Decision Making Under Risk

Probability

Historical Data (relative frequency) (e.g Insurance)

Cause and Effect Models (e.g. casinos, weather forecasting)

Subjective Probability

Often, the decision maker must rely on subjective probabilities. Don't dismiss this as mere intuition. If you are familiar with the domain in which the problem arises, you can assess very useful, if imperfect probabilities. In fact, there is a sense in which all probabilities are subjective, since the only absolutely "objective" probability that something will happen is 1 if it happens, or 0 if it doesn't. Any other probability is an expression of the incomplete information possessed by a particular decision maker at a particular point in time.

Expected Monetary Value (EMV)

The **EMV** criterion is fundamental to DMUR.

S_i The i[th] State of Nature from our list of possible States of Nature.

A_j The j[th] Action from our list of possible decisions.

$P(S_i)$ The probability that S_i will occur.

V_{ij} The Payoff to the decision maker if he or she chooses A_j and S_i occurs.

EMV_j Expected Monetary Value. The long term average payoff if we could repeat the same decision many times under the same circumstances.

$$EMV_j = \sum_{i=1}^{n} P(S_i) \times V_{i,j}$$

The EMV criterion chooses the decision alternative which has the highest EMV. We'll call this EMV the Expected Value Under Initial Information (EVUII) to distinguish it from what the EMV might become if we later get more information. ***Do not*** make the common student error of believing that the EMV is the payoff that the decision maker will get. The actual payoff will be the $V_{i,j}$ for that alternative (j) and for the State of Nature (i) that actually occurs.

Expected Monetary Value is a ***long run*** criterion.

a decision maker (individual or institution) who consistently applies the EMV criterion will do better in the long run than one who does not,

provided that the amount of money at risk is small relative to the decision maker's total wealth.

If the amount at risk is substantial, then we need to consider the possibility that a few bad outcomes could drive total wealth to zero. When total wealth is zero, you no longer have access to good outcomes. This concept is sometimes called the case of Gambler's Ruin.

Introduction to Risk Analysis

One way to evaluate the risk associated with an Alternative Action by calculating the variance of the payoffs. Depending on your willingness to accept risk, an Alternative Action with only a moderate EMV and a small variance may be superior to a choice that has a large EMV and also a large variance. The variance of the payoffs for an Alternative Action is defined as

$$Variance_j = \sum_{i=1}^{n} P(S_i) \times (EMV_j - V_{i,j})^2$$

Given Action j, we square the difference between each payoff and the EMV, weight it with the probability of the State of Nature that leads to that payoff, and add up those products. Most of the time, we wand to make EMV as large as possible and variance as small as possible, Unfortunately, the maximum-EMV alternative and the minimum-variance alternative are usually not the same, so that in the end it boils down to an educated judgment call.

Value of Information

Suppose that although you couldn't *control* the future, you could *foresee* the future with perfect accuracy. That's what Certainty is all about. If you could foresee the future perfectly, then you would have Perfect Information. Perfect Information is certainly the very best kind of information. Imperfect Information can't be as valuable.

In many situations, you can at least get *bette*r information before you commit to an Alternative Action, often at a price. Before tooling up to produce a radical new product, you can do consumer research using prototype units. Before buying land for a plant, you can survey its neighbors to learn whether they will oppose your plans. Race tire manufacturers often rent a race track for a day so that teams can test to see if a new tire design is actually faster than the existing model.\

We will begin our discussion of the value of information with perfect information because it is much easier to assess than the more realistic case of imperfect information/

Expected Value of Perfect Information

The **E**xpected **V**alue of **P**erfect **I**nformation (**EVPI)** provides an absolute upper limit on the value of additional information (ignoring the value of reduced risk). It measures the amount by which you could improve on your best EMV if you had perfect information. It is the difference between the **E**xpected **V**alue **U**nder **P**erfect **I**nformation (**EVUPI**) and the EMV of the best action (EVUII).

$$\text{evUpi} = \sum_{i=1}^{n} P(S_i) \times \max_{j=1}^{m}(V_{i,,j})$$

$$\text{EVUII} = \max_{j=1}^{m} (EMV_j)$$

$$\text{EVPI} = \text{evUpi} - \text{EVUII}$$

The Expected Value of Perfect Information measures how much better you could do on this decision, averaging over repeating the decision situation many times, if you could always know what State of Nature would occur, just in time to make the best decision for that State of Nature. Remember that it does ***not*** imply control of the States of Nature, just perfect prediction. Remember also that it is a long run average. It places an upper limit on the value of additional information.

Back to Roger's Problem

Roger is not actually in a state of ignorance. He is in a state of risk. He has a pretty good idea what the weather can be like in March. He's not worried about snow. If it snows, he gets to put on the event the next weekend. His concern is just with rain and with cold. He had a subordinate call the Weather Bureau to get the facts. In the last 100 years, the weekend he has the event scheduled was a cold, wet one 10 times, a cold, dry one 15 times, a warm, wet one 40 times, and it was a warm, dry weekend 35 times. He is willing to assume that those figures tell him the probabilities of the events.

He'll try the EMV criterion to see what it tells him.

NET PAYOFF	Cold Wet	Cold Dry	Warm Wet	Warm Dry	EMV	Variance/ 1,000,000
No Busch, no MARTA	($375,000)	($212,500)	$112,500	$2,062,500	$697,500	1,028,259
Busch, no MARTA	($550,000)	($355,000)	$35,000	$2,375,000	$737,000	1,480,694
MARTA, no Busch	($225,000)	($533,750)	$457,500	$1,968,750	$769,500	**895,980**
MARTA and Busch	($270,000)	($640,500)	$549,000	$2,362,500	**$923,400**	1,290,211
Probability	0.1	0.15	0.4	0.35		

The combination of MARTA and the Busch support race has the highest Expected Monetary Value by a healthy margin.

But using MARTA with no Busch race is the least risky choice by a substantial amount.

The first two choices (the "plain vanilla" race and the Busch, no MARTA option) are clearly inferior to the MARTA, no Busch option since they have lower EMV <u>and</u> higher variance (less average payoff and more risk).

If Roger does go with MARTA and Busch, remember that he won't make **$923,400**. He either will lose $270,000 or $640,500 ***or*** he will make $549,000 or $2,362,500. Choosing via EMV is "going with the odds". Those who "go with the odds" come out ahead in the long run, *if* they can afford to stay in the game. That's how the casinos at Lost Wages make their money.

Roger has to accept some risk, but he doesn't have to like it. If he can reduce the risk at an affordable cost, he certainly would consider it. That's where the EVPI comes in. The EVPI can tell him the most he should consider paying for new information about the weather next March. And that would be for perfect foreknowledge of the weather, something he isn't going to get in this lifetime. Clearly, better information about next March's weather is valuable, but not that valuable. And he has no motive to pay the full value of the information. He'll want to pay less so that his bottom line can improve.

Roger's Expected Value of Perfect Information

Let's imagine that Roger can get a sealed envelope containing a perfect weather forecast for race day. How much better off would he be with the envelope than without it? While the envelope remains sealed, Roger doesn't know what it contains. He *does know* that the chance that it has a Cold Wet forecast is 0.1, the chance that it has a Cold Dry forecast is 0.15, the chance that it as a Warm Wet forecast is 0.4, and the chance that it has a Warm Dry forecast is 0.35.

NET PAYOFF	Cold Wet	Cold Dry	Warm Wet	Warm Dry	EMV	Best EMV
No Busch, no MARTA	($375,000)	($212,500)	$112,500	$2,062,500	$697,500	
Busch, no MARTA	($550,000)	($355,000)	$35,000	$2,375,000	$737,000	
MARTA, no Busch	($225,000)	($533,750)	$457,500	$1,968,750	$769,500	
MARTA and Busch	**($270,000)**	**($640,500)**	**$549,000**	**$2,362,500**	**$923,400**	**$923,400**
Ideal	*($225,000)*	*($212,500)*	*$549,000*	*$2,375,000*	*$996,475*	***$996,475***
Probability	0.1	0.15	0.4	0.35	Difference	$73,075

EVUPI=$996,475 EVUII =$923,400 EVPI=$73,075

If Roger could get the magic envelope, he would be, on average, $73,075 better off than he is in the real world. This is because his expected return without the magic envelope is $923,400 while with it his average return is $996,475, or $923,400 + $73,075.

Roger's Expected Value Under Perfect Information is $996,475. Exactly what does that mean? **IF** it could really happen this way, it means that if it is going to turn out to be:

a cold, wet weekend then he will know in time to choose MARTA, no Busch and only lose $225,000.

a cold, dry weekend then he will know in time to choose no Busch, no MARTA and only lose $212,500.

a warm, wet weekend then he will know this in time to choose MARTA and Busch, and clear $549,000.

a warm, dry weekend then he will find this out in time to choose Busch, no MARTA, earning $2,362,500.

On the average, this would be $73,075 better than doing without perfect information. This is his EVPI for this decision.