

MIXED STRATEGIES IN 2X2, TWO PERSON, ZERO SUM GAMES

Consider the following 2x2 two person zero sum game

		P(C1)=q	P(C2)=1-q	
		C1	C2	EU of row
P(R1) = p	R 1	a	b	qa + (1-q)b
P(R2) = 1-p	R 2	c	d	qc + (1-q)d
	EU of column	pa + (1-p)c	pb + (1-p)d	

If there's no saddle point, then

Row's best strategy is to choose p so that the payoff is equal regardless of which column the opponent chooses

$$pa + (1-p)c = pb + (1-p)d$$

which is equivalent to

$$pa - pb = (1-p)d - (1-p)c$$

$$p(a-b) = (1-p)(d-c)$$

$$\frac{p}{1-p} = \frac{d-c}{a-b} = \Omega(R1)$$

the odds in favor of Row playing R1. so the probability that Row will play R1 is

$$p = \frac{\Omega(R1)}{1+\Omega(R1)}$$

Col's best strategy is to choose q so that the payoff is equal regardless of which row the opponent chooses

$$qa + (1-q)b = qc + (1-q)d$$

which is equivalent to

$$qa - qc = (1-q)d - (1-q)b$$

$$q(a-c) = (1-q)(d-b)$$

$$\frac{q}{1-q} = \frac{d-b}{a-c} = \Omega(C1)$$

the odds in favor of Col playing C1. so the probability that Col will play C1 is

$$q = \frac{\Omega(C1)}{1+\Omega(C1)}$$

Note that if these probabilities come out negative or greater than 1, it means that there is a saddle point and therefore a pure solution.