Bayes Theorem in Odds Form

Bayes Theorem in Probability Form:

$$P(E\,|X) = \frac{P(E\&X)}{P(X)} = \frac{P(X|E)P(E)}{p(X)} \qquad P(notE\,|X) = \frac{P(X\,|notE)P(notE)}{p(X)}$$

Bayes Theorem in Odds Form (odds in favor of E):

Prior Odds in favor of event E: $\Omega(E\,) = \dfrac{P(E)}{P(notE)}$

Likelihood ratio of datum X relative to event E: $R(X)) = \dfrac{P(X\,|E)}{P(X\,|notE)}$

Posterior Odds in favor of event E:

$$\Omega(p\,|X) = \frac{P(E\,|X)}{P(notE\,|X)} = \frac{P(X\,|E)P(E)/P(X)}{P(X\,|notE)P(notE)/P(X)}$$

$$= \left(\frac{P(X|E}{P(X\,|notE)}\right)\left(\frac{P(E)}{P(notE)}\right)\left(\frac{1/P(X)}{1/P(X)}\right) = LR(X)\Omega(E)$$

Application to Daltex

Suppose you are very unsure about the prior probability that Daltex will go public, but you are confident of the likelihoods based on the historical performance of the research firm.

Let Ω(GoPub) be the unknown prior odds in favor of Daltex going public.

Based on the payoffs, the critical odds are $\frac{\$55,000-\$52,500}{\$100,000-\$55,000} = \frac{2,500}{45,000} = .0556$

the likelihood ratios are:

$$LR(SayGo) = \frac{P(SayGo|GoPub)}{P(SayGo|NoGoP)} = \frac{0.99}{0.5} = 1.98$$

$$LR(SayNO) = \frac{P(SayNO|GoPub)}{P(SayNO|NoGoP)} = \frac{0.01}{0.5} = 0.02$$

The posterior odds in favor of Daltex going public are

Ω(GoPub|SayGo) = 1.98 Ω(GoPub) Ω(GoPub|SayNO) = 0.02 Ω(GoPub)

If Ω(GoPub) is below 0.0556/1.98 = 0.028, then Ω(GoPub|SayGo) and Ω(GoPub|SayNO) are both $<$ 0.0556 so don't buy Daltex no matter what the consultant says. The information is of no value.

If Ω(GoPub) is above 0.0556/0.02 = 2.778, then Ω(GoPub|SayGo) and Ω(GoPub|SayNO) are both $>$ 0.0556 so buy Daltex no matter what the consultant says. The information is of no value.

If Ω(GoPub) is between 0.028 and 2.778, then you should buy Daltex if consultant says Go and not if consultant says No. The information is valuable.