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## **Functional Form in Regression Models of Tobin's $q$**

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### **Abstract**

The Box-Cox transformation is used to compare alternative functional forms of market value equations. Based on evidence from a panel of 480 publicly-traded U.S. manufacturing companies and two additional data sets used previously in the literature, the semilog form of a Tobin's  $q$  equation is found to be strongly preferred to the commonly estimated linear form. We provide illustrations in which inferences can be affected by the choice of functional form.

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## Introduction

The use of market value rather than accounting-based measures of profitability has become widespread in empirical analyses of firm profitability. The most widely used measure has been Tobin's  $q$ , defined as the market value of the firm divided by the replacement costs of assets.<sup>1</sup> Most empirical studies estimate a linear rather than semilog or double log form of a  $q$  equation.<sup>2</sup> In this note, we employ the Box-Cox transformation to compare profitability equations using  $q$  and  $\ln(q)$  as dependent variables. Following analysis of these results, we examine estimates of union-nonunion differences in market value using a balanced panel of 480 firms during 1972-1980, and use alternative data sets to explore inferences based on the use of linear and semilog forms of  $q$  equations.

### Estimation and Testing for Functional Form

Theory does not dictate a specific functional form for a  $q$  equation. Both additive and multiplicative functional forms have been estimated previously, although the former clearly has been the predominant form. Studies employing a linear model (e.g., Salinger, 1984; Hirschey, 1985; Montgomery and Wernerfelt, 1988) begin by specifying some form of an additive market value model. For example, let  $M$  be the market value of the firm (the value of equity plus debt),  $V_K$  the value of the physical capital stock,  $V_I$  the value of the intangible capital stock, and  $V_O$  the value associated with other factors (market structure and collusive arrangements, Ricardian rents, unions, disequilibrium, etc.). If firm market value,  $M$ , is the sum of its components,  $V_K + V_I + V_O$ , it follows,

$$(1) \quad M/V_K = 1 + V_I/V_K + V_O/V_K.$$

The value of intangible capital is measured by estimates of the R&D and advertising stocks, and  $V_K$  by an estimate of the net capital stock,  $K$ :

$$(2) \quad V_I/V_K = \tau_1(R\&D/K) + \tau_2(ADV/K) + \varepsilon_1.$$

Other value determinants are approximated by

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<sup>1</sup> Widely cited studies are Lindenberg and Ross (1981), Salinger (1984), Smirlock et al. (1984), and Hirschey (1985). McFarland (1988) provides a comparative evaluation of  $q$  and the accounting rate of return.

<sup>2</sup> Exceptions include Griliches (1981), Jaffe (1986), and Hirsch (1990, 1991). None of these provides an analysis of the appropriate functional form. A number of studies consider the inclusion of squared, inverse, and interaction terms on the right side of a  $q$  equation.

$$(3) \quad V_O/V_K = \alpha + \Sigma\beta X + \varepsilon_2,$$

where  $X$  includes measures of market structure, firm size, unionization, and demand shifts. Letting  $K$  be the estimate of  $V_K$  and substituting (2) and (3) into (1), we obtain the following linear  $q$  equation:

$$(4) \quad q = M/K = (1 + \alpha) + \tau_1(R \& D/K) + \tau_2(ADV/K) + \Sigma\beta X + (\varepsilon_1 + \varepsilon_2).$$

Alternatively, a semilogarithmic  $q$  equation can be derived in which market value determinants have a multiplicative effect on  $q$ .<sup>3</sup> Let

$$(5) \quad M = p(V_K + V_I),$$

where  $p$ , the price at which the tangible and intangible capital stock is valued, is equal to  $\exp(\alpha' + \Sigma\beta X + \varepsilon_2')$ . Thus it is assumed that factors included in  $X$  affect the valuation of the firm's asset base multiplicatively. Substituting the expression for  $p$  into (5) and dividing by  $V_K$ , we obtain:

$$(6) \quad M/V_K = (1 + V_I/V_K) \exp(\alpha' + \Sigma\beta X + \varepsilon_2').$$

Taking logarithms and noting that for small values of  $x$ ,  $\ln(1+x) \approx x$ :

$$(7) \quad \ln(q) = V_I/V_K + \alpha' + \Sigma\beta X + \varepsilon_2'.$$

Letting

$$(8) \quad V_I/V_K = \tau_1'(R \& D/K) + \tau_2'(ADV/K) + \varepsilon_1'$$

and substituting (8) into (7), the semilogarithmic specification of a  $q$  equation is obtained:

$$(9) \quad \ln(q) = \alpha' + \tau_1'(R \& D/K) + \tau_2'(ADV/K) + \Sigma\beta X + (\varepsilon_1' + \varepsilon_2').$$

Below, we compare the linear (equation 4) and semilog (equation 9) forms of a Tobin's  $q$  equation.<sup>4</sup>

A multiplicative functional form might be expected on several grounds. Intangible capital has a fixed cost and thus is likely to have multiplicative rather than additive effects on value. For example, economies of scale imply that advertising expenditures are likely to have a larger dollar impact on the earnings of larger

<sup>3</sup> This derivation is similar to Jaffe (1986, p. 990), except that in his model variables in  $X$  are logged. A double log model is not specified here since most of our right-hand-side variables take on zero or negative values.

<sup>4</sup> The semilog form can be obtained as an approximation to the additive model (Griliches, 1981; Montgomery and Wernerfelt, 1988). In equation (4), take logarithms and use the approximation  $\ln(1+x) \approx x$ , where  $x$  is everything to the right of 1. The semilog approximation of an additive model may therefore be appropriate when values of  $q$  are close to unity. Company values of  $q$ , however, may diverge widely from unity. "Good" estimates from a semilog specification, therefore, may imply that it is a good approximation of an additive model or that the underlying value generating process is multiplicative. Subsequent evidence is provided on this issue.

businesses (see Ravenscraft [1983] for evidence). Similarly, it is plausible that R&D expenditures generate larger absolute increases in market value among high-valued companies.

Unions tax some share of the rents and quasi-rents associated with tangible and intangible capital, market power, and disequilibrium (Hirsch, 1991). Again, multiplicative rather than additive effects on market value are plausible. Whereas a large additive union tax implies that small companies are unlikely to survive unionization, a proportional union tax on earnings makes survival more likely (for related evidence see Hirsch [1990]). Finally, market structure variables that lead to wider price-cost margins imply larger increases in value for firms with larger output.

Apart from these theoretical arguments, a semilog  $q$  equation may be preferable to the linear form because the logarithmic transformation dampens the influence of observations with extreme and mismeasured values of  $q$  ( $q$  is unbounded above). Measurement error results from difficulties in estimating the replacement cost of the capital stock. High values of  $q$  need not imply measurement error, however, since some companies realize a high valuation of tangible assets owing to large stocks of intangible capital and special firm advantages.

Because the optimal functional form cannot be determined a priori, we test for functional form by estimating an equation that subjects the dependent variable,  $q$ , to the Box-Cox transformation defined by  $q^{(\lambda)} = (q^\lambda - 1)/\lambda$  if  $\lambda \neq 0$  and  $q^{(\lambda)} = \ln(q)$  if  $\lambda = 0$ .

Our model is:

$$(10) \quad q_{it}^\lambda = \beta_1 + \beta_2 X_{2it} + \beta_3 X_{3it} + \dots + \beta_k X_{kit} + \varepsilon_{it}.$$

Subscripts  $i$  and  $t$  designate firm and year, respectively, the  $X$ 's represent explanatory variables, the  $\beta$ 's denote regression coefficients, and  $\varepsilon_{it}$  is distributed approximately  $N(0, \sigma^2)$ . Values of  $\lambda$  and  $\beta$  are estimated by maximum likelihood and likelihood ratio tests are applied to  $\lambda$ . A value of  $\hat{\lambda} \approx 0$  would approximate a semilog form in which  $\ln(q)$  is the appropriate dependent variable, whereas  $\hat{\lambda} \approx 1$  would approximate a linear form. The statistic  $x_1^2 = 2[L(\hat{\lambda}) - L(\lambda)]$  provides an asymptotic test of the null hypotheses  $H_0: \lambda = 0$  and  $H_0: \lambda = 1$ , where  $L$  denotes the value of the log-likelihood.

The primary data set used in our analysis was developed by merging three files: (1) the *R&D Master File* (Cummins, et al., 1985), a panel of firm-level data on publicly-traded manufacturing companies (2) firm-level union coverage information collected in a survey by Hirsch, and (3) industry data matched to firms' primary SIC code listed on Compustat. Fuller description of the data set and variables is provided in Hirsch (1991). For the analysis here, we focus on a balanced panel of 480 companies over the years 1972-1980.

The dependent variable,  $q$ , is measured by market value (equity value plus debt) divided by the net inflation-adjusted capital stock. Right-hand variables include the R&D stock divided by the net capital stock ( $R\&D/K$ ); the advertising stock divided by capital ( $ADV/K$ ); the proportion of a firm's workforce covered by a collective bargaining agreement in 1977 ( $UN$ ); firm size as measured by employment ( $L$ ) in millions; company age ( $AGE$ ) in hundreds of years; the company 2-year annualized logarithmic growth in sales ( $GROWTH$ ) and the industry 4-year logarithmic growth in sales ( $I-GROWTH$ ); industry union coverage ( $I-UN$ ), concentration ( $I-CR$ ) and import penetration ( $I-IMPORT$ ), all taking on values between 0 and 1; and dummies for year and approximate 2-digit industry (21 categories).

Table 1 presents estimates of the  $\beta$  vector when the linear ( $\lambda = 1$ ) and semilog ( $\lambda = 0$ ) forms of the  $q$  equation are assumed. Qualitative results are roughly similar. Market valuation of firms' assets is positively associated with R&D and advertising intensity, firm size, firm and industry sales growth, and industry concentration; it is negatively related to union coverage and age. A standard  $F$  test shows that industry and year dummies are each jointly significant. The Box-Cox log-likelihood function is higher in the semilog than in the linear equation ( $\bar{R}^2$ 's cannot, of course, be compared across equations with different dependent variables).<sup>5</sup>

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<sup>5</sup> The intercept of the linear model (equation (4)) should be close to unity if  $\alpha$  is small, while the intercept in the semilog model (equation (9)) should be close to zero if  $\alpha'$  is small. Intercepts vary with year and industry, being both above and below unity (zero) in alternative year-industry combinations in the linear (semilog) models; the tendency, however, is for intercepts to be less than unity (zero). Note also that because  $K$  is measured with error, its presence in the denominator on both sides of the equation may bias coefficient estimates. Coefficients are highly similar, however, when  $K$  is instrumented with lagged values, or when the dependent variable is the log of market value and  $\ln(K)$  is moved to the right side (its coefficient is 1.040).

Next we estimate  $\lambda$  and  $\beta$  simultaneously by maximum likelihood. The log-likelihood function is maximized at  $\hat{\lambda} = -0.30$ , “close” to the semilog value of  $\lambda = 0$ , and far from the linear value  $\lambda = 1$ . The test statistic  $x_1^2 = 2[L(\hat{\lambda}) - L(\lambda)] = 8960.21$ , overwhelmingly rejects  $H_0: \lambda = 1$  (the critical value at the 0.01 level is 6.63). The hypothesis  $H_0: \lambda = 0$  is also rejected, with  $x_1^2 = 418.48$ . Although neither functional form can be accepted as optimal, the semilog form is strongly preferred to the linear form.<sup>6</sup>

Further analysis reveals that the results are not sensitive to the choice of independent variables. In specifications deleting, in turn, the union; union and industry; and union, industry, and growth variables; values of  $\hat{\lambda}$  are -0.30, -0.32 and -0.33, respectively. Thus,  $\hat{\lambda}$  is highly robust across “dense” and “sparse” specifications.

Amemiya and Powell (1981) have questioned whether maximum likelihood estimation of  $\lambda$  is robust, arguing in favor of the nonlinear two stage least squares (N2SLS) estimator of  $\lambda$ . We check our estimates by applying the N2SLS estimator with quadratic terms of the non-dummy explanatory variables added to the right hand variables to make up the instruments. The N2SLS estimate of  $\lambda$  is -0.43, not very different from the ML estimate of -0.30.

A partial explanation for the superiority of the semilog over the linear functional form could be that the logarithmic transformation dampens extreme values of  $q$ . To explore this possibility directly, we estimate values of  $\lambda$  following exclusion of observations with high values of  $q$ . When we exclude 140 company-years with values of  $q$  at least two standard deviations greater than the mean in the relevant year (leaving  $n = 4180$ ), we obtain  $\hat{\lambda} = -0.17$ , as compared to  $\hat{\lambda} = -0.30$  for the full sample. Exclusion of 300 observations with values of  $q$  at least one standard deviation above ( $n = 4020$ ) results in  $\hat{\lambda} = -0.04$ . These results imply that it is *not* the case that the superiority of the semilog over the linear models is the result of dampening outliers.

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<sup>6</sup> As illustrated by Seaks and Layson (1983), heteroskedasticity can seriously bias the test toward rejection of the linear and acceptance of the semilog model. Our results are highly similar, however, when we employ their weighted Box-Cox model (BCEW) with weighting by firm size. In work not shown, no evidence favoring either the linear or semilog form is found when an accounting profitability measure (earnings/assets) is the dependent variable.

The results are probed further by omitting companies with  $q$  values *close* to unity. We find  $\hat{\lambda} = -0.41$  when the sample excludes 801 company-years with  $q$  between 0.8 and 1.2 ( $n = 3519$ ). This result suggests again the robustness of our basic finding, and rules out the possibility that the strong performance of the semilog model is due to its being a good approximation to a linear model when  $q$  is close to unity (see footnote 4).

The  $q$  equations and  $\hat{\lambda}$  are next estimated separately by year. Annual regressions provide a vehicle for checking the robustness of the pooled results, and allow us to purge the error term of serial correlation within firms across years. Table 2 presents estimates of  $\hat{\lambda}$  by year, as well as the  $\chi^2_1$  values for testing the hypotheses  $\lambda = 0$  and  $\lambda = 1$ . In all years the linear form is decisively rejected. Estimates of  $\hat{\lambda}$  are always below zero with an average value of -0.26 over the 1972-1980 period, and -0.20 for the years 1975-1980.<sup>7</sup> Although  $H_0: \lambda = 0$  is always rejected, the values of  $\chi^2_1$  for  $H_0: \lambda = 1$  are thirteen to sixty-five times as large as the test statistics for  $H_0: \lambda = 0$ . Further, the Amemiya-Powell N2SLS estimator provides results similar to MLE, the average value of  $\hat{\lambda}$  for the separate yearly regressions being -0.11. Thus, the annual regressions reinforce the conclusion that the semilog form is preferred if one wishes to choose a simple model from within the class of models covered by the Box-Cox transformation.

Because all but four of our right-hand-side variables ( $L$ ,  $AGE$ ,  $I-UN$ , and  $I-CR$ ) take on zero or negative values, we have focused on the semilog rather than a double log specification (see Jaffe, 1986). If we estimate optimal transformation parameters  $\hat{\lambda}_L$  and  $\hat{\lambda}_R$  for both the left-hand side of the equation (with all other variables included as previously), we obtain the values  $\hat{\lambda}_L = -0.308$  and  $\hat{\lambda}_R = -0.228$ , much closer to the double log ( $\hat{\lambda}_L = 0$ ,  $\hat{\lambda}_R = 0$ ) than the semilog ( $\hat{\lambda}_L = 0$ ,  $\hat{\lambda}_R = 1$ ) specification.<sup>8</sup> Annual estimates of  $\hat{\lambda}_R$ , however, exhibit an unstable pattern and large standard errors. Estimates of  $\hat{\lambda}_L$  are estimated far more

<sup>7</sup> Estimates of  $\lambda$  during 1972-1974 cause us to suspect specification bias for this period. Conventional  $F$ -tests reveal far greater stability in slope coefficients across time in the  $\ln(q)$  than in the  $q$  equations,  $F(240,4041) = 1.49$  in the former, and  $F(240,4041) = 4.18$  in the latter case.

<sup>8</sup> The BOXCOX command in LIMDEP version 6.0 was employed. Values of  $\hat{\lambda}_L$  are estimated far more precisely than  $\hat{\lambda}_R$ . The estimated standard errors from LIMDEP are 0.015 and 0.146, respectively.

precisely and are largely invariant to the transformation of the right-hand-side variables. Our results can be interpreted as providing strong support for some form of a multiplicative model; the choice between a simple semilog or double log specification is less clear.

### **Further Examples and the Effects of Functional Form on Inferences**

Two natural questions are whether our results, which strongly reject the linear model, can be confirmed with other data sets, and whether choice of functional form can affect a researcher's inferences. We provide three illustrative examples.

First, yearly estimates of union-nonunion differentials in  $q$  for the semilog and linear forms (holding constant the same set of regressors as in table 1) are presented in table 2. The coefficients on  $UN$  from the linear equations suggest a much larger intertemporal variability in the union-nonunion differential than do the  $UN$  coefficients from the semilog equations. The coefficient of variation across years in  $\hat{\beta}_{UN}$  is 87.1% using the linear model, as compared to 40.3% using the semilog model. This greater variability from the linear model is likely to result because the union “tax” on market value is more closely approximated by a percentage than by an absolute differential in  $q$ . Table 2 also provides estimates of the percentage union-nonunion differential in  $q$ , comparing nonunion companies to unionized companies with 50 percent coverage ( $UN = 0.50$ ). The percentage differential is approximated by  $[\exp(0.50 \hat{\beta}_{UN}) - 1] \cdot 100$  in the semilog case, and by  $(.50 \hat{\beta}_{UN} / \bar{q}_{NU}) \cdot 100$  in the linear case, where  $\hat{\beta}_{UN}$  is the coefficient on  $UN$  and  $\bar{q}_{NU}$  is mean  $q$  in year  $t$  among nonunion companies.

The issue of functional form is explored next with a data set previously used by Hirsch (1990). This second data set comprises companies with information not only on union coverage, but for which market share and industry concentration variables for 1977, weighted by firms’ sales across 4-digit SIC categories, were made available from Hirschey (1985). The ML estimate of  $\lambda$  is -0.42 from an unbalanced panel of 2175 company-years over the 1972-80 period (the N2SLS estimate is  $\hat{\lambda} = -0.32$ ). The specification of the model is identical to that in table 1, with  $I-CR$  deleted and weighted concentration ( $WCR$ ) and market share ( $WMS$ ) variables added. Other included variables are  $R\&D/K$ ,  $ADV/K$ ,  $L$ ,  $AGE$ ,  $GROWTH$ ,  $I-GROWTH$ ,  $I-$

*UN*, *I-IMPORT*, *IND*, and *YEAR*. Coefficients and  $|t|$  values for *UN*, *WMS*, and *WCR* only are shown below.

$$(11) \quad q: \quad -0.173UN + 0.434WCR - 0.035WMS \quad \bar{R}^2 = 0.408$$

(1.62)      (2.04)      (0.09)

$$(12) \quad \ln(q): \quad -0.252UN + 0.328WCR + 0.418WMS \quad \bar{R}^2 = 0.580$$

(5.26)      (3.45)      (2.33)

The sensitivity of the *WMS* result with respect to the use of  $q$  or  $\ln(q)$  reinforces the conclusion that inferences can be affected by the choice of functional form.<sup>9</sup>

Finally, we reestimated  $q$  models presented in Stevens (1990), using a 1972 firm-level data set kindly provided to us by the author (these results are available on request). The semilog form again is found to be strongly preferable to the linear form. Moreover, inferences about the strength and interdependence of market share and concentration effects on  $q$  are affected by the choice of functional form.

### Conclusion

Using three data sets, the semilog form of a Tobin's  $q$  equation is found to be strongly preferred to the commonly estimated linear form. Although neither form is close enough to the optimal form to avoid rejection at conventional levels, the  $\ln(q)$  models are invariably much closer to the optimal form in terms of both the log likelihood function and the estimated value of the transformation parameter  $\lambda$ . The results are robust with respect to specification, omission of observations with high values of  $q$ , and omission of observations with  $q$  close to unity. Estimation with alternative data sets suggests that our conclusions may be generalized, and that inferences can be affected by the choice of the linear or semilog forms of the  $q$  equation.

An issue not examined in this note is whether the choice of functional form is affected by simultaneity (for an exploration of this problem, see Spitzer (1977)). Theory and evidence indicate the

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<sup>9</sup> In contrast to Hirschey (1985), here concentration is found to significantly affect  $q$ . Our sample is a subset of companies used by Hirschey, who estimated his model for 1977 only. Besides differences in the sample, there are differences in specification and in measurement of  $q$  and other variables. In addition, the impact of concentration and market share weakened during the 1970s.

possible endogeneity of such factors as R&D, advertising, growth, market structure, and union coverage. Papers estimating simultaneous models have utilized sparse specifications and identified key equations through statistically necessary restrictions; our preference has been to present estimates from a dense specification of a single equation model. Because our functional form results are found to be so invariant to how sparse or dense the specification, we think it is likely that our conclusions would not be substantially affected by a simultaneous equation framework that adds instrumental variables.

The results reported here support the proposition that firm and industry characteristics have multiplicative rather than additive effects on the market valuation of company assets, and provide a strong presumption for employing  $\ln(q)$  rather than  $q$  in future empirical work. At a minimum, researchers should examine the sensitivity of their findings to the choice of functional form.

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**Table 1: Tobin's  $q$  Regression Results, 1972-80 Panel**

Variable	Dependent Variable	
	$q$	$\ln(q)$
<i>UN</i>	-0.237 (3.46)	-0.288 (8.42)
<i>R&amp;D/K</i>	0.729 (6.43)	0.333 (5.88)
<i>ADV/K</i>	0.100 (1.11)	0.055 (1.23)
<i>L</i>	0.335 (1.12)	0.568 (3.81)
<i>AGE</i>	-0.117 (2.13)	-0.063 (2.31)
<i>GROWTH</i>	1.317 (13.23)	1.016 (20.42)
<i>I-GROWTH</i>	0.444 (1.35)	0.275 (1.67)
<i>I-UN</i>	-0.188 (1.00)	-0.200 (2.13)
<i>I-CR</i>	0.596 (4.94)	0.377 (6.25)
<i>I-IMPORT</i>	-0.010 (0.04)	0.155 (1.13)
<i>IND</i> ( $F_{20, 4281}$ )	24.66	34.97
<i>YEAR</i> ( $F_{8, 4281}$ )	63.76	89.05
$\bar{R}^2$	0.316	0.447
$\lambda$	1.0	0.0
<i>L</i>	-6093.60	-1822.74

The ML estimate of  $\lambda$  is -0.30 with  $L = -1613.50$ .  $n = 4320$  and  $|t|$  shown in parentheses.

**Table 2: Estimates of  $\hat{\lambda}$  and Union-Nonunion  $q$  Differentials By Year, 1972-1980**

	$\hat{\lambda}$	$x_1^2(0)$	$x_1^2(1)$	$\bar{q}_{NU}$	$\bar{q}_U$	$q$			$\ln(q)$		
						$\hat{\beta}_{UN}$	$ t $	% $q$ -diff	$\hat{\beta}_{UN}$	$ t $	% $q$ -diff
1972	-0.36	62.35	957.97	3.429	1.587	-0.660	-1.60	-9.6	-0.386	-3.08	-17.6
1973	-0.35	63.07	902.82	2.022	1.079	-0.229	-0.89	-5.7	-0.329	-2.64	-15.2
1974	-0.42	63.95	808.99	1.029	0.673	0.018	-0.14	0.9	-0.130	-1.29	-6.3
1975	-0.25	27.72	630.41	1.140	0.733	-0.103	-0.80	-4.5	-0.214	-1.98	-10.2
1976	-0.22	14.57	430.15	1.239	0.815	-0.139	-1.26	-5.6	-0.222	-2.34	-10.5
1977	-0.18	8.38	343.86	1.101	0.739	-0.131	-1.60	-6.0	-0.196	-2.4	-9.3
1978	-0.22	11.20	360.35	1.087	0.686	-0.124	-1.60	-5.7	-0.212	-2.62	-10.1
1979	-0.19	10.01	423.41	1.293	0.696	-0.394	-4.30	-15.2	-0.446	-5.17	-20.0
1980	-0.15	9.87	650.43	1.551	0.722	-0.469	-3.54	-15.1	-0.431	-4.39	-19.4
1972-80	-0.30	418.48	8960.21	1.543	0.859	-0.237	-3.46	-7.7	-0.288	-8.42	-13.4

For each year  $n = 480$ . Annual regressions include all the same variables used in the regressions of table 1 except that the year dummies are omitted here. The pooled 1972-80 regression includes the year dummies. The % $q$ -diff columns represent estimates of the percentage differential in  $q$  between union companies with 50 percent coverage and otherwise similar nonunion companies. The method of calculation is shown in the text.