Problems

1) Briefly define (not describe) the following terms in your own words:

DMUC	EMV	Prior Probability
DMUI	EVUII	Likelihood
DMUR	EVUPI	Joint Probability
Payoff Table	EVPI	Marginal Probability
Regret	EVII	Posterior Probability
Maximax	EVSI	Objective Probability
Maximin	EOL	Subjective Probability
Minimax Regret	Certainty	Bayes Theorem
LaPlace-Bayes	Ignorance	Critical Probability
State of Nature	Risk	Marginal Analysis
Alternative	Decision Tree	

2) Maureen and Charles own a Flower Shop. As you might guess, florists must deal with a highly seasonal business (relax - we aren't jumping back to Time Series Decomposition). The availability of stock is somewhat seasonal, though much less so since the development of fast air freight. More importantly, a lot of customs and holidays generate some very "peaky" demands for specific flowers. St. Valentine's Day is one of these.

To a florist, Valentine's means red roses. Lots of red roses if it is to be a successful selling period. This year, roses cost Maureen and Charles about $20/dozen in bulk. Since theirs is a full service florist, they dress up each dozen with greenery and Valentine - specific florist paper. Altogether, each dozen gets about $10 worth of perishables. They sell the roses to lovesick swains at $80/dozen. Leftover roses and ancillaries, if any, will give them the loveliest dumpster in town.

With their fine service, Charles and Maureen agree that 800 dozen is their most probable demand during the Valentines selling period. The worst case scenario they can envision is that only 450 dozen will be demanded. If everything breaks in their favor, they might face a demand for as many as 1200 dozen. How many dozen roses should they order from their supplier?

3) Your friend Tenfour Goodbuddy is an independent trucker whose truck is empty after his most recent haul. He has found a deal to take a load to Birmingham, with a return load included, for a total of $3000. Another deal would give him a load to Charlotte for $2500. Tenfour thinks there's a 50-50 chance of finding a $2500 return load from Charlotte. Otherwise he must return empty, with no revenue. Assume that the cost of a round trip to either city is the same, and that he sees no other loads presently available.

Sometimes in the past he has called the Thunder Road Truckemup Stop in Charlotte to get information. His friend there is very chatty, so it always costs him at least $10 for the call. 90% of the time when he had ended up with a return load, TRTS had told him "It's busy". 80% of the time when he had ended up with no return load (a "load of postholes"), TRTS had said "It's slow".

To summarize, the available probabilities are:

P(RL) = .5 P(B|RL) = .9 P(S|RL) = .1

P(NL) = .5 P(B|NL) = .2 P(S|NL) = .8

B = busy S= slow RL = return load NL = no return load

a) The above scenario implies a decision tree. Neatly draw that tree. Include all decisions and chance occurrences, and all payoffs. You may ignore the $10 cost of the call (and perhaps deal with it later).

b) Compute P(B), P(S), P(RL|B), P(RL|S), P(NL|B), and P(NL|S)

c) Solve the decision tree, and state the resulting decision rule. How much, if anything, is the $10 phone call to Charlotte worth?

4) Cal Swift inherited a small but basically empty factory building and a substantial investment portfolio from his cousin Tom, the famous inventor. He has been working day and night on his own invention, a camcorder that records directly to Compact Disc. It actually involves no new discoveries, so he can only protect it under Trade Secret law; he cannot patent it. He figures he only has 2 years to sell it, once he puts it on the market, because it will just not be all that hard for good engineers to figure out how to duplicate its function. As he sees the market, he will probably be able to sell them to distributors for $350 F.O.B. his factory door; the retail price obviously will be much higher.

One real question is whether the CD player industry will support it by providing TV converters for their players. Another question is whether consumers will really want the device, since although the images will be very high quality, the camcorder will be quite bulky and the disks cannot be edited. Given these factors, he supposes that during his 2 year window of opportunity he might be able to sell 1000, 10,000, 50,000, 100,000, or 250,000 SwiftCorders. He wishes that he knew which, but he really doesn't. *(Hint: don't these amount to States of Nature?)*

He sees 5 alternative ways that he might produce SwiftCorders. With no investment in manufacturing equipment, he could subcontract the manufacturing to *Mitsybitsy Heavy Industries* in Japan, at cost to him of $250 per unit at any volume. If he bought $50,000 worth of equipment for his factory, he could hire low cost workers (homeless, college students, and such) to hand make the SwiftCorders at a unit cost of $220. By adding a small machine shop to his factory at a cost of $2050,000 he could reduce his dependence on purchased parts, cutting the unit cost to $205. If he set up a regular assembly line in his plant at a cost of $750,000 he could drop unit cost all the way down to $160. And if he wants to go all the way, he can spend $2,000,000 more than that for a completely automated facility. That would slash unit cost to a rock bottom $125. No matter how he does it, he figures that the true market value of whatever equipment he buys will be ***zero*** after 2 years. In effect, his alternatives have 2 year fixed costs of $0, $50,000, $250,000, $1,000,000, and $3,000,000.

Much of this problem is far easier using a spreadsheet than with paper and a calculator, but it is your choice. Both ways is probably best, but either is acceptable.

a) This isn't Decision Analysis, but it often relates: At what unit volume does each manufacturing method break even?

b) List the Alternatives and States of Nature that Cal faces.

c) Construct the Payoff Table.

d) Assume that Cal is truly uncertain about demand. Show how he should choose an alternative under each of the following decision making criteria: Maximax, Maximin, LaPlace - Bayes.

e) Also construct the Regret Table. Show Cal's proper decision under the Minimax criterion.

f) Cal isn't all that clueless. On reflection and after some study, he concludes that there is a 20% chance that he will only sell 1000, a 30% chance that he will sell 10,000, and a 35 % chance that it will really take off and he will sell 50,000 SwiftCorders. If he's really lucky, he thinks there is a 10% chance of selling 100,000 and 5% chance of hitting the jackpot and selling a quarter of a million SwiftCorders. Given this information, what criterion should he apply? Apply it, and show the result.

g) Suppose that he could get more information on the potential demand for SwiftCorders. What is the absolute most he should pay for such information? By what name do we call that number? Realistically, what options might Cal have for improving his demand estimates?

h) Is it possible to figure a range of demand levels for each manufacturing method where it is the most profitable method? How would you do it? (You are *not required* to do it, but you might find it instructive to do it.)

5) Roget Pinky (our IndyCar entrepreneur from the first part of the Chapter) faced this situation:

	Cold Wet	Cold Dry	Warm Wet	Warm Dry
No Busch, no Marta	($375,000)	($212,500)	$112,500	$2,062,500
Busch, no Marta	($550,000)	($355,000)	$35,000	$2,375,000
Marta, no Busch	($225,000)	($533,750)	$457,500	$1,968,750
Marta and Busch	($270,000)	($640,500)	$549,000	$2,362,500
Probability	0.1	0.15	0.4	0.35

He could hire a Bonnie Day, a long range weather forecaster, to develop a long term forecast of next March's weather at Road Alpharetta. This lady uses incredible laser and satellite technology. Roget knows that a number of successful promoters of outdoor events use her services, despite the fact that they are very expensive. Given that she forecasts many months in advance, she cannot provide much detail. She only tells promoters that the weather will be ***Excellent, Fairly Good,*** or ***Awful.*** After several telephone calls to promoters and sanctioning bodies, he has assembled this picture of her performance record for her most recent 260 forecasts:

Events	**Weather**	**Forecast**
5	**Cold , Wet**	**Excellent**
10	**Cold, Dry**	**Excellent**
15	**Warm, Wet**	**Excellent**
80	**Warm, Dry**	**Excellent**
10	**Cold, Wet**	**Fairly Good**
30	**Cold, Dry**	**Fairly Good**
25	**Warm, Wet**	**Fairly Good**
15	**Warm, Dry**	**Fairly Good**
35	**Cold, Wet**	**Awful**
10	**Cold, Dry**	**Awful**
20	**Warm, Wet**	**Awful**
5	**Warm, Dry**	**Awful**

These forecasts were for events all over the U.S. at all times of year, so Roget knows that the information must be interpreted very carefully.

a) Construct a complete set of conditional probabilities describing her performance as a long term forecaster. You should come up with probabilities like $P(Excellent|Cold, Wet) = \frac{5}{5+10+35} = \frac{5}{50} = .10$.

b) Now Roget must decide if he should engage her services. That gives Roget a sequential decision problem. Set up the decision tree, assuming that she will work for free.

c) Solve the decision tree. What is the absolute most he should pay Bonnie for a forecast for this event?

d) Would your answer to the above question change if Roget were highly risk averse? Why or why not?

e) Suppose Bonnie has just changed her technology so that she can nail it completely every time. She now guarantees that if she does not get the high temperature within 3 degrees and the rainfall within $\frac{1}{4}$ inch, she will refund her fee and cover the opportunity cost of a wrong decision resulting from her forecast. Now what is the most Roget should pay?

6) LaLa Lovely is a romantic actress. Mega Studios wants to sign her for a movie to be filmed next spring. The Turnip Network wants her to star in a mini-series to be shot during the same period. Turnip has offered her a fixed fee, but Mega wants to give her a percentage of the Gross. Unfortunately, as usual, the Gross is not certain. She may choose either offer but not both.

The situation breaks down to this:

	State of Nature		
DECISION	**Small Gross**	**Medium Gross**	**Great Gross**
Mega Studios	200,000	1,000,000	3,000,000
Turnip Network	900,000	900,000	900,000
Probability	**0.3**	**0.6**	**0.1**

LaLa must decide soon. Who should she sign with? What's the most she should consider paying for a forecast of the Gross?

7) Continuing with LaLa's problem, Srivastiva and Associates is a well established Hollywood firm specializing in advising film figures and investors on the potential success of cinematic efforts. They can have the script for the film examined by a panel of experts, who would return either a favorable (F) or unfavorable (U) verdict. Srivastiva's customary fee for this service is $50,000, cash in advance. Their "Track Record" over the past 5 years has been as follows:

P(F|Small)=	**0.2**	**P(U|Small)=**	**0.8**
P(F|Medium)=	**0.5**	**P(U|Medium)=**	**0.5**
P(F|Great)=	**0.9**	**P(U|Great)=**	**0.1**

a) Assuming that LaLa hires Srivastiva, what will be the values of the joint and posterior probabilities?

b) This problem requires a decision tree. Please develop it.

c) Should LaLa engage the services of Srivastiva and Associates?

d) Including Srivastiva's fee, what is LaLa's Expected Value of Sample Information?

8) A very famous imaginary Casino game called the Saint Petersburg (before it was Leningrad) Paradox highlights tThe limitations of Expected Monetary Value as a decision criterion. are highlighted by a very famous imaginary Casino game called the Saint Petersburg (before it was Leningrad) Paradox. You must pay the Casino a predetermined sum for the privilege of playing each time. Our question is going to be this: What would be a "fair" amount to pay the Casino for each round of the Paradox. Alternatively, what is the most that a "rational" gambler would pay to play?

The essence of the game is as follows: Once you have paid your ante, you and the croupier each toss a fair coin. If you match, he pays you 2 rubles and you will both toss again. If you fail to match, he pays you nothing, and the game is over. If you toss again and match, he pays you an *additional* 4 rubles, and you will get to play the next toss. If you fail to match, the game is over and you keep your 2 rubles. If you won the second round, then you get to go a third time. This time, if your coins match, you win an *additional* 8 rubles and get to continue. Otherwise the game is over. His payoff to you doubles on every round until you lose.

From the viewpoint of having reached any given round, the probability of winning that round is simply $\frac{1}{2}$, but before the start of the game, the probability of both reaching *and* winning the n^{th} round is $(\frac{1}{2})^n$. The payoff from winning that round is 2^n rubles. The EMV of the first round is $\frac{1}{2} \times 2 = 1$, the EMV of the second round is $(\frac{1}{2})^2 \times 2^2 = \frac{1}{4} \times 4 = 1$, and the EMV of the n^{th} round is $(\frac{1}{2})^n \times 2^n = 1$ ruble.

a) Show that the EMV of the game is ∞. *(Hint: If the first round is worth $1, the first 2 rounds are worth $2, and the first n rounds are worth $n, show what all potentially possible rounds are worth.)*

b) Show that the variance of the payoff is ∞.

c) Comment briefly on the implications of this.

d) (Optional, but maybe some fun) DATA +, on the network, has a simulation of the Saint Petersburg Paradox game. Play it, and see if you can draw a conclusion concerning a "fair" ante.

9) Guido's EZ Loan Company pigeonholes credit applicants as Good and Bad risks. On the average, 10% of applicants are Bad risks. Guido uses a sophisticated computerized credit scoring model to attempt to discriminate between these 2 groups. A study of their experience in using the model yielded interesting results. Guido's gives credit to gGood risks are given credit 90% of the time. Bad risks get credit only 20% of the time.

Assume that a person selected at random from potential applicants to Guido's applies for credit and is granted a loan. What is the probability that Guido will have to break his legs?

10) Ajax Electronics Company must decide whether to invest in a new system for home music. The system, called chip audio, has no moving parts so it is potentially more reliable than discs or tape. The Net Present Value of the investment that would be required would be $400,000.

In the home entertainment industry, a technology either "catches on" or it doesn't; if only a minority of customers prefer a technology, then the recording studios do not use it and there is little or nothing to play on the system. This is what happened to Betamax and video discs in the home video market. If chip audio catches on, then the present value of the future net profits from the investment will be $600,000 but it the technology does not catch on the $400,000 will be wasted. If Ajax does not invest in chip audio, then there is no effect on their profit.

Payoff Table for Chip Audio Decision

	Chip Audio will catch on	**Chip Audio will not catch on**
Invest in Chip Audio	**$600,000**	**-$400,000**
Don't Invest in it	**$0**	**$0**

Ajax is a large enough firm that they are comfortable with the level of risk that this decision involves. They estimate that there is a 35% chance that Chip Audio will catch on. Should Ajax invest in Chip Audio?

11) For $10,000, Ajax can put together a prototype system with the "look and feel" of the final Chip Audio system, and get the reaction of a group of potential consumers. Experience has shown that these groups are somewhat more likely to err on the positive side than the negative; if the product is one that will in fact "catch on," there is a 90% chance the group's rating will be favorable and a 10% chance that it will be unfavorable. On the other hand, if the product is one that really will not "catch on," the group is equally likely to be favorable or unfavorable.

Should Ajax spend the $10,000 to build the prototype and get consumer reactions? What is the maximum they should be willing to spend for this?

12) Ajax Electronics has decided to do the prototype and focus group study to assess the demand for chip audio. They need to pick a market research firm to carry out the study using their prototype audio system. Ajax management has decided that the market research firm they choose must have experience doing over 80 similar studies in the past. In addition, they want a firm that has experience evaluating both "winners" and "losers." Hence, they insist that the selected firm be one that has evaluated at least 40 products that eventually caught on, and at least 40 that did not.

Just 2 market research firms meet the minimum standards set by Ajax management. *Boiler Plate Associates* has done 100 similar studies in the past, and so has *Smoke & Mirrors.* The detailed performance records of the two firms are as follows:

Boiler Plate and Associates

	Cases where Product later Succeeded	**Cases where Product later Failed**	**Total Cases**
Favorable Report	**36**	**30**	**66**
Unvavorable Report	**4**	**30**	**34**
Total Cases	**40**	**60**	**100**

Smoke & Mirrors

	Cases where Product later Succeeded	**Cases where Product later Failed**	**Total Cases**
Favorable Report	**54**	**20**	**74**
Unvavorable Report	**6**	**20**	**26**
Total Cases	**60**	**40**	**100**

Which Market Research firm should Ajax Electronics choose? Why?