ESSENTIALS OF DECISION MAKING UNDER GENERALIZED UNCERTAINTY[1]

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I. Introduction: Uncertainty, Fuzziness, and Optimization

One of the essential characteristics of a decision-making situation is the amount and quality of information that the decision maker has available at the onset of the problem. Luce and Raiffa [1958, p.13] developed a three-tiered classification of decision making based on the amount of knowledge, or information, possessed by the decision maker. If full knowledge regarding options, outcomes, and the various states of the world is available, the task of making a decision becomes a straightforward process of selecting the action whose outcome maximizes the decision criteria. Completely deterministic problems of this nature are classified as "decision making under certainty". The solution procedure involves the evaluation and optimization of the decision criteria, such as maximizing a utility function or minimizing a loss function.

The next lower level of knowledge regarding actions, outcomes, and states of the world is called decision making under risk. In this case, any given action of the decision maker leads to a probability distribution of possible outcomes which is known by the decision maker. Knowledge of the outcome's probability of occurrence is valuable because it allows the evaluation of the decision criteria through probabilistic optimization. This can be performed either through mathematical analysos of probability functions, or through Monte Carlo simulation techniques. The action selected for implementation is that for which the expected value of the decision criteria is most favorable.

Uncertainty results from an even further reduction in the quality and quantity of the decision maker's information level.

> "We shall say that we are in the realm of decision making under:...(c) uncertainty if either action or both has as its consequence a set of possible specific outcomes, but where the probabilities of these outcomes are completely unknown or are not even meaningful." [Luce and Raiffa, p.13]

Risk and uncertainty can also be combined to yield a mixed classification in which experimental evidence plays an important role in influencing the decision maker's actions by reducing original uncertainty to risk or even to certainty.

There is an important conceptual distinction between uncertainty in the Luce and Raiffa sense and fuzziness. Uncertainty implies that there are specific, although unknown, outcomes or sets of outcomes associated with each action that the decision maker can take. This conception of uncertainty assumes that a stochastic process underlies the connection between the actions and outcomes. Although this stochastic process may not be completely known to the decision maker, there is no question that it does exist uniquely.

Fuzziness, on the other hand, is qualitatively different. Fuzziness stems more from the concept of imprecision which comes about because the decision maker is not able to clearly distinguish between groups of possible outcomes. This difficulty is more consistent with the real world situation presented by multiple attribute, multiple objective decision problems expressed in linguistic rather than mathematical form. The complexity of the decision situation makes it impossible to clearly delineate the optimal course of action.

Bellman and Zadeh [1970] distinguished between statistical randomness and fuzziness by asserting that the former has to do with uncertainty regarding membership or nonmembership of an object in a set. In statistical analysis we are concerned that an object belongs to a set either perfectly or not at all, but we do not know which. Fuzziness, however, relates to sets where grades, ranging from full membership to full nonmembership, are possible. In fuzzy analysis, we are concerned with whether the object belongs strongly, moderately, or weakly to the set.

Optimization, in the sense of conventional statistical decision theory, is the process of searching through all possible values of the decision variable(s), evaluating all of them in terms of the decision criteria to determine the course of action that gives the highest critical value. "Soft" optimization is defined in an analogous manner. The difference is that the level of knowledge regarding the courses of action and the outcomes is lower. Consequently, any claim of optimality based on qualitatively and quantitatively inferior levels of information will, in general, be less concrete than that of statistically based evaluations.

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II. Obstacles to Certainty

The role of the decision maker falls into Simon's [1977] trichotomy: an intelligence phase, a design phase, and a choice phase. In the intelligence phase the decision maker searches the environment for information relating to the potential or immediate problem that is of concern. In the design phase the objectives are diagnosed and formulated, and the alternatives are identified and evaluated. The choice phase involves the selection of the best alternative course of action and its subsequent implementation.

Decision problems of any seriousness generally involve features which serve to complicate the process of achieving an "optimal" solution. Four such features have been identified by Bunn [1984] as follows: A) uncertainty about alternatives; B) uncertainty about consequences; C) uncertainty about preferences; and D) sequentiality, the dependence of future actions and consequences on decisions made in the past.

In order to know for certain what to do, three conditions must be satisfied. First, the decision maker must comprehend all of the alternative courses of action from which to choose. Second, the consequences of each alternative course of action must be known. Third, the decision maker must know which set of consequences is preferable to any other achievable set. In addition, since these conditions must often be met in an environment of multiple interacting decisions, consistency must be maintained throughout a complex decision process.

II A. Uncertainty About Alternative Courses of Action.

Comprehension of the set of alternative courses of action can be limited in three ways: failure of imagination, immensity of choice, and imprecision of specification. Failure of imagination simply means that relevant alternative courses of action exist which the decision maker is unaware of. Unless we can be certain that all possible alternative courses of action have been enumerated, we cannot be certain that the one we select is indeed optimal.

Sometimes it is possible to specify all available choices in an abstract (intensive) way, but the resulting set is too large to be extensively listed, or at least too large to be exhaustively evaluated. When this immense set of alternative courses of action can be represented as a continuum of real numbers or vectors, there are many well-known tools, such as mathematical programming, to proceed more or less efficiently to an optimal solution. However, in other cases the large number of alternative courses of action is due to a combinatoric explosion rather than a real continuum. If such combinatoric problems are to be solved at all, heuristic methods of search must be used. These heuristics typically do not afford proofs of optimality, so a decision made in this manner is uncertain.

The process of limiting and coping with uncertainty in the set of alternative courses of action has not generally been the focus of paradigms for decision making under uncertainty. The majority of rational models of decision making that have been developed have concentrated on the choice phase. Relatively little attention has been spent on either the intelligence or the design phases. The requirements for the intelligence phase of problem solving vary more significantly from problem to problem than does uncertainty about consequences or about preferences. Mintzberg, Raisinghani, and Theoret [1976] suggest that, of the three, the choice phase may be the least important in the decision making process. They describe the design phase as the heart of the decision making process. In their review of 25 cases of strategic decision making, 22 cases revealed that a considerable amount of time was spent in the design phase of the decision process and that in 21 of those cases it dominated the time spent in the intelligence and choices phases.

Two activities characterize the design phase of decision making: the search for a ready made, existing solution and, failing that, the design of a customized methodology or the modification of an existing solution to fit the new situation. Often the search process is used to narrow the field of potential alternatives to a number that can be realistically dealt with in the time and resource constraints. The search is often heuristically based.

Uncertainty regarding what courses of action are potential candidates for choice often results from the relationship between instructions issued by higher level management and their interpretation by those who are to execute them. The fuzziness comes about through the imprecision of the instruction statement. For example, the descriptors in the statement "... information is to be significantly relevant, accurate, and timely" may have considerably different interpretations when viewed from the perspective of the manager issuing the instruction and the subordinate charged with collecting data and presenting it.

Dimitrov and Driankova [1975] discuss the problem of social decision making when there are many actors involved in the process. In a complex organization, policies and instructions emanating from higher levels impose fuzzy constraints on decisions taken at lower levels. Dimitrov and Driankova present a computer program that develops a preference profile of the alternatives under consideration based on the individual preferences and comprehension of an instruction from a higher level manager. Using Arrow's criteria for a socially acceptable solution [Arrow, 1970], the computer based procedure manipulates the profiles of the individual members of the decision making coalition to produce a choice rule for the coalition as a whole.

II B. Uncertainty About Consequences

When we cannot predict with certainty what outcome will follow from a given course of action, we usually model this situation using the concept of "states of the world". We hypothesize that the outcomes we receive depend on two things: on which course of action we select and on the current values of one or more variables called "state variables". If we knew the values of the state variables, we would know the outcomes of each alternative course of action; if we do not know these values for certain, we must make an uncertain choice. (However, see Fishburn [1969] for an alternative discussion of decision analysis based on conditional probability distributions of utilities given actions and observations, with no explicit reference to states of the world.)

Considerable analysis and specific background knowledge of the domain of the decision in question are necessary to enumerate the relevant set of states of the world. Once these are enumerated, the next step is to marshal whatever information is available regarding the relative likelihood of these states. Several levels of information have been studied. The lowest level of information is when the states of the world are specified but no information about their relative degree of possibility or probability is known. With more information the second level is attained, in which some states of the world are known to be more possible than others (incomplete order); a third level is reached when states can be put in a complete weak order from most to least possible; that is, for any two states the decision maker can say which one is more possible than the other unless they are of exactly equal possibility. The fourth level of information allows the specification of approximate statistical probabilities for all states of the world using fuzzy real numbers, and the fifth level is when an exact specification of the probability distribution over states, using (crisp) real numbers, can be made.

Game theory can be viewed as a sixth level of information, in which a rational opponent's actions, while unknown in advance, will be completely determined by our own actions and the payoff structure of the game.

II C. Uncertainty About Preferences

The view of preferences most generally accepted among economists is that utilities are measurable by a complete weak order, corresponding to level three information. An individual will always either be able to specify one of a pair of outcomes as better than the other, or else be strictly indifferent between the two. In this view, it is meaningless to assign numbers to the utilities of outcomes, and hence no arithmetic can be performed on them. From an information content point of view, it is clearly equivalent to talk about ordinal gains, in which the best outcome ranks first, and ordinal losses, in which the worst outcome ranks first. A more sophisticated view of ordinal utilities postulates that it is not a static outcome that is valued, but rather the difference between an actual outcome and a standard or ideal outcome. On this basis, well substantiated by studies of human behavior, it is possible to talk about an ordinal theory of regrets in the context of decision making under uncertainty. The regret associated with a particular (state - action) pair is defined by the difference between the outcome of that particular (state - action) pair and the outcome of the best possible action for that particular state.

A well-established minority view, however, holds that meaningful numeric measures of an individual's utilities for outcomes can be generated. The most sophisticated variants of this theory derive from the work of Von Neumann and Morgenstern [1947]. In these approaches, utility is measured on an interval scale anchored by specific, context-dependent "best" and "worst" outcomes, and utilities for intermediate outcomes are determined by betting preferences in hypothetical lotteries. More recently, work has been done using fuzzy numbers rather than crisp numbers to represent these utilities [Watson et al, 1979; Freeling, 1980]; this can be a very valuable way to handle the fact that some of the hypothetical choices between bets are much easier to make than others in the Von Neumann - Morgenstern methodology. Fuzzy utilities come into play even more directly when the outcomes themselves are only vaguely known in advance. When utilities are measured by crisp or fuzzy real numbers, it is possible to compute regrets by subtracting the utility of the outcome of each (state - action) pair from the utility of the best possible action for that particular state.

II D. Sequentiality

A very important and widely-studied class of problems arises when it is possible to break a decision process down into stages so that later decisions are made in the light of information gained in earlier stages of the process. In fact, we may often choose to perform experiments or otherwise take actions designed deliberately to obtain information about the states of the world; typically this information is both imperfect and costly, so that a major part of our burden as decision makers is knowing when to seek information and when to make a substantive decision on the basis of what is already known.

A multistage problem can be diagrammed by a decision tree with alternating choice and chance nodes: at each choice node that we encounter in working through the tree we must pick one of several alternative action branches, while at each chance node that we encounter the unknown state of the world will determine which one of several possible outcome branches we will observe.

The notion of the decision tree enters into the design phase activities as the decision maker proceeds to the final customized solution. The process involves an initial vague (fuzzy) conception of the required solution. A sequence of nested search and design steps are evaluated as the decision maker procedes through the tree. At any time the process can cycle back to a previous point in the design if the line of inquiry proves fruitless; this serves to trim the tree and more sharply focuses the solution as the process evolves. "Thus a solution crystalizes, as the designers grope along, building their solution brick by brick, without knowing what it will look like until it is completed." [Mintzberg et al, p.256]

For analytic purposes, however, it is convenient to transform a multistage problem into an equivalent single-stage problem in "normal form" [Raiffa, 1968]. The first step to convert the problem into normal form is to define all possible "strategies" for moving through the decision tree. To specify a possible strategy, begin by selecting one alternative action at the first decision node of the decision tree. This action branch will lead to a chance node, each of whose branches in turn will lead to another choice node. For each of these possible second choice nodes, we must specify what action branch our strategy would dictate, and so on through the tree. The normal form decision tree will have only one choice node, with one branch for each possible strategy derived from the original tree. (A multistage specification of a decision problem and its corresponding tree are called the "extended form" to distinguish them from the normal form specification of the same problem.)

The second step in normalization is to respecify the set of possible states of the world. To do this, we must enumerate all possible combinations that can be formed by selecting one outcome branch from each chance node. Knowledge of the background of the specific problem-situation is essential here to avoid a combinatoric explosion; while the total number of combinations is likely to be unmanageably large, many combinations will be physically impossible because of identity or dependency between the variables being observed at the corresponding chance nodes.

The last step in converting a problem into normal form is to determine the utility of each strategy defined in step one under each state of the world defined in step two. This involves working through the extended form of the tree for each (strategy - state) pair, using the strategy to decide all choice branches and the state of the world to decide all chance branches, and accumulating all the gains and losses associated with the various partial actions and outcomes. The result is a shorter but wider tree; a satisfactory or optimal solution of the structurally simpler normal form of the problem is guaranteed to yield a satisfactory or optimal strategy for traversing the extended form of the problem.

III. Methodologies for Decision Making Under Generalized Uncertainty

III A. Decision Analysis Typology

The uncertainty components and the levels of information described in the previous section can be used to develop a framework for establishing a typology of decision analysis methodologies. This typology is presented in Figure 1 on the following page. The horizontal axis represents the quality of knowledge about the decision maker's utility or preference. The vertical axis depicts the quality of knowledge about states of the world. In this case the generalized uncertainty is composed of the courses of action and the consequences.

This typology associates specific decision analysis method with some of the combinations of information lquality eiht respect to the preferences/utilities and the states (which determine the consequences of actions). Use of a specific decision analysis method at an inappropriate information level, either too low or too high, for that method has serious implications. Misapplication of a procedure will require the decision maker either to make unwarranted assumptions in order to meet the information needs of the model, or to ignore relevant available information because the selected model is not capable of dealing with it. Either case leads to suboptimal results.

Figure 1: Decision Analysis Typology

Uncertainty/ Possibility Representation \ Utility Representation	Numeric	Fuzzy Numeric	Unified Total Order (Complete Weak)	Unified Partial Order (Incomplete Weak)	Distinct Partial Order
Numeric	Statistical Decision Analysis				
Fuzzy Numeric		Fuzzy Decision Analysis			
Unified Total Order (Complete Weak)			Possibilistic and Revised Possibilistic Decision Analysis		
Unified Partial Order (Incomplete Weak)				L-Fuzzy Risk Minimization	
Distinct Partial Orders					Commensurate L-Fuzzy Risk Minimization
No Relative Possibility Information	Game Theory with mixed strategies; Maximum Entropy		Ordinal Game Theory		

In the following subsections, some of the paradigmatic problems which arise from specific combinations of information about states of the world and about utility are examined in detail. Because the nature and amount of information about the relative possibilities of states of the world that can be usefully applied to decision making depends strongly on the nature and amount of available information about preferences, the techniques corresponding to each of the above levels will be discussed in the context of uncertainty about preferences. First, techniques which require crisp or fuzzy real numbers to measure utility are examined, then techniques that arise when utility is ordinal are considered in some detail.

III B. Decision Analysis With Numeric or Fuzzy Numeric Utilities

When the utility of the outcome of each alternative course of action under each possible state of the world is specified by a real number, these utilities can be combined with numeric probability measures to compute expected values. The chosen course of action is that for which the expected value is best. If the utilities and/or the probabilities are known only approximately, they can be represented as fuzzy numbers. Fuzzy expected utilities can be calculated by the extension principle of fuzzy mathematics; this process reduces to ordinary arithmetic when the operands are crisp.

B.1 No relative possibility information. With numeric utilities, minimax loss and maximax gain approaches are simple matters of numeric comparisons, while the regret measures needed for the minimax regret approach can be found by subtracting the utility of each outcome from the best utility obtainable in the relevant state of the world. Another approach, unique to the situation with numeric utilities and no information about relative possibilities, is the maximum entropy approach. In this approach, all possible states of the world are treated as equally probable in the absence of information to the contrary; operationally, this means simply taking the average across states of the world of the utilities which might arise from each alternative course of action, and choosing that course of action for which this average is best.

Game Theory, mentioned previously as a possible sixth level of information, also offers some insight for the decision maker under the conditions of numeric or fuzzy numeric utlities and no information regarding the likelihood of the different states. Minimax loss defines the optimal pure strategy in the presence of an intelligent adversary (or under the pessimistic assumption that the universe behaves as one). However, in some games a mixed strategy, one in which the decision maker's choice between pure strategies is regulated by chance with an optimal probability distribution [Jones 1980] offers a greater potential for a favorable outcome. An example is bluffing in poker, which is optimally used some, but not all, of the times a player has a losing hand.

B.2 Ordinal Possibilities. If by "ordinal possibilities" it is meant only that some possibilities are known to be greater than others, there is little advantage to combining this information with fuzzy or crisp numeric measures of utilities. However, if we also know just a little more, for instance that one state has a probability of more than .5 or that state s' is more than three times as likely as state s, these constraints allow meaningful bounds to be placed on the expected value of the outcome of each alternative course of action. Smith's [1980] "textured sets" approach demonstrates how linear programming techniques can be used to find the maximum and minimum possible expected values of each alternative course of action subject to linear constraints on the probabilities of the possible states of the world. Bartree [1971] also proposed a means of employing a linear programming approach. This procedure involves "fuzzifying" the objective function and changing the constraint functions into inequalities. Feasible solutions, rather than a single optimal solution, are developed in the course of the analysis. Any course of action whose maximum expected utility is less than the minimum expected utility of another can then be eliminated from further consideration.

Another approach, developed by Fishburn [1965], uses maximum expected utility as the decision criterion. Dealing with consequences of uncertainty involves the use of imprecise measures of probability. This procedure uses sets of inequalities and bounds and assesses their impact on the ordinal ranking of the expected utilities of the alternative courses of action. This procedure relieves the decision maker from having to assign explicit numbers to the probabilities of the states of the world.

B.3 Fuzzy or Crisp Numeric Utilities. The most commonly discussed technique for decision making under imperfect knowledge is statistical decision analysis [Raiffa, 1968]. In this technique, the imperfect information about the state of the world is represented by a probability distribution over the set of such states, and the utility of each strategy given each state of the world is expressed on an interval scale after the manner of Von Neumann and Morgenstern [1947]. The expected value of each strategy is found by multiplying the corresponding utilities and probabilities and adding the products; the strategy whose utility is greatest (or, equivalently, whose disutility is least) is the one that is chosen.

It is sometimes possible to specify the utility and probability information required by statistical decision analysis, but only in an approximate way. If the degree of imprecision in the estimates of probability or of utility is relatively small, statistical decision analysis provides for the use of sensitivity analysis, in which the numerical inputs are "perturbed" about their original values and the analysis is re-done to see whether the final decision changes.

Fuzzy statistical decision analysis, as presented by Watson et al [1979] and by Freeling [1980], can be viewed as an extension of sensitivity analysis to the case where the degree and qualitative shape of the imprecision need to be considered throughout the entire analysis of a decision. A major goal of this approach is to represent the imprecision of each value explicitly, and to manipulate these imprecise values in such a way as to determine the degree and nature of the resulting imprecision in the final decision. In order to accomplish this, fuzzy decision analysis uses linguistic and graphical techniques to elicit probabilities and utilities in the form of fuzzy numbers [Dubois and Prade, 1979].

The "extension principle of fuzzy mathematics" [Zadeh, 1965; Dubois and Prade, 1979] allows any mathematical operations that can be performed on real numbers to be performed on fuzzy numbers as well. Fuzzy statistical decision analysis makes extensive use of this principle to compute a fuzzy number representing the statistical expected value of each alternative course of action given the fuzzy probabilities and utilities in the input. The course of action for which this fuzzy number is highest is chosen; the method also specifies the degree of confidence that this action is actually the best, by measuring the degree to which the highest expected utility is clearly higher than the next-highest as opposed to the degree to which these two fuzzy numbers overlap. (It is in the assessment of confidence that Freeling differs from Watson et al).

III C. Decision Analysis With Ordinal Utility

C.1 No Relative Possibility Information. When we have no information about the relative likelihood of the various states of the world, we must make our decision on the basis of the utilities of the outcomes of the various (state - action) pairs together with a fundamental choice of philosophies. The "optimistic" philosophy in such a situation is to choose the course of action whose best possible outcome is better than that of any other (Maximax algorithm). The "pessimistic" philosophy, in contrast, seeks to reduce losses by choosing the course of action whose worst possible outcome is better (or less bad) than the worst possible outcome of any other course of action (Minimax loss algorithm).

The Ordinal Game Theory approach limits the utilities of the players' outcomes to an ordinal preference scale. This restriction allows only the consideration of pure (deterministic) strategies, since the choice probabilities of mixed strategies ultimately depend on subtractions and divisions carried out on numeric utilities.

The minimax regret approach steers a course between the extremes of optimism and pessimism. Outcomes are ordered in terms of regret rather than actual gains or losses, and that course of action is selected for which the worst possible regret is less bad than the worst possible regret for any other course of action. This approachhas the effect of focusing attention primarily on those states of the world for which our choices have the greatest effect. In contrast, minimax focuses on the most dangerous states of the world and maximax on the most promising ones.

C.2 Ordinal Possibilities. If only ordinal information about utilities is available, then whatever information is known about the relative possibility or probability of the various possible states of the world is also most appropriately expressed in an ordinal manner. The lowest level of information, whether in utilities or in possibilities, places the limits on the appropriate decision analysis model. Selecting an approach for the higher level of knowledge will require assumptions about the less well defined conditions, thereby introducing added uncertainty.

The Commensurate Ordinal Decision Analysis algorithm [Whalen, 1984b] uses two distinct ordinal scales, one for disutility (loss or regret) and the other for possibility. These scales define three L-fuzzy sets [Goguen, 1967]: the set of poor outcomes, the set of possible states of the world, and the set of risky exposures. An "exposure" is an ordered pair consisting of an outcome and the state of the world in which that outcome occurs; its membership in the set of risky exposures is also defined by an ordered pair consisting of the poorness of the outcome and the possibility of the state.

The inputs to the algorithm are two complete or incomplete rank orderings: one ranking the poorness of all possible outcomes, and the other ranking the possibilities of all possible states of the world. It is also necessary to specify, by means of a decision tree in normal form, which outcomes are associated with which states of the world and which strategies. The algorithm then determines the fuzzy set of risky exposures and uses this to eliminate suboptimal strategies using a series of dominance criteria which are successively more powerful, but less robust. These criteria, discussed below in turn, are complete dominance, global riskiest-states dominance, and pairwise riskiest-states dominance. Typically, commensurate ordinal risk minimization alone will not be sufficient to narrow the range of alternative strategies to just one optimal solution, but it can be very useful as a preliminary screening procedure. Given the results of a commensurate ordinal decision analysis, the decision maker is better prepared to seek additional information about just those states and actions identified as critical, or to use informal or intuitive methods to pick a final course of action from the resulting "short list".

C.2.1 Complete Dominance: A strategy A is completely dominated by another strategy A' if for all possible states of the world s the disutility D(A|s) arising from strategy A is worse than or equal to the disutility D(A'|s) arising from strategy A' when s is the state of the world, and the inequality is strict for at least one s. This is essentially the Pareto rule; a strategy is dominated by another if it is possible to improve one criterion of the outcome without worsening any other criterion. The different criteria in this case are the conditional outcomes given the different possible states of the world. Note, however, that mixed strategies are undefined when utilities are ordinal; a strategy can only be dominated by a specific other strategy, not by the convex combinations of two or more which are possible in numerical utility theory.

C.2.2 Global Riskiest-States Dominance. In this method, each alternative strategy A has associated with it a nonfuzzy partition of the set of possible states of the world into two subsets RA and RAc. For each state s in RAc there exists a state s* in RA which is both more possible than s and leads to worse outcome for strategy A than s. Thus, any doubts about strategy A caused by worry that s might be the true state of the world must be milder than the doubts about A caused by worry that s* might be. RA is called the set of riskiest states for strategy A.

The set R is defined as the set union of RA across all alternative strategies A. R is thus the set of states of the world which belong to the set of riskiest states for any strategy. The complement set Rc, the set of states not in R, is the set of states which are neither very likely nor ever very poor, regardless of what strategy is selected.

The "global riskiest-states dominance criterion" is evaluated by deleting the states in Rc from consideration and eliminating any strategies which are completely dominated on just those states of the world in R. A strategy A is global-riskiest-states dominated by another strategy A' if D(A|s) > D(A'|s) for all s in R and the inequality is strict for at least one s in R. In effect, we are saying that A' completely dominates A if we ignore the "unimportant" states of the world in Rc.

C.2.3 Pairwise Riskiest-States Dominance. For each pair of alternative strategies A and A', RAA' is defined to be the set union of the two sets of states of the world which are in the riskiest sets, RA and RA', for either of the strategies A and A'. The strategy A is then pairwise riskiest-states dominated by strategy A' if the outcome of A_ under state of the world s is worse than the outcome of A', also under s, for all s in RAA' and the inequality is strict for at least one s in RAA'. The argument in this case is that two strategies can be compared taking into consideration only those states which are risky ones for one or the other action, ignoring any states which may be risky for some extraneous third alternative as well as the unimportant states in Rc.

Clearly, any strategy which is completely dominated is also dominated according to the global riskiest-states criterion and any strategy which is dominated according to the latter is also dominated according to the pairwise riskiest-states criterion. Nevertheless, it is useful to know the most robust criterion under which a specific strategy can be eliminated, since each of the three criteria differs from its predecessor by making stronger assumptions and discarding more information as "unimportant".

C.2.4. Other Ordinal Techniques: The assumptions of the L-Fuzzy Risk Minimization algorithm [Whalen, 1984a] differ from those of the Commensurate Ordinal Decision Analysis algorithm by allowing direct comparisons between the grade of membership of an outcome in the set of bad outcomes on one hand and the grade of membership of a state of the world in the set of possible states on the other. The two weak orders are unified using fuzzy logic: the degree to which it is true that the exposure (A,S) is a risky one is equal to the degree to which the propositions "The outcome of A given S is bad" and "S is a likely state" both are true. Thus, the riskiness of an exposure is defined as the lesser of the poorness of the outcome and the likelihood of the state. As in the Commensurate Ordinal Decision Analysis algorithm, the incompletely ordered lattice structure of the L-fuzzy risk minimization algorithm allows many comparisons to remain undefined, concentrating the decision maker's attention on just those few comparisons which actually affect the course of the decision making process. Furthermore, the user has the option of refusing to make any given requested comparison. In this case, the algorithm continues to pass through the decision tree, and in many instances the difficult comparisons which the user has declined to make can be rendered moot by further analysis. If the user's refusals do in fact make a final solution impossible, the algorithm will identify several alternative unresolved pairs of memberships such that at least one of these difficulties must be resolved by the user before analysis can continue.

A further simplification arises when the truth values of the propositions are assumed to follow a complete weak order, so that for any pair of propositions about likelihoods or utilities it is possible either to identify which one is truer or to be sure that the two propositions are equally true. Placing propositions in a complete weak order of truth-values is equivalent to assigning them to a fuzzy set of true propositions whose membership grades are measured on the unit interval from zero to one; this is Zadeh's [1965] original formulation of a fuzzy set, sometimes referred to as a z-fuzzy set to distinguish it from the more general L-fuzzy set. The Revised Possibilistic Decisionmaking algorithm [Whalen, 1984a] corresponds to the L-fuzzy decisionmaking procedure when the sets are z-fuzzy. Yager's [1980] original Possibilistic Decisionmaking procedure differs in that it seeks to maximize the possibility of a good outcome rather than to minimize the possibility of a bad one.

V. General Multiple Facet Optimization

The above discussion centered around problems which have one important simplifying feature: the utility of any single possible outcome was viewed as a unit. We will now relax this simplifying assumption and state a unified theoretical framework for the resulting broader class of problems.

The current literature on utility theory devotes much concern to conditions which make numeric utility measurements or even ordinal utility comparisons difficult. These conditions include: multicriterion or multi-attribute decision making, in which outcomes are valued along several dimensions; discount theory, in which costs and benefits occur over a long period of time after the decision is made; and social decision making, in which several different stakeholders' interests must be respected.

These problems, along with the problem of uncertainty about the state of the world, can be subsumed in a general mathematical structure, which we will call the general multiple facet decision problem. In this abstract problem, we have a number of possible courses of action to choose from; the value of each strategy depends on a number of different facets, some of which may be more important than others.

In multicriterion or multiattribute decision making, each facet is one of the criteria or attributes that different choices are being judged on, and the relative importance of each facet depends on the importance weight given to that attribute or criterion. The multiple facet approach can be viewed as an extension of multicriterion decision making to situations which have traditionally been viewed as distinct topics.

In discount theory, each facet is the net cost or benefit accruing at a particular point in time, and the relative importance of each facet is the degree of discount to be applied to events at that point in time; the farther into the future an event is, the more it is discounted and thus the lower the relative importance of the facet.

In social decision making, the various facets of an alternative course of action are the utility assessments of that course of action by the various interested individuals and groups, and the relative importance of the each facet may be associated with the "clout" of each interested party. In a pure democracy, the clout of a facet depends on the number of persons it represents; in other situations, it may mean rhetorical skill, financial resources, or political or military power, depending on the circumstances and mores surrounding the decision making process.

In the problems considered in Sections I-IV above, the different facets of a given course of action consist of the outcomes of that course of action under the different possible states of the world, and the relative importance of each facet depends on the relative possibility or probability of the corresponding state of the world.

In principle, any problem involving facets of any of the above types, or a combination of types, can be treated by the methods in Section III discussed in the context of facets formed by uncertain states of the world. Levels of knowledge about the importance of the criteria, about the discount to be applied to future periods, or about the clout of interested parties, respectively, take the place of information about the relative possibility of states of the world in such an analysis. Obviously, treating these different decision making problems under a single theoretical framework closely resembling traditional views of multicriterion decision making does not remove the need for considerable situation-specific work in unraveling these and other difficulties in any specific situation. However, recognizing the structural commonalities between the problems will allow any methodological advance in one field to be readily transported to the others.

V. Conclusion

The complexities found in real world decisions are becoming better represented in models of decision making. Consideration of the psychological difficulties imposed on the decision maker when assessing multiple preferences in a complex environment has led decision scientists to develop methodologies which are better suited to operating with the limited quality and amount of knowledge characterizing applied decision making. This trend also benefits from the ever increasing sophistication of mathematical techniques that allow more and more to be done with less and less.

One of the most important stages of making a decision is the early choice of what formal model (if any) will be used to structure the remainder of the decision process. Different decision models make different assumptions about the nature of the alternative actions, goals (utilities), and other considerations for evaluating the alternative actions. Early choices among models, made on the basis of the general appropriateness of their assumptions to the case in point, deeply affect the subsequent analysis by determining the way in which the relevant data will be collected and defined in the future.

Because of this effect on the structuring of a decision, it is important to have available a wide variety of techniques with differing assumptions. Furthermore, these techniques need to be classfied within an integrative framework according to the nature of their assumptions. Only then can we be confident of choosing a model which makes the most effective possible use of the available data without introducing the distortions which result from a mismatch between the data and the algorithm (e.g. treating nominal or ordinal scale data as if it were measured on a ratio scale).

The methodologies discussed in this chapter constitute a subset of the various possible assumptions about the kinds of information that can be obtained and used. Once a new practical decision problem has been identified as belonging to the general class of multiple facet problems, the quality and quantity of the data associated with the new problem can be compared with the information presented herein to select the best model around which to structure the processes of estimating numeric probabilities or relative possibilities, assessing utilities, and arriving at a final decision.

The goal is to maximize the efficient use of whatever information is actually available while minimizing the need for introducing arbitrary assumptions of questionable precision. For example, if the information actually available in a given problem situation were just sufficient to satisfy the requirements of the L-fuzzy risk minimization algorithm, then to use a less information-intensive algorithm such as minimax regret would require ignoring real information which might be critical to an optimal decision. Using a more information- intensive technique, such as statistical decision analysis, would require introducing arbitrary assumptions about cardinal measurement scales which might distort the solution enough to lead to a suboptimal decision. In general, a problem situation will not fit the assumptions of any one model exactly. In such a case, a good strategy might be to bracket the problem by comparing the results of using two techniques: the most information-intensive technique whose assumptions are totally satisfied by the situation (but which does not use all the available information); and the least information-intensive technique which uses all the available information (but which also requires some additional assumptions). If the two "bracketing" techniques agree on a single decision alternative, that alternative may be adopted with some confidence; if the two techniques disagree, their respective recommendations may be compared more intensively as a "short list" from which the final action is to be selected.

Further advances in enriching and guiding the choice of methodologies for soft optimization can take three separate directions: development and refinement of individual techniques; systematic comparisons of their characteristics; and development of tools to aid in the selection of appropriate techniques for a particular problem.

One advantage of the conceptual framework used in this chapter is that it can suggest important gaps in the spectrum of techniques, and thus serve as a stimulus to the development of additional techniques which may fit some practical problems better than the ones currently in place. Table 1 identifies some combinations of utility and possibility representations. The decision methodologies associated with specific combinations are optimal only for the cells within which they are placed. The empty cells offer opportunities for the development of useful potential additions such as hybrid systems combining information at different levels; e. g. ordinal numbers and real numbers. In addition to investigating the technical efficiency of new and existing techniques, research is also needed regarding their potential for user acceptance; any decision making methodology which imposes major conceptual shifts on its intended users will be accepted only very slowly regardless of its other merits, as witnesses by the histories of Bayesian statistics and, more recently, fuzzy mathematics.

The framework of this paper provides a starting point for the systematic comparison of techniques in terms of their basic assumptions regarding uncertainty. However, in order to provide really effective guidance as to what techniques ought to be used in a particular situation, it is also necessary to have a body of knowledge comparing the difficulty of use and the quality of results using each technique in a variety of situations. Such a body of knowledge exists only in fragmentary form at present, and needs to be expanded and systematized using both axiomatic analysis and experimental studies with realistic problems and user populations. An initial step in this direction is that of Whalen [1986]. This involved the use of the entropy concept as a vehicle for comparing the effectiveness of various decision methodologies operating with fixed amounts of initial information.

As the number of techniques in the collection and the number of criteria for selection become larger, the difficulty of choosing a technique using reports from the literature, such as this one, becomes greater. This suggests a third avenue of research; the development of an "intelligent index" to help a decision maker to find the technique which best matches his or her perception of this problem. Since the choice of technique must be made very early in the decision process, at a time when the problem is still relatively ill-structured, a fuzzy ordinal approach to such an index seems appropriate. An example of such an approach is the fINDex program [Whalen and Schott, 1985]. This is an expert system program utilizing fuzzy linguistics to interactively assist in the selection of a forecasting technique given user specified constraints.

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