.00	
Assembly	4	8	720	720
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	Production Schedule Widgets	20.0000	0	$100.00	$50.00	$62.50
C2	Production Schedule Gadgets	80.0000	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D4	Assembly Value	720	$4.17	720	240	480
D5	Painting Value	120	$41.67	120	240	30

Effect of One Additional Assembly Hour

	Widgets	Gadgets		
Production Schedule	19.91667	80.16667	Value	Constraint
Contribution	$100.00	$75.00	$8,004.17	
Assembly	4	8	721	721
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	Production Schedule Widgets	19.9167	0	$100.00	$50.00	$62.50
E3	Production Schedule Gadgets	80.1667	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F5	Assembly Value	721	$4.17	721	239	481
F6	Painting Value	120	$41.67	120	240.5	29.875

Effect of One Additional Assembly Workstation

	Widgets	Gadgets		
Production Schedule	19.33333	81.33333	Value	Constraint
Contribution	$100.00	$75.00	$8,033.33	
Assembly	4	8	728	728
Painting	2	1	120	120

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	Production Schedule Widgets	19.3333	0	$100.00	$50.00	$62.50
E3	Production Schedule Gadgets	81.3333	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F5	Assembly Value	728	$4.17	728	232	488
F6	Painting Value	120	$41.67	120	244	29

Effect of One Additional Painting Workstation

	Widgets	Gadgets		
Production Schedule	25.33333	77.33333	Value	Constraint
Contribution	$100.00	$75.00	$8,333.33	
Assembly	4	8	720	720
Painting	2	1	128	128

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	Production Schedule Widgets	25.3333	0	$100.00	$50.00	$62.50
E3	Production Schedule Gadgets	77.3333	0	$75.00	$125.00	$25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F5	Assembly Value	720	$4.17	720	304	464
F6	Painting Value	128	$41.67	128	232	38

"Allowable" Increases and Decreases

Quantity	Amount it Increases or Decreases BY	Optimal Decision	Change in Optimal Payoff
Objective Coefficient of Decision Variable "X"	Less then "Allowable"	Unchanged	Quantity of X times the change in its objective coefficient
	Equal to "Allowable"	An infinite number of solutions (including the old one)	Quantity of X times the change in its objective coefficient
	More than "Allowable"	Changed	Changed (Rerun to find the new Optimal Payoff)
Right-Hand-Side of Binding Constraint (Shadow Price Not = 0)	Less then "Allowable"	Changed	Shadow Price of the constraint times the change in its RHS
	Equal to "Allowable"	Changed	Shadow Price of the constraint times the change in its RHS
	More than "Allowable"	Changed	Changed (Rerun to find the new Optimal Payoff)
Right-Hand-Side of Non-Binding Constraint (Shadow Price = 0)	Less then "Allowable"	Unchanged	Unchanged
	Equal to "Allowable"	Unchanged	Unchanged
	More than "Allowable"	Changed	Changed (Rerun to find the new Optimal Payoff)