## Critical Odds, Critical Probabilities

Suppose you must choose between action A and Action B when the probability of state S is p and the probability of state T is 1-p.

<table>
<thead>
<tr>
<th>State S</th>
<th>State T</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr(S)=p</td>
<td>Pr(T)=1-p</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Action A</th>
<th>AS</th>
<th>AT</th>
<th>p*AS +(1-p)*AT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action B</td>
<td>BS</td>
<td>BT</td>
<td>p*BS +(1-p)*BT</td>
</tr>
</tbody>
</table>

Without loss of generality, assume outcome AS is better than BS, the only nontrivial case is then when outcome BT is better than AT.

- **Action A better than Action B**
  \[
  p*AS +(1-p)*AT > p*BS +(1-p)*BT \\
  p*AS -p*BS > (1-p)*BT -(1-p)*AT \\
  p*(AS-BS) > (1-p)*(BT-AT)
  \]
  \[
  \frac{p}{1-p} > \frac{BT-AT}{AS-BS}
  \]

- **Action B better than Action A**
  \[
  p*AS +(1-p)*AT < p*BS +(1-p)*BT \\
  p*AS -p*BS < (1-p)*BT -(1-p)*AT \\
  p*(AS-BS) < (1-p)*(BT-AT)
  \]
  \[
  \frac{p}{1-p} < \frac{BT-AT}{AS-BS}
  \]
CALCULATING CRITICAL ODDS

Let \( \Omega = \frac{BT - AT}{AS - BS} \) = \( \frac{\text{advantage of B given T}}{\text{advantage of A given S}} \) = \( \frac{\text{regret of A given T}}{\text{regret of B given S}} \)

\( \Omega = \) "Critical Odds"

Note that \( \Omega \) is a pure value judgment, completely independent of any consideration of the probability of state S or T.

The more regrettable A would be if T happens, the higher the probability of S needs to be to justify choosing A.

CONVERTING FROM CRITICAL ODDS TO CRITICAL PROBABILITY

\[
\begin{array}{c|c}
\frac{p}{1-p} > \Omega & \frac{p}{1-p} < \Omega \\
P > (1-P)\Omega & P < (1-P)\Omega \\
P > \Omega - p\Omega & P < \Omega - p\Omega \\
P + p\Omega > \Omega & P + p\Omega < \Omega \\
p > \frac{\Omega}{1+\Omega} & p < \frac{\Omega}{1+\Omega} \\
\end{array}
\]

Choose Action A if \( p > \frac{\Omega}{1+\Omega} \)  
Choose Action B if \( p < \frac{\Omega}{1+\Omega} \),
### School Closing for Snow

<table>
<thead>
<tr>
<th></th>
<th>Snowstorm</th>
<th>No Storm</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pr(S)=p</td>
<td>Pr(T)=1-p</td>
<td></td>
</tr>
<tr>
<td>Close schools</td>
<td>AS =</td>
<td>AT =</td>
<td>p*AS +</td>
</tr>
<tr>
<td></td>
<td>Typical Snow Day</td>
<td>Wasted day</td>
<td>(1-p)*AT</td>
</tr>
<tr>
<td>Keep Schools</td>
<td>BS =</td>
<td>BT =</td>
<td>p*BS +</td>
</tr>
<tr>
<td>Open</td>
<td>Hardship, Potential Accidents</td>
<td>Typical School Day</td>
<td>(1-p)*BT</td>
</tr>
</tbody>
</table>

Let $\Omega = \frac{BT - AT}{AS - BS} = \frac{\text{Harm of wasting a good day}}{\text{Harm of opening on a bad day}}$

It is possible to estimate this ratio without putting numbers on AS, AT, BS, and BT. For example, if the harm of opening on a bad day is twice as bad as the harm of wasting a good day, then $\Omega = 1/2$, which means $\frac{\Omega}{(1+\Omega)} = 1/3$ and schools should be closed if the probability of a snowstorm exceeds 1/3.