

A Very Simple Linear Maximization Problem

Widgets and Gadgets (Unpainted)

A widget takes 4 hours to assemble

A gadget takes 8 hours to assemble

If $W = \#$ widgets produced and $G = \#$ gadgets produced,
the number of hours of assembly time required is $4W + 8G$

A maximum of 720 assembly hours are available per eight hour day since your factory can only accommodate 90 workers in the assembly area.

$$4W + 8G \leq 720$$

Each widget earns \$50 contribution to profit and overhead. The contribution of gadgets varies from month to month; some months it is as low as \$20 per gadget while other months it is as high as \$110.

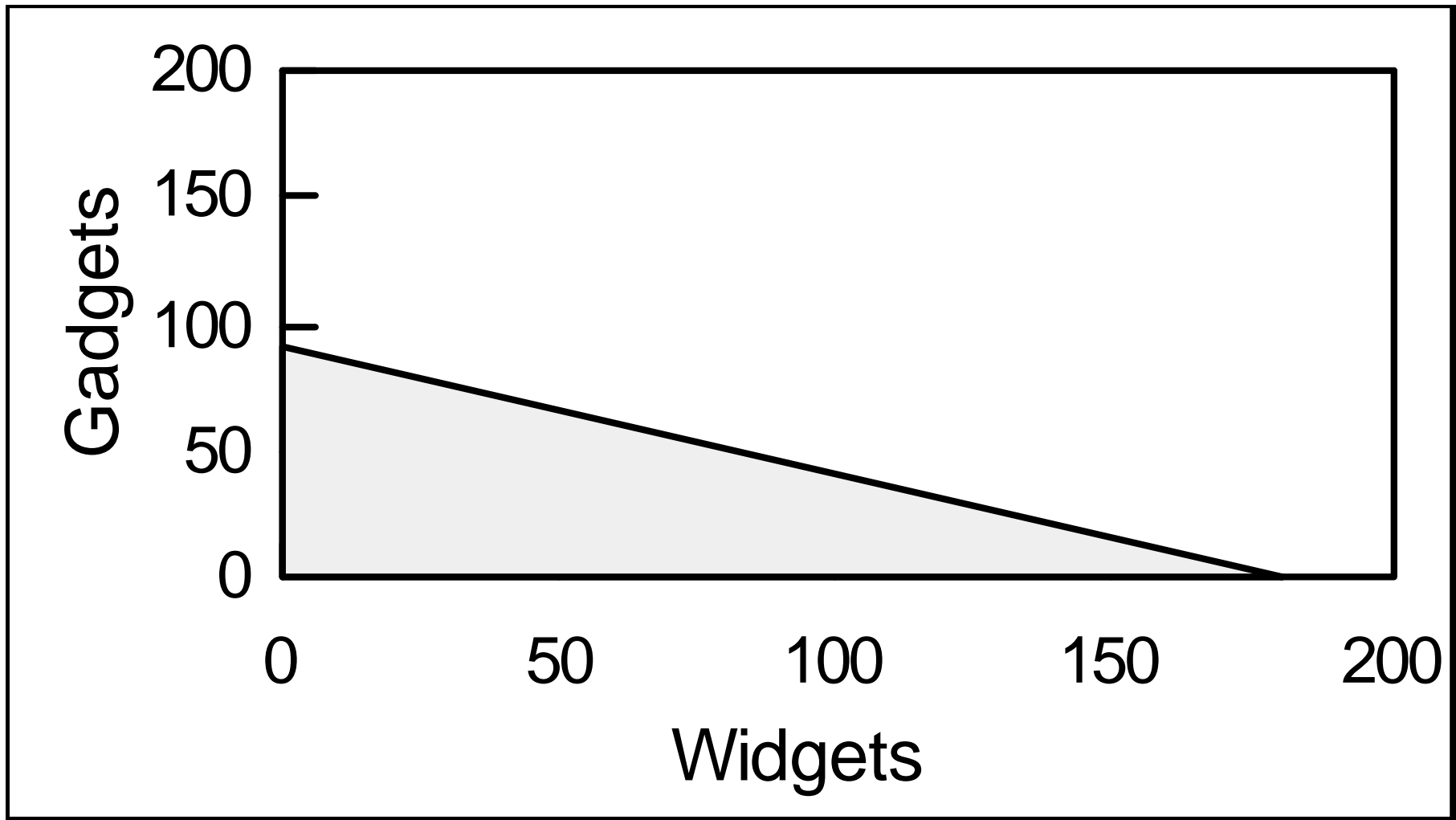
All widgets and gadgets are sold immediately at the going rate; none are kept in inventory.

QUESTION:

how many widgets and gadgets should you produce per day?

ANSWER: if the contribution per gadget is more than \$100, produce 90 gadgets and no widgets. If the contribution per gadget is less than \$100, produce 180 widgets and no gadgets.

(This problem is so simple that there's really no need for anything as powerful as linear programming, but it illustrates the barest essentials of an LP problem.)



Widgets and Gadgets (Painted)

You have acquired the facilities to paint your widgets and gadgets. (There is no longer any market for unpainted ones).

Your assembly constraints are as before: $4W + 8G \leq 720$ since each widget takes 4 hours to assemble, each gadget takes 8 hours to assemble, and the assembly department can accommodate up to 90 workers yielding 720 assembly hours per day.

Each widget takes 2 hours to paint, and each gadget takes 1 hour to paint. The paint department can accommodate up to 15 painters, yielding 120 painting hours per day. Thus, $2W + 1G \leq 120$ It is not necessary to use the full capacity if partial utilization is more profitable.

The net contribution for a painted widget is \$100; the net contribution for a painted gadget varies, some months as low as \$40 but other months as high as \$210.

QUESTION: How many widgets and gadgets should you produce per day?

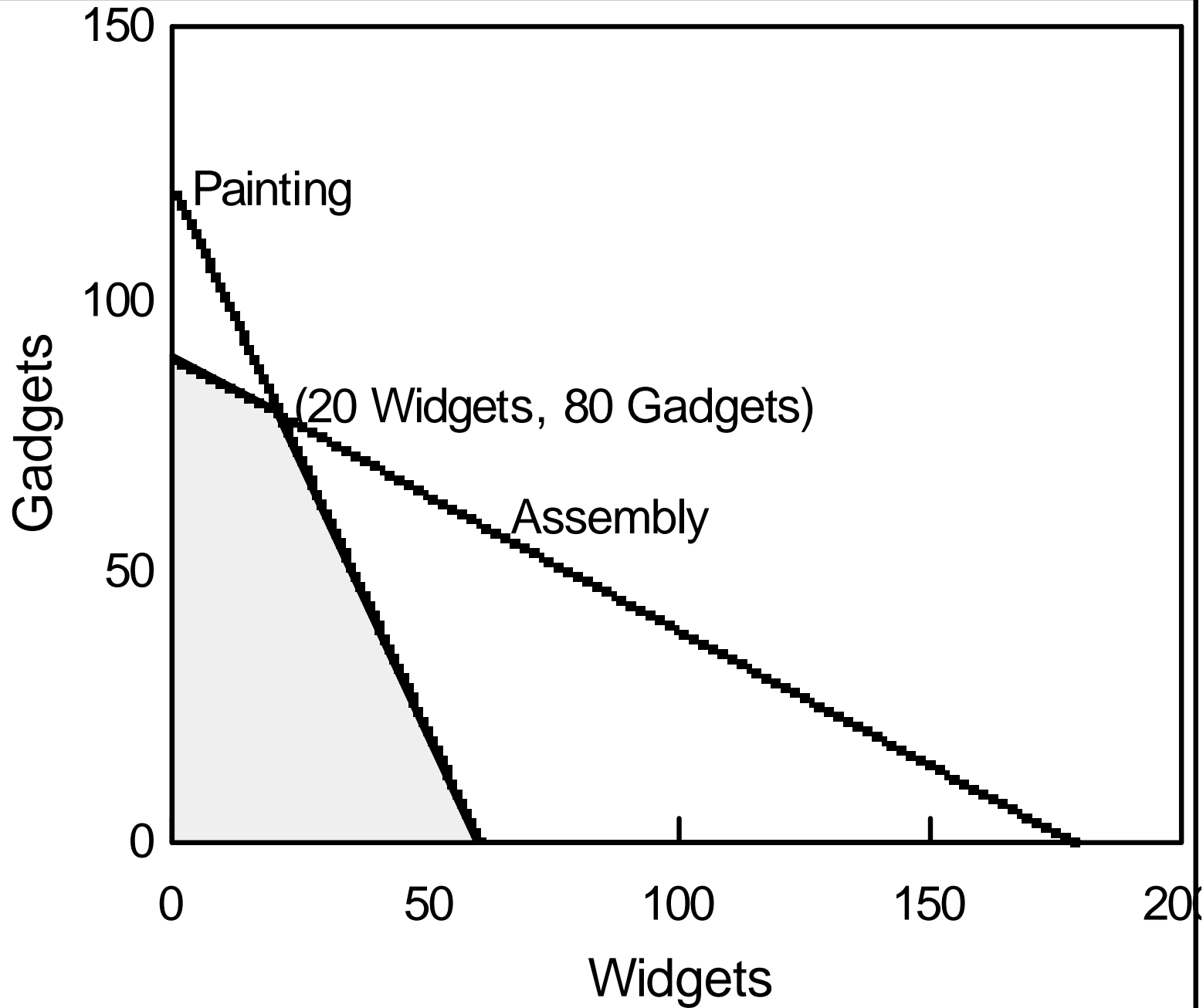
ANSWER:

If the contribution per gadget is less than \$50, make 60 widgets and no gadgets. (60 is the most widgets you can paint; there is some idle assembly capacity)

If the contribution per gadget is between \$50 and \$200, make 20 widgets and 80 gadgets. (This is the only way to completely utilize both assembly and painting capacity)

If the contribution per gadget is over \$200, make 90 gadgets and no widgets. (90 is the most gadgets you can assemble; there is some idle painting capacity.)

This problem is a bit more complicated, though still far simpler than the ones LP was invented for. The graph below and the LINDO printouts on the following pages illustrate the solution when the going price for a gadget is \$75. By the end of this course, you will be able to set up and interpret problems like this almost effortlessly!



Output for Widgets and Gadgets

MAX 100 WIDGETS + 75 GADGETS

SUBJECT TO 2) 4 WIDGETS + 8 GADGETS <= 720
 3) 2 WIDGETS + GADGETS <= 120

OBJECTIVE FUNCTION VALUE 1) 8000.000000
 VARIABLE VALUE REDUCED COST
 WIDGETS 20.000000 .000000
 GADGETS 80.000000 .000000

ROW SLACK OR SURPLUS DUAL PRICES
 2) .000000 4.166667
 3) .000000 41.666670

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	50.000000	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	720.000000	240.000000	480.000000
3	120.000000	240.000000	30.000000

Effect of One Additional Assembly Hour

MAX 100 WIDGETS + 75 GADGETS

SUBJECT TO 2) 4 WIDGETS + 8 GADGETS <= 721
 3) 2 WIDGETS + GADGETS <= 120

OBJECTIVE FUNCTION VALUE 1) 8004.16700
 VARIABLE VALUE REDUCED COST
 WIDGETS 19.916670 .000000
 GADGETS 80.166660 .000000

ROW SLACK OR SURPLUS DUAL PRICES
 2) .000000 4.166667
 3) .000000 41.666670

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	50.000000	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	721.000000	239.000000	481.000000
3	120.000000	240.500000	29.875000

Effect of One Additional Painting Workstation

MAX 100 WIDGETS + 75 GADGETS
 SUBJECT T 2) 4 WIDGETS + 8 GADGETS <= 720
 3) 2 WIDGETS + GADGETS <= 128

OBJECTIVE FUNCTION VALUE 1) 8333.33300

VARIABLE	VALUE	REDUCED COST
WIDGETS	25.333330	.000000
GADGETS	77.333340	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	4.166666
3)	.000000	41.666670

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	49.999980	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	720.000000	304.000000	464.000000
3	128.000000	232.000000	38.000000

Effect of One Additional Assembly Workstation

MAX 100 WIDGETS + 75 GADGETS
 SUBJECT TO 2) 4 WIDGETS + 8 GADGETS <= 728
 3) 2 WIDGETS + GADGETS <= 120

OBJECTIVE FUNCTION VALUE 1) 8033.33300

VARIABLE	VALUE	REDUCED COST
WIDGETS	19.333330	.000000
GADGETS	81.333340	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	4.166667
3)	.000000	41.666670

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	50.000000	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	728.000000	232.000000	488.000000
3	120.000000	244.000000	29.000000

Effect of a Redundant Constraint

MAX 100 WIDGETS + 75 GADGETS
 SUBJECT TO 2) 4 WIDGETS + 8 GADGETS <= 720
 3) 2 WIDGETS + GADGETS <= 120
 4) WIDGETS + GADGETS <= 101

OBJECTIVE FUNCTION VALUE 1) 8000.00000

VARIABLE	VALUE	REDUCED COST
WIDGETS	20.000000	.000000
GADGETS	80.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	4.166666
3)	.000000	41.666670
4)	1.000000	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	49.999980	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	720.000000	12.000000	480.000000
3	120.000000	3.000000	30.000000
4	101.000000	INFINITY	1.000000

Effect of a Degenerate Constraint

MAX 100 WIDGETS + 75 GADGETS
 SUBJECT TO 2) 4 WIDGETS + 8 GADGETS <= 720
 3) 2 WIDGETS + GADGETS <= 120
 4) WIDGETS + GADGETS <= 100

OBJECTIVE FUNCTION VALUE 1) 8000.00000

VARIABLE	VALUE	REDUCED COST
WIDGETS	20.000000	.000000
GADGETS	80.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	4.166667
3)	.000000	41.666670
4)	.000000	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WIDGETS	100.000000	50.000000	62.500000
GADGETS	75.000000	125.000000	25.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	720.000000	.000000	480.000000
3	120.000000	.000000	30.000000
4	100.000000	INFINITY	.000000

"Allowable" Increases and Decreases

Quantity	Amount it Increases or Decreases BY	Optimal Solution	Amount the Optimal Value changes BY
Objective Coefficient of Variable "X"	Less than "Allowable"	Unchanged	Quantity of X times change in its objective coefficient
Objective Coefficient of Variable "X"	Equal to "Allowable"	Infinite # of solutions (inc.luding the old one)	Quantity of X times change in its objective coefficient
Objective Coefficient of Variable "X"	More than "Allowable"	Changed	Changed (Rerun to find the new OV)
RHS of Binding Constraint	Less than "Allowable"	Changed	Dual price of the constraint times the change in its RHS
RHS of Binding Constraint	Equal to "Allowable"	Changed	Dual price of the constraint times the change in its RHS
RHS of Binding Constraint	More than "Allowable"	Changed	Changed (Rerun to find the new OV)
RHS of Non-Binding Constraint	Less than "Allowable"	Unchanged	Unchanged
RHS of Non-Binding Constraint	Equal to "Allowable"	Unchanged	Unchanged
RHS of Non-Binding Constraint	More than "Allowable"	Changed	Changed (Rerun to find the new OV)

Dimensional Analysis for Widgets & Gadgets:

$$\text{MAX } 100 \text{ W} + 75 \text{ G}$$

$\frac{\text{Contribution Widgets}}{\text{Per Widget}} + \frac{\text{Contribution Gadgets}}{\text{Per Gadget}}$

SUBJECT TO

$$2) \quad 4 \text{ W} + 8 \text{ G} \leq 720$$

$\frac{\text{Asmb. Hr. Widgets}}{\text{Per Widget}} + \frac{\text{Asmb. Hr. Gadgets}}{\text{Per Gadget}} \leq \text{Asmb. Hr.}$

$$3) \quad 2 \text{ W} + 1 \text{ G} \leq 120$$

$\frac{\text{Paint Hr. Widgets}}{\text{Per Widget}} + \frac{\text{Paint Hr. Gadgets}}{\text{Per Gadget}} \leq \text{Paint Hr.}$

OBJECTIVE FUNCTION VALUE 1) 8000.00000

Contribution: $20 * \$100 + 80 * \75

VARIABLE	VALUE	REDUCED COST
W	20.000000	.000000
Widgets		$\frac{\text{Dollars per}}{\text{Required Widget}}$
G	80.000000	.000000
Gadgets		$\frac{\text{Dollars per}}{\text{Required Gadget}}$

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	4.166667
Unused Assembly Hours		$\frac{\text{Contribution}}{\text{Per Asmb. Hr.}}$
3)	.000000	41.666670
Unused Painting Hours		$\frac{\text{Contribution}}{\text{Per Paint Hr}}$

9 January 10, 2002