

# Two Person Zero Sum Games

Mr. Row chooses R1, R2, R3, or R4 without knowing what Mr. Col chooses.

Mr. Col chooses C1, C2, C3, or C4 without knowing what Mr. Row chooses.

Then the choices are revealed and Mr. Col pays Mr. Row the indicated dollar amount.

	C1	C2	C3	C4
R1	\$0	\$1	\$7	\$7
R2	\$4	\$1	\$2	\$10
R3	\$3	\$1	\$0	\$25
R4	\$0	\$0	\$7	\$10

## Dominance Approach:

Mr. Col's Reasoning:

I certainly won't play C4

because that's my highest cost no matter what Mr. Row plays.

Mr. Row knows I won't play C3, so he won't play R3

because R2 is always better for him unless I play C4

Since Mr. Row won't play R3, I won't play C3

because that's worse than C2 unless Row plays R3

Mr. Row knows I won't play C3, so I know he'll play R2

because that's his best move given that I won't play C3 or C4

Since I know Mr. Row will play R2, I'll pay C2

to minimize the amount I have to pay him.

## Saddle Point Approach

	C1	C2	C3	C4	Minimum of Row
R1	\$0	\$1	\$7	\$7	\$0
R2	\$4	\$1	\$2	\$10	\$1 Maximin Row
R3	\$3	\$1	\$0	\$25	\$0
R4	\$0	\$0	\$7	\$10	\$0
	\$4	\$1	\$7	\$25	<-- Maximum of Column
	Minimax Column				

If Mr. Row plays the maximin row R2 and Mr. Col plays the minimax column C2, neither player can unilaterally improve his position.

Note (R2,C2) is the maximum of its column and the minimum of its row.

This game has a saddle point, but dominance reasoning won't work

	C1	C2	C3
R1	\$8	\$8	\$7
R2	\$0	\$10	\$4
R3	\$9	\$0	\$1

**Attempt at Dominance Reasoning:**

If Mr. Col plays C1, he'll wish he hadn't if Mr. Row plays R1 or R3

If Mr. Col plays C2, he'll wish he hadn't if Mr. Row plays R2.

If Mr. Col plays C3, he'll wish he hadn't if Mr. Row plays R2.

	C1	C2	C3	Minimum of Row
R1	\$8	\$8	\$7	\$7 Maximin Row
R2	\$0	\$10	\$4	\$0
R3	\$9	\$0	\$1	\$0
	\$9	\$10	\$7	<-- Maximum of Column
			Minimax Column	

**Saddle Point Reasoning:**

If Row plays the maximin row R1 and Col plays the minimax column C3, neither player can unilaterally improve his position.

Note (R1,C3) is the maximum of its column and the minimum of its row.

Mr. Row's reasoning: If I play Row 1, even if Mr. Col finds out or guesses correctly, the worst I can do is to gain \$7, but if I play row 2 or 3 I might end up with nothing.

Mr. Col's reasoning: If I play Column 3, even if Mt. Row finds out or guesses correctly the worst I can do is to lose \$7, but if I play column 1 or 2 I might end up losing \$9 or \$10.

## Stone, Paper, Scissors: A Game With No Saddle Point

(-\$1) means Mr. Row pays \$1 to Mr. Col.

	Stone	Paper	Scissors	Minimum of Row
Stone	\$0	(\$1)	\$1	(\$1)
Paper	\$1	\$0	(\$1)	(\$1)
Scissors	(\$1)	\$1	\$0	(\$1)
	\$1	\$1	\$1	<-- Maximum of Column

The best solution is to play completely at random and break even on the average in the long run.

If you show a pattern in repeated play, your opponent can take advantage of that pattern and beat you on average in the long run.

There are stone paper scissors tournaments; they're all about psyching out the other person.