Interpretations of Probability

a. The Classical View: Subsets of a set of equally likely, mutually exclusive possibilities

- **Problem of infinite or indefinite sets**
- **Problem of "equally likely"**

b. The Relative Frequency View

- **Propensity**

c. Subjective Views

- **The connection between belief, desire, and action**
- **Dutch Books**

d. Coherence and Conditionalization

Fair Bets

A bet on an event E is a contract in which person A bets Against the event and person F bets For the event:

- **Person A promises to pay $x to person F if event E occurs, and**
- **Person F promises to pay $y to person A if event E does not occur.**
- **The Stake of the bet is the sum of the absolute values of the two amounts, x + y.**
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A bet is fair from an individual's point of view if the individual believes its expected value, xP(E) - yP(not E), equals zero.
P(E) and P(~E) are that person's subjective probabilities.

Betting Odds

The Betting Odds Against an Event E, Ω(~E), equals x/y to 1 against.

If a bet where	**person A pays \$x to person F if E occurs** **F pays \$y to A if E does not occur**	**is a fair bet**

then so is a bet where person A pays \$Ω(~E)=$^x/_y$ to person F if E occurs
for every dollar that person F pays to person A if E does not occur.

The Betting Odds in Favor of an Event E, W(E), equals y/x to 1 in favor.
Colloquially, when someone says "the odds of that happening are 100 to 1" they mean the fair betting odds against it happening are 100 to 1 against, meaning that the probability is approximately .01 (technically, 1/101 or .00990099)

Simplified Bookmaking

A bookmaker posts betting odds $\Omega(\sim E)$ against various events E.

If you bet for an event E, he agrees to pay you $\$\Omega(\sim E)$ if E occurs for every dollar you agree to pay him if E does not occur.

If you bet against E, he agrees to pay you $\$\Omega(E)$ if E does not occur for every dollar you agree to pay him if E occurs.

If $P(E)=p$, $\$\Omega(E)={}^{p}/_{(1-p)}$, and $\$\Omega(\sim E)={}^{(1-p)}/_{p}$, then the EMV of every bet is zero -- a "fair bet."

If $\$\Omega(E)<{}^{p}/_{(1-p)}$, and $\$\Omega(\sim E)<{}^{(1-p)}/_{p}$, that's the bookie's profit margin.

However, we will simplify the situation by assuming that the bookmaker will take a fair bet in any amount for or against any event E as long as you pay him a betting fee, separate from the winnings or losses, of one dollar per bet.

We will also assume that the bookie's offerings are complete, in the sense that he posts betting odds against all combinations of a pair of simple events p and q. (The "de Finetti closure.")

The Bookie's Subjective Probabilities

If the bookmaker believes that the probability that an event E will occur is P(E),

and he posts betting odds against E of $\$\Omega(\sim E) = \dfrac{1-P(E)}{P(E)}$

and he posts betting odds for E of $\$\Omega(E) = \dfrac{P(E)}{1-P(E)}$

then the EMV to him of a customer's one-dollar bet against E equals

$$P(E)*\$1 - (1-P(E))*\$W(E) = \$P(E) - (1-P(E))*\$\,\frac{P(E)}{1-P(E)} = \$P(E) - \$P(E) = \$0$$

and the EMV of to him a customer's one-dollar bet on E equals

$$-P(E)*\$\Omega(\sim E) + (1-P(E))*\$1 = -P(E)*\$\,\frac{1-P(E)}{P(E)} + \$(1 - P(E)) = \$0$$

If a one-dollar bet has an EMV of zero, so does a bet in any other amount.

Since the bookmaker collects $1 for every bet up front, he is money ahead on average to accept any such bet as long as his probability estimates are correct.

"Making Book"

Even if the probability estimates are wrong, if the bookmaker has many customers whose spectrum of opinions are not too far different from his, he may put together a collection or "book" of bets where the winnings of customers who guess ri ght are covered by the losses of customers who guess wrong, and the bookmaker pockets the dollar-per ber fees.

This is called "making his book;" it is a variety of perfect hedge.

"Dutch Book"

Given posted betting odds against an event E, $\Omega(\sim E)$, you can find the equivalent subjective probability of the event,

$P(E) = 1-P(\sim E) = 1-{}^{\Omega(\sim E)}/_{(1+\Omega(\sim E))}$.

If a bookmaker's posted betting odds correspond to a set of subjective probabilities that are inconsistent with the laws of probability theory, then a customer can make a Dutch Book against the bookmaker. In a Dutch Book, the customer takes money away from the bookmaker at no risk to himself.

(However, this only works if you can make negative bets, forcing the bookie to use the odds he gives the public and let you take the odds he uses for himself. Real bookies are not likely to agree to this!.)