Optimal Public Debt Redux*

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Abstract

We examine the role played by government investment in infrastructure in determining the optimal quantity of public debt in a heterogeneous agent economy with incomplete insurance markets. Calibrating our model to the key aggregate and distributional moments of the U.S. economy for the period 1990-2015, we show that (i) the inclusion of infrastructure, and (ii) transitional dynamics between stationary states critically affect the characterization of the optimal level of public debt. Our results indicate that the inclusion of public infrastructure in the model specification implies a lower optimal debt level relative to the specification without infrastructure, both when comparing stationary equilibria and when accounting for transitional dynamics. When welfare comparisons are made by comparing stationary equilibria, we find that it is optimal for the government to accumulate assets (public surplus). However, once transitional dynamics are accounted for, accumulating debt becomes optimal, with the optimal share implied by our model being significantly higher than the average public debt-GDP ratio for the U.S. observed during our sample period.

Keywords: Infrastructure, public investment, heterogeneous agents, public debt, welfare, transitional dynamics.

JEL Classification: E2, E6, H3, H4, H6

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1 Introduction

What is the optimal amount of public debt? This important question has received a lot of attention recently, especially in the aftermath of the global financial crisis of 2008-2009. In a traditional representative agent model, the quantity of public debt is irrelevant for private decision making as long as the intertemporal budget constraint of the government satisfies the transversality condition (thereby ensuring that the government does not run a Ponzi scheme against the private sector). However, in a context where households receive idiosyncratic shocks that cannot be perfectly insured, public debt can have important consequences for agents' decisions. Higher levels of public debt can be detrimental to welfare by crowding out private investment, leading to lower wages, output and consumption in equilibrium. At the same time, public debt can also relax borrowing constraints for households by increasing liquidity in the economy, thereby facilitating consumption smoothing and improving aggregate welfare. *A priori*, it is therefore not clear whether it is optimal for the government to accumulate debt or assets (surplus) in equilibrium. This paper analyzes this important public policy question by embedding two key features in a calibrated heterogeneous agent framework: (i) the government's provision of productive public goods such as infrastructure, and (ii) transitional dynamics of the economy when the government's debt policy changes. Both of these features have not been analyzed systematically in the existing literature and, as we will subsequently demonstrate, fundamentally alter the characterization of the optimal level of public debt.

In their seminal paper studying public debt in the United States, Aiyagari and McGrattan (1998) show that the optimal level of debt is positive and approximately two-thirds of GDP which, in fact, was very close to the actual share of public debt in the United States during the post-war period. Additionally, they find that the welfare profile is relatively flat near the optimum, suggesting small welfare losses from deviating from this level. Three critical issues that Aiyagari and McGrattan (1998) abstracted from are: (i) matching the wealth and earnings distribution for U.S. households, (ii) accounting for transitional dynamics when computing the welfare effects associated with a change in debt policy, and (iii) accounting for the composition of government spending, specifically on productive public goods such as infrastructure. Using a labor productivity shock process that generates endogenous income and wealth distributions that more closely match the U.S. data, Rohrs and Winter (2017) find that the optimal level of public debt is actually a surplus when welfare effects are computed by comparing stationary equilibria. On the other hand, Desbonnet and Weitzenblum (2012) find that once the transition path is accounted for, there may exist significant welfare gains from increasing the level of public debt, though they do not characterize what the new optimum should be.[1] Furthermore, much of the existing literature on optimal public debt has only taken into account government consumption and transfers when characterizing government spending. Both of these components are modeled as being wasteful, with no consequences for the economy's productive capacity. By contrast, public infrastructure, as embodied in an economy's stock of roads, transportation networks, ports, power and electricity generation, etc., has important consequences for the productivity of private factors such as capital and labor.[2] These productivity effects are especially important in models where agents face idiosyncratic shocks and lack access to complete insurance markets, as changes in factor prices distort savings, consumption, and labor supply decisions, thereby affecting aggregate welfare. Moreover, since the benefits of government investment in infrastructure for private factors of production accrue only gradually over time (as its stock accumulates), its consequences for the optimal level of public debt cannot be correctly analyzed unless the transition path between steady states is considered.

The primary contribution of our paper is two-fold. First, we show that the introduction of public infrastructure into the aggregate production structure of a heterogeneous agent economy calibrated to be consistent with U.S. wealth and earnings inequality fundamentally alters the calculation of the optimal quantity of public debt. Second, we show that there are stark differences between the short-run and long-run welfare consequences of a change in a country's debt policy, which in turn leads to dramatically different conclusions for the optimal share of public debt once the transition path between stationary states is internalized. These issues have not been addressed simultaneously in the existing literature on optimal public debt.

The starting point of our analysis is the development of two distinct model environments: a standard environment where output is produced using only private capital and labor, and an alternative environment where we introduce a government-provided stock of pubic infrastructure which enters the aggregate production function and generates positive spillovers for

[1]See Floden (2001) and Peterman and Sager (2016) for an assessment of the role of optimal transfers and life-cycle effects on optimal debt, respectively. There are also a few recent papers that examine optimal tax policy that relate to this literature (see Acikgoz (2013), Dyrda and Pedroni (2016), and Bakis et al. (2015)). However, unlike our work, these papers do not consider the simultaneous inclusion of public infrastructure and transitional dynamics when computing welfare effects in a model that is calibrated to replicate the degree of inequality in the U.S.

[2]There is a large literature that investigates the growth-enhancing feature of public infrastructure (Aschauer, 1989; Barro, 1990; Glomm and Ravikumar, 1994). The distributional effects of infrastructure investment have also been studied more recently (Chatterjee and Turnovsky, 2012; and Gibson and Rioja 2017a, 2017b).

private capital and labor. Both model specifications are calibrated to generate the same stationary equilibrium and replicate key aggregate and distributional characteristics of the U.S. economy for the sample period 1990-2015. While we do not target the debt-GDP ratio when calibrating the baseline stationary equilibrium, we follow the existing literature and adjust the debt-GDP ratio exogenously when computing counter-factual stationary equilibria that would result under alternative debt policies (see Aiyagari and McGrattan 1998, among others). Also following the previous literature, the government adjusts the income tax rate to maintain their budget constraint as debt is varied exogenously.

With the baseline and counterfactual stationary equilibria pinned down for both model specifications, we proceed by computing the welfare impact of moving from the baseline equilibrium to each of the counterfactual equilibria. We find that the optimal share of public debt is a surplus when welfare is computed by comparing stationary equilibria. Specifically, the implied welfare-maximizing level of public debt in the model specification with infrastructure is a surplus of about 175 percent of GDP. On the other hand, the specification without public infrastructure implies a much smaller surplus, at about 75 percent of GDP. Not only is the optimal surplus substantially larger when infrastructure is included, but the welfare effects around the optimum are also sizeable, relative to the specification without infrastructure.

One caveat to the above result is that this calculation of the optimal level of public debt (or surplus) incorporates only the *long-run* welfare change between stationary equilibria. In other words, the underlying assumption is that any change in the level of debt is accompanied by an instantaneous "switch" to the new stationary equilibrium. While an understanding of these long run costs and benefits of increasing or reducing public debt is no doubt important, it is implausible to assume that the economy adjusts instantaneously to its new long-run equilibrium. In fact, a change in debt policy will lead to a gradual transition over time, as macroeconomic aggregates such as capital, consumption, labor supply, and output adjust to the change in policy (and the accompanying changes in other parts of the government's budget). Such changes can fundamentally affect the calculation of welfare, in both the short run as well as the long run. Consequently, our next step is to characterize the optimal level of public debt after accounting for welfare changes along the entire transition path. Here, we find that the long-run result of an optimal surplus no longer holds. Once the transition path is accounted for, the optimal level of public debt is again positive, implying that the government should now be a net borrower in equilibrium. As before, the optimal level of public debt is lower in the specification with infrastructure relative to when it is absent. In the specification that includes public infrastructure, we find that the implied optimal level of public debt is about 100 percent of GDP, which is higher than the average share of public debt in the U.S. over our sample period 1990-2015.

To understand better the sharp contrasts in the calculation of optimal public debt when considering stationary equilibria and transition paths, we need to focus on both the role of public infrastructure and the consequences of the changing composition of the government's budget as the level of public debt is altered. Consider first the long run welfare effects. A reduction in public debt means that private capital will be crowded in. Now, the presence of public infrastructure, given its positive spillover for both private factors, will further boost the productivity of capital and labor. This will tend to reduce the precautionary savings motive for households, as they can now rely more on labor income. Consequently, this leads households to increase consumption and reduce asset holdings, thereby increasing welfare. On the other hand, the reduction in government debt also tends to reduce liquidity in the economy and tightens borrowing constraints for credit-constrained households, which tends to reduce consumption and welfare in the economy. Overall, the welfare increase from the reduced precautionary savings motive dominates, with the presence of public infrastructure further strengthening this effect. This leads the government to accumulate assets (a public surplus) at the optimum.

By contrast, the transitional welfare effects work very differently. For example, when the tax rate is adjusted to satisfy the government's budget constraint, reducing public debt leads to a modest reduction in the tax rate in the long run. However, it requires a substantial increase in the tax rate in the short run in order to facilitate the debt reduction. This tax increase worsens welfare as it reduces the after-tax return on capital and labor income of agents. Therefore, *increasing* public debt is welfare enhancing in the short run: this allows the government to temporarily reduce taxes, which increases after-tax returns on capital and labor. Over time, this leads to a more indebted government at the optimum, in sharp contrast to the case where an economy can instantaneously "switch" from one stationary equilibria to another. The presence of public infrastructure does, however, tend to temper the increase in public debt by providing an additional channel through which households can increase their flow of income from capital and labor. This in turn raises tax revenues for the government, thereby slowing down the accumulation of debt. The inclusion of public infrastructure in the model specification therefore implies a lower level of optimal debt relative to the standard model without infrastructure, both when comparing steady states as well as accounting for transitional dynamics.

The rest of the paper is organized as follows. Section 2 describes the analytical framework. Section 3 describes the calibration and computational procedure. Section 4 presents our results, and Section 6 concludes.

2 Analytical Framework

We consider an economy populated by a continuum of infinitely-lived households. The key feature of these households is that they face incomplete insurance markets, as in Aiyagari (1994), i.e., they are unable to purchase perfect insurance against the realization of an idiosyncratic labor productivity shock. Therefore, though all households are identical *ex-ante*, their inability to insure against the labor productivity shock makes them heterogeneous *ex-post*. There is a government in the economy which spends tax revenues on three types of public goods - a wasteful public consumption good, a lump-sum transfer, and the economy's stock of public infrastructure which generates productivity spillovers for private factors. The government can also sell or purchase bonds, resulting in a public debt or surplus.

2.1 Households

Households in this economy choose their rate of consumption, c, and time allocation between labor and leisure to maximize a per-period utility function given by:

$$U(c,l) = \frac{\left[c^{\eta}(1-l)^{1-\eta}\right]^{1-\sigma}}{1-\sigma} \tag{1}$$

where l denotes the allocation of the household's unit time endowment to labor supply. Households are identical *ex-ante*, but receive idiosyncratic shocks to their labor productivity, ϵ, at the beginning of each period. While agents cannot perfectly insure against these fluctuations, they can partially insure against them by accumulating a stock of assets, a, that pay out a market-determined interest rate, r. These assets are comprised of private capital, k, which households rent to the representative firm in the economy, and holdings of government bonds, b, such that a household's portfolio is given by $a = k + b$. Therefore, the presence of incomplete markets generates a precautionary motive for savings, causing households to accumulate wealth during periods of high productivity, in order to compensate for periods where adverse productivity shocks are realized. Over time, this mechanism leads to an endogenous distribution of wealth across households. In maximizing the per-period utility (1), households are constrained by an intertemporal budget constraint

$$c + a' \leq [1 + (1 - \tau)r]a + (1 - \tau)wl\epsilon + TR \tag{2}$$

where a' denotes the household's stock of wealth in the next period, τ represents the income tax rate, w is the real wage rate, and TR represents a lump-sum transfer received from the government. We assume that the household-specific productivity shock, ϵ, follows a Markov process with a transition matrix given by $\pi(\epsilon'|\epsilon)$. The household's maximization problem can then be written as:

$$V(a, \epsilon) = \max_{c,l,a'} \left[U(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon)V(a', \epsilon') \right] \tag{3}$$

subject to the intertemporal budget constraint (2), along with the restriction that $a' \geq \underline{a}$, where $\underline{a}$ denotes the borrowing constraint faced by households.

2.2 Firms

The representative firm in this economy produces a flow of final output using a standard neoclassical technology and three inputs, namely aggregate capital, K, aggregate labor, L, and the stock of public infrastructure, K_G:

$$Y = K_G^{\phi} K^{\alpha} L^{1-\alpha}, \ \phi \in (0, 1) \text{ and } \alpha \in (0, 1) \tag{4}$$

In the production function (4), the stock of public infrastructure, K_G, generates positive spillovers for the firm's production, with an output elasticity of ϕ. The representative firm is competitive and, in maximizing its flow of profits, takes all market prices and the stock of public infrastructure as exogenously given. The firm's problem can be written as:

$$\max_{K,L} K_G^{\phi} K^{\alpha} L^{1-\alpha} - wL - (r + \delta_K)K \tag{5}$$

where δ_K is the rate of depreciation of private capital. The optimality conditions for the firm's problem pins down the equilibrium real wage and return on capital:

$$w = (1-\alpha)K_G^{\phi}\left(\frac{K}{L}\right)^{\alpha} \tag{5a}$$

$$r = \alpha K_G^{\phi}\left(\frac{K}{L}\right)^{\alpha-1} - \delta_K \tag{5b}$$

2.3 Government

The government raises revenue by levying a tax on household income, and spends on transfer payments, TR, a public consumption good, G_c, and investment in the economy's stock of infrastructure, G_I. The government can also issue or purchase instantaneous one-period bonds, B (which are held by households). The government's flow budget constraint can be written as:

$$G_c + TR + G_I + rB = \tau(wL + rA) + B - B' \tag{6}$$

As explained further in a subsequent section, we assume that each category of public expenditure is proportional to aggregate output when obtaining the baseline stationary equilibrium for the model economy:

$$G_c = g_c Y;\, TR = g_{TR} Y;\, G_I = K_G' - (1-\delta_G)K_G = g_I Y \tag{7}$$

where g_c, g_{TR}, $g_I \in (0,1)$ and δ_G denotes the depreciation rate of infrastructure.

2.4 Equilibrium

To analyze the optimal level of debt, we consider both stationary and transitional equilibria. We define these equilibrium concepts in the following two subsections.

2.4.1 Stationary Equilibrium

A stationary equilibrium in this economy is characterized by a value function, $v(a,\epsilon)$, time-invariant decision rules, $a'(a,\epsilon)$, $l(a,\epsilon)$, and $c(a,\epsilon)$, a time-invariant joint distribution of individual states, $F(a,\epsilon)$, factor prices, w and r, government policy variables, τ, g_c, g_{TR}, and g_I, and a vector of aggregates, A, K, L, C, K_G, G, TR, B, such that:

1. Given factor prices, the decision rules solve the household's problem

2. Factor prices satisfy the firm's FOCs

3. Aggregate assets, consumption, and labor are derived from individual decision:

$$A = \sum_{\epsilon} \int_a a'(a,\epsilon) f(a,\epsilon) da$$
$$C = \sum_{\epsilon} \int_a c(a,\epsilon) f(a,\epsilon) da$$
$$L = \sum_{\epsilon} \int_a \epsilon l(a,\epsilon) f(a,\epsilon) da$$

4. The asset market clears:

$$A = K + B$$

5. The goods market clears:

$$C + \delta K + G_c + G_I = Y$$

6. The government's budget constraint is satisfied:

$$G_c + TR + G_I + rB = \tau[wL + rA]$$

7. The distribution of individual states is stationary:

$$F(a',\epsilon') = \sum_{\epsilon} \pi(\epsilon'|\epsilon) F(a'^{-1}(a',\epsilon),\epsilon)$$

2.4.2 Transitional Equilibrium

We are also interested in accounting for the welfare changes that accrue over the transition path as the economy converges to its new stationary equilibrium following a change in debt policy. As such, we adopt t-notation to distinguish between different time periods within the transition phase, and specify the exogenous path for the economy's debt-to-GDP ratio. A transitional equilibrium for this economy is a sequence of time varying decision rules, $\{a'_t(a,\epsilon), l_t(a,\epsilon), c_t(a,\epsilon)\}_{t=0}^{T}$, a sequence for the time varying distribution of individual states, $\{F_t(a,\epsilon)\}_{t=0}^{T}$, sequences of factor prices, $\{w_t, r_t\}_{t=0}^{T}$, sequences of government polices, $\{\tau_t, g_{c,t}, g_{TR,t}, g_{I,t}\}_{t=0}^{T}$, and sequences of aggregate variables, $\{A_t, K_t, L_t, C_t, G_t, K_{G,t}, TR_t, B_t\}_{t=0}^{T}$, such that:

1. The economy at time $t = 0$ is consistent with our initial stationary equilibrium.

2. The economy at time $t = T$ is consistent with our terminal stationary equilibrium.

3. The decision rules solve the household's problem for each time period.

4. The factor prices satisfies the firm's FOCs for each time period

5. Aggregate assets, consumption, and labor at each point in time are determined by:

$$A_t = \sum_{\epsilon} \int_a a'_{t-1}(a,\epsilon) f_t(a,\epsilon) da$$
$$C_t = \sum_{\epsilon} \int_a c_t(a,\epsilon) f_t(a,\epsilon) da$$
$$L_t = \sum_{\epsilon} \int_a \epsilon l_t(a,\epsilon) f_t(a,\epsilon) da$$

6. The asset market clears each period:

$$A_t = K_t + B_t$$

7. The goods market clears each period:

$$C_t + K_{t+1} - (1-\delta)K_t + G_{c,t} + G_{I,t} = Y_t$$

8. The government's budget constraint is satisfied in every period:

$$G_{c,t} + TR_t + G_{I,t} + r_t B_t = \tau_t[w_t L_t + r_t K_t] + B_{t+1} - B_t$$

9. The distribution of individual states evolves as:

$$F_{t+1}(a,\epsilon) = \sum_{\epsilon} \pi(\epsilon'|\epsilon) F_t(a^{-1}(a_{t+1},\epsilon),\epsilon)$$

3 Calibration and Solution

As a first step, and to understand better the role played by public infrastructure, we compare our benchmark specification (with infrastructure) to a specification where infrastructure is absent. For the specification without infrastructure, the production function is given by:

$$Y = \Phi K^{\alpha} L^{1-\alpha},\ \Phi > 0 \text{ and } \alpha \in (0,1) \tag{8}$$

where Φ denotes an exogenously specified aggregate level of productivity for the representative firm. In this specification, government spending entails only the public consumption good, G_c, and transfers, TR. The two model specifications are then calibrated to yield identical stationary equilibria using data from the United States for the period 1990-2015. These baseline equilibria serves as the starting point of our analysis, with the model specifications being evaluated at an annual frequency.

Table 1 reports the model's parameters along with their corresponding empirical targets. The rate of time preference, β, is set to 0.95 in order to match a steady-state interest rate (or return on capital) of approximately 4 percent. We set the parameter σ in the utility function (1) to 1.5, which yields an intertemporal elasticity of substitution of about 0.67, within the range of estimated values reviewed by Guvenen (2006). The relative share of consumption in the utility functions, η, is set to 0.36 to match the aggregate share of time allocated to work in the United States during the sample period, which is about 30 percent. The output elasticity of private capital, α, is set to its standard value of 0.3, while the output elasticity of public infrastructure, ϕ, is set to 0.14, consistent with the average estimates from U.S. studies listed in Bom and Ligthart's (2014) meta-analysis.[3] Correspondingly, in the specification that does not include infrastructure, we calibrate the TFP parameter, Φ, in (8) to ensure that both specifications yield the same productivity index (see (4)). While we set the depreciation rate of private capital to the standard annual rate of 10 percent, the corresponding depreciation rate for public infrastructure is chosen to match the observed public capital-GDP ratio in the data.[4]

The remaining parameters found in Table 1 relate to government policy (g_c, g_I, g_{TR}, and τ), and must be set differently depending on the model specification considered (with vs without infrastructure). Starting with our baseline specification with infrastructure, we set g_I, g_c, and g_{TR} to 0.04, 0.15, and 0.09 respectively, matching their corresponding average shares observed in the data. On the other hand, in the specification without infrastructure, we increase the share of government consumption in GDP to 0.19 so that total government spending remains identical across the two model specifications, and equal to the corresponding average in the data. Lastly, the income tax rate, τ, is set so the government's tax revenue

[3]Another important consideration is the sensitivity of the optimal debt calculations with respect to the output elasticity of public infrastructure, ϕ. While our benchmark calibration sets $\phi = 0.14$, we consider a lower value of $\phi = 0.05$ as well. Our computations suggests that the more productive public capital is, the lower is the optimal share of public debt. Intuitively, when ϕ is large, the precautionary savings motive generated by public investment is also large, causing the government to accumulate a lower level of debt relative to when ϕ is small. In the limit, as $\phi \to 0$, the model specification approximates the case where infrastructure is absent, as shown in the welfare comparison in Figure 2. While we do not report these results here, they are available on request.

[4]Our results are not sensitive to small variations in these depreciation rates.

as a share of GDP, TS, is approximately 31 percent of output.[5]

$$TS = \frac{\tau(wL + rA)}{Y} = 0.31 \tag{9}$$

With all policy parameters pinned down, the level of public debt in the baseline stationary equilibrium is backed out from the government's budget constraint.

In order to assess the fit of our baseline calibration, Table 2 compares the consumption-output, private capital-output, and public debt-output ratios derived endogenously from our model to that found in the U.S. data. Inspection of Table 2 indicates that our model's values are reasonably close to the U.S. averages, with a consumption-output ratio of 59 percent (versus 66 percent in the data), a private capital-output ratio of 2.15 (versus 2.20 in the data), and a public debt-output ratio of 72 percent (versus 70 percent in the data). Given these baseline stationary-equilibria, we examine the following questions: (i) what are the steady-state optimal shares of public debt in the two model specifications?, and (ii) how do these optima differ when the transition paths between stationary states are accounted for when computing welfare effects?

3.1 Income Shock Process

Following Rohrs and Winter (2017) we set $\underline{a} = 0.3Y$ so that households can borrow up to 30 percent of steady-state output, and adopt the following shock process:

$$\epsilon = [0.055, 0.551, 1.195, 7.351]$$

$$\pi = \begin{pmatrix} 0.940 & 0.040 & 0.020 & 0.000 \\ 0.034 & 0.816 & 0.150 & 0.000 \\ 0.001 & 0.080 & 0.908 & 0.012 \\ 0.100 & 0.015 & 0.060 & 0.825 \end{pmatrix}$$

This shock process was estimated following Castaneda et al. (2003) and yields model estimates of income and wealth distributions that align well with the U.S. data.[6] Table 3 shows

[5] During the sample period 1990 - 2015, tax revenue as a share of GDP averaged 28 percent. However, we chose to target a tax share of 31 percent to account for the fact that our baseline stationary equilibrium abstracts from budget deficits that averaged 3 percent during this time period.

[6] Data from the 2007 Survey of Consumer Finances is used to compute a measure of net financial assets excluding longer term investments such as property, vehicles, business ownership, residential mortgages, and automobile loans. We choose to not re-estimate the shock process as the values reported in Rohrs and Winter (2017) allow our model to match the data sufficiently well.

the distribution of net financial assets from the 2007 SCF: the top 20 percent of households hold more than 90 percent of net financial assets, while the bottom 80 percent's share is slightly below 9 percent, indicating a highly unequal distribution of wealth in the U.S. economy. Table 3 also shows that the baseline model with infrastructure generates a reasonably good fit to the distribution of wealth and income reported in the U.S. data, especially for the top three quintiles.[7]

3.2 Welfare

An important aspect of our analysis is the measurement of economic welfare from an underlying menu of government policies. Following Aiyagari and McGrattan (1998), Floden (2001), we adopt the following utilitarian social welfare function:

$$\Gamma = \sum_{\epsilon} \int_{a} V(a, \epsilon) f(a, \epsilon) da \tag{10}$$

The welfare measure in (10) can be interpreted as the welfare level of the average individual in the economy. In reporting welfare changes in subsequent sections, we use a compensating variation measure, which quantifies the units of consumption that need to be transferred between two steady-states (say, generated by two different policies or shocks), such that the average individual is indifferent between these steady states. This leads to the following compensating variation measure for welfare changes:

$$\Delta\Gamma = 1 - \left[\frac{\sum_{\epsilon} \int_{a} V_0(a, \epsilon) f_0(a, \epsilon) da}{\sum_{\epsilon} \int_{a} V_1(a, \epsilon) f_1(a, \epsilon) da} \right]^{\frac{1}{\eta(1-\sigma)}} \tag{11}$$

where the subscripts 0 and 1 refer to the baseline (pre-shock) and new (post-shock) steady states. If $\Delta\Gamma > 0$, the average agent would prefer being at the new equilibrium without compensation for the change. On the other hand, if $\Delta\Gamma < 0$, then $\Delta\Gamma$ units of consumption is required to make the agent indifferent between the two steady states. This then becomes a measure of welfare loss across the two steady states.

While the compensating variation measure described in (11) provides the long run welfare costs of varying debt policies, this measure ignores the short run dynamics that occur as the economy transitions between steady states. Accounting for these transitional dynamics in

[7] The model specification without infrastructure also fits the U.S. wealth and income distributions well. Results available from authors upon request.

our welfare measure is very important because adjustments to the debt level will impact variables differently depending on the time horizon considered. For example, in the long run reducing debt will reduce the tax rate in the economy, as a lower debt level implies a lower debt servicing burden. However, in the short run, taxes must be increased significantly in order to pay down the debt. Differences such as these imply that the welfare effects of debt policies may be very different in the short and long run.[8] To this end, we also compute the welfare effects including transitional dynamics as:

$$\Delta\Gamma = 1 - \left[\frac{\sum_{\epsilon}\int_{a}\sum_{t=0}^{T}\beta^{t}U(c_0(a,\epsilon),l_0(a,\epsilon))f_0(a,\epsilon)da}{\sum_{\epsilon}\int_{a}\sum_{t=0}^{T}\beta^{t}U(c_t(a,\epsilon),l_t(a,\epsilon))f_{t-1}(a,\epsilon)da}\right]^{\frac{1}{\eta(1-\sigma)}} \quad (12)$$

where $c_t(a,\epsilon)$, $l_t(a,\epsilon)$, and $f_{t-1}(a,\epsilon)$ denote the decision rules for consumption and leisure and the wealth density known at time t in the transition path while $c_0(a,\epsilon)$, $l_0(a,\epsilon)$, and $f_0(a,\epsilon)$ denote the same values for the initial stationary equilibrium.

4 Results

In the next several sections we present the results of our paper in steps. Throughout our counterfactual experiments, we assume that the government adjusts the tax rate, τ, in order to satisfy their budget constraint as public debt is varied. Therefore, when computing alternative equlibria and associated transition paths, we relax the restriction that $TS = 0.31$ used in our baseline calibration to determine the initial value of τ, and instead, determine τ endogenously using the government's budget constraint. This strategy is consistent with the existing literature, and facilitates a direct comparison of our results with those found in previous studies.

4.1 Adjustment in Aggregates Across Stationary Equilibria

In this section, we characterize the stationary equilibrium relationship between the level of public debt and the economy's key aggregate and distributional variables. Figure 1 shows how the level of these variables change as the share of public debt in GDP changes from its benchmark level, with the income tax rate adjusting to satisfy the government's budget constraint. The red line indicates the equilibrium relationships for the model specification with public infrastructure, while the black line represents the specification without infrastructure.

[8] See Desbonett and Witzenbaum (2012) for further details on this issue.

The productivity-enhancing role of public infrastructure raises both the marginal product of labor and capital, enabling the economy to produce more output. As the government reduces the ratio of public debt to GDP, it crowds in private capital with the real rental rate falling which, in turn, feeds back into higher output. This mechanism can be easily seen in Figure 1: the lower is public debt, the larger is the gap between the flow of aggregate output, the real wage rate, and the stock of private capital between the two model specifications. The higher flow of output also enables agents to increase consumption at a faster rate in the model with public infrastructure, though diminishing returns set in at negative levels of public debt. The effect of public debt on productivity can be seen clearly in the first panel of Figure 1, which presents the TFP coefficient in the production function for both model specifications.[9] As the debt level is reduced, K_G increases, allowing productivity in the baseline specification to rise. However, in the alternative specification in which infrastructure is absent, this productivity-enhancing feature of debt reduction is ignored. This has implications for the optimal level of debt in the two model specifications, which we will explore further in the next section.

Figure 1 also plots the wealth Gini coefficient as a function of the share of public debt in GDP. [10] Here, we find an inverse relationship between wealth inequality and the steady-state share of public debt: for both model specifications, as the steady-state share of public debt declines, wealth inequality increases.[11] Both specifications yield very similar levels of long-run inequality for a given level of public debt, with the specification that includes public infrastructure generating slightly lower levels of wealth inequality relative to the specification without infrastructure. As the long-run stock of public debt declines, it tightens the borrowing constraint for households. For households that face a binding borrowing constraint, this implies less access to private capital. On the other hand, for wealthier households that are not credit-constrained, this implies the opposite: the reduction in public debt crowds in private capital for their portfolios. Consequently, wealth (and income) inequality increases in equilibrium.

[9] In the baseline specification with infrastructure, productivity is K_G^{ϕ} and in the alternative specification productivity is given by Φ, where Φ is fixed so that both specifications yield the same output level at the baseline steady-state equilibrium.

[10] To compute the Gini coefficients we start by computing the wealth and income shares at the centile level, and we use these values to recover a discrete Lorenz curve. The gap between this discrete Lorenz curve and the 45 degree line is used to compute the Gini coefficients.

[11] Since wealth may take on negative values, the gini coefficient may actually exceed 1.

4.2 Optimal Public Debt: Comparing Stationary Equilibria

In this section, we ask the following question: what is the optimal share of public debt in GDP for the U.S. when welfare effects are computed ignoring the transition path between stationary states, and how does the presence of public infrastructure influence these results? The first panel of Figure 2 characterizes the optimal share of public debt for both model specifications (with and without infrastructure) under this scenario. The results in this panel only account for the steady-state welfare change that occurs as the economy instantly "switches" from the benchmark equilibrium level of debt to the various alternative debt levels, without taking into account the transition path of endogenous variables.

Inspection of the first panel of Figure 2 clearly demonstrates that the optimal share of public debt in output depends critically on whether infrastructure is included or excluded from the model specification. For the specification *without* infrastructure, our results indicate that the government should run a net *surplus* of about 75 percent of GDP. However, the model specification *with* public infrastructure implies a very different welfare-maximizing share of public debt: in this case, the optimal share of public debt is a much larger surplus, of about 175 percent of GDP. This is an important result, suggesting that by not including infrastructure in the model specification, previous studies on this issue may have over-stated the optimal level of public debt by a significant amount. Further, we also note that the welfare profile implied by the model *without* infrastructure is very flat around the optimum, indicating that changing public debt from this level does not lead to significant long-run welfare gains for the economy. On the other hand, the welfare profile for the specification including infrastructure is relatively steeper around the optimum, indicating that the welfare loss from deviating from this optimal level is non-trivial.

The intuition behind the different levels of optimal public debt across the two specifications can be explained as follows. When the government provides a productive public good like infrastructure, it acts as a complement to private factors in the production function, thereby raising the return to private capital and labor. Consequently, this reduces the precautionary savings motive for households, allowing them to sell their claims on the government. This causes households to be net debtors and the government to be a net creditor to the private sector at the optimum. Consequently, the government accumulates assets and runs a surplus, until diminishing returns to infrastructure set in, leading to the optimum. When infrastructure is absent from the model specification, the channel through which the government provision of public goods affects the household's precautionary savings motive is absent and, hence, diminishing returns to capital set in much earlier (i.e., at a higher share of public debt or lower surplus in GDP).

4.3 Optimal Public Debt: Accounting for the Transition Path

An important caveat with the results presented in the previous subsection is that the welfare computation ignores the transitional adjustment that occurs between stationary equilibria, as public debt changes. To address this shortcoming, the bottom panel of Figure 2 presents welfare profiles for the case where the transition path between stationary equilibria is fully internalized when computing the welfare effects. In sharp contrast to the results shown in the top panel of Figure 2, we now see that once transitional dynamics are internalized, the optimal share of public debt is *positive*. However, whether public infrastructure is included in the model specification matters: for the specification with public infrastructure, the optimal share of public debt is now around 100 percent of GDP, while the specification that excludes public infrastructure implies a higher share of public debt, at around 110 percent of GDP. An interesting aspect of these results is that after including infrastructure and accounting for transitional dynamics in our welfare calculations, our model generates an optimal share of public debt that is significantly larger than its corresponding average (about 70 percent) observed in our sample period of 1990-2015. This comparison remains robust to extending our sample range further back to the entire post-war period for the U.S. economy.

An important question that arises in this context is: why is the optimal level of public debt positive when transitional dynamics are accounted for, while it is negative when one assumes the economy can move instantly between stationary equilibria? The intuition lies in the fact that the steady-state welfare comparison incorporates only the long-run effects of a change in the share of public debt, while ignoring the short run implications of changes in the composition of the government's budget and key macroeconomic variables. For example, an increase in the share of public debt allows the government to reduce the income tax rate in the short run in order to satisfy the government's budget constraint. This, in turn, raises the after-tax return on both capital and labor, thereby increasing both the flow of consumption and output. The increase in public debt also relaxes the borrowing constraint, which further increases welfare along the transition path, by allowing poorer households to smooth consumption over time. These short run welfare gains accumulate in transition, but must eventually trade-off against the long-run welfare losses, as the tax rate must be increased in the long-run to sustain the higher level of debt, and diminishing returns sets in for both public and private factors. However, the short-run welfare gains dominate the long-run losses, leading to an optimal level of debt that is positive in equilibrium. The presence of public infrastructure does, however, tend to reduce the optimal level of public debt, even when transitional dynamics are considered. This follows from the results presented in the previous section. When public infrastructure is included, the long run welfare costs associated with increasing public debt are amplified, as increasing the debt level crowds out both private capital and public infrastructure. Therefore, the inclusion of infrastructure leads to an optimal level of debt that is lower than that implied by the specification without infrastructure.

In order to better understand the short-run effects that occur during the transition between stationary states, we turn our attention to the transition paths for key aggregate variables in our model economy. Figure 3 presents the model's transitional dynamics when the economy moves from the baseline stationary equilibrium to the one that results under the optimal share of public debt. For clarity of exposition, we only plot the transition paths for the model specification that includes infrastructure. Inspection of the bottom panel of Figure 3 shows the strong effect the policy has on the income tax rate. Increasing debt leads to a sharp reduction in the tax rate in the short run (the periods immediately following the policy change). This reduction in taxes, coupled with the fact that an increase in debt relaxes the liquidity constraint for poor households, allows agents to increase consumption, savings (in the form of capital), and output on average. However, this tax cut is short lived, as the government has to raise the tax rate subsequently to sustain the higher level of public debt. Consequently, this reverses the process of capital accumulation and, in the long-run, reduces the equilibrium level of output, private capital, and consumption.

The welfare and distributional implications of the debt policy can be seen in the top row of Figure 3. The panel labeled *Instantaneous Welfare* plots the period-by-period welfare effects in consumption equivalent units while the panel labeled *Intertemporal Welfare* represents the cumulative effects.[12] Inspection of these panels indicates that instantaneous welfare initially falls but then spikes to near 5% of consumption before decaying gradually to zero.[13] This leads to an intertemporal welfare measure that starts negative but increases at a decreasing rate over the entire transition path. Lastly, with respect to inequality, we see that the wealth Gini declines both in the short-run and in the long run, but reaches its minimum value during the periods immediately following the policy change.

[12]For example, at $t = 5$, *Instantaneous Welfare* presents the welfare effect of moving between $t = 4$ and $t = 5$ in the transition path, while *Intertemporal Welfare* presents the entire welfare effect accumulated up to $t = 5$.

[13]The initial reduction in welfare is simply an artifact of the fixed state variables at the time the policy is changed.

5 Conclusions

In this paper, we have revisited an important policy issue that has recently received a lot of attention, namely the optimal share of public debt in an economy populated by heterogeneous agents. Previous studies, starting with the seminal work of Aiyagari and McGrattan (1998), have shown that this optimal share is positive and, at least for the United States, possibly around two-thirds of GDP. While recent work has focused on the underlying income process used to calibrate these models to the data, an important issue that has been neglected is the role of government investment in the economy's stock of infrastructure. Arguably, public infrastructure, by generating productivity benefits for private capital and labor, can help reduce the precautionary savings motive when households face idiosyncratic shocks and incomplete insurance markets. Another critical issue are the welfare consequences of a change in the stock of public debt along the transition path between steady states in the presence (or absence) of infrastructure in the model specification. Our results indicate that these considerations can have important consequences for the optimal quantity of public debt in the economy.

We introduce public infrastructure into a workhorse heterogeneous agent model that is calibrated to match the key aggregate and distributional moments of the United States for the period 1990-2015. To understand better how our model relates to the previous literature, we compare our baseline specification to one without infrastructure. When welfare effects are computed by comparing stationary equilibria, the optimal debt is a large public surplus, indicating that aggregate welfare maximization requires the government to be a net lender to the private sector. Moreover, we find that this optimal public surplus is significantly larger than those derived by previous studies that ignored public infrastructure. However, this finding of an optimal surplus is completely reversed once the welfare effects that occur along the transition path between stationary states are considered. We also demonstrate that the presence of public infrastructure in the context of a heterogeneous agent model lowers the implied welfare-maximizing share of public debt on GDP, relative to the standard model specification that ignores this channel. These results are mainly driven by the differential trade-offs generated for the household's precautionary savings motive in the short run and the long run. From a distributional perspective, we find an inverse relationship between public debt reduction and wealth and income inequality, driven primarily by how the reduction in public debt affects the borrowing constraint for households for whom it is binding.

In our present setup, infrastructure investment follows a fixed exogenous rule whereby the government is assumed to spend a certain fraction of output on infrastructure investment every period. Such an assumption is consistent with the existing optimal debt literature which makes similar assumptions regarding government consumption and transfers. However, given the connection between infrastructure and public debt, one may be interested in endogenizing the government's infrastructure investment decision. Doing so would require solving a fully specified Ramsey plan for the government. While there is a unique solution to the Ramsey plan in the context of a representative agent model with perfect insurance markets, several complications arise when markets are incomplete. Specifically, when markets are incomplete and transitional dynamics are considered, the solution to the Ramsey plan becomes non-unique, and sensitive to underlying assumptions regarding the time-path followed by policy variables (see Shin, 2006 and Dyrda and Pedroni, 2016).[14] While fully addressing this issue is beyond the scope of the present paper, future research into this area may be of interest to academics and policy makers alike.

In summary, our paper contributes to the existing literature on the optimal quantity of public debt in two important ways. First, we show that ignoring the role of public investment in infrastructure may lead to a significant over-statement of the optimum quantity of debt. Second, we show that the characterization of the optimum is incomplete if one ignores the transitional adjustment path and the policy target used by the government to satisfy its budget constraint. Needless to say, there are several related issues that we do not yet consider, such as the internal composition of public spending between new investment and maintenance, and a richer tax structure that differentiates between tax rates on capital and labor income, among other things. These are important issues that we hope to address in future research.

[14] To gain insight into the qualitative implications of endogenizing infrastructure investment we re-solved our model under the assumption that the government varies infrastructure investment (not the tax rate) to maintain their budget constraint as debt is varied. This experiment indicates that the optimal level of debt is lower under such a policy, thought the welfare profiles are qualitatively consistent (results available from authors upon request).

Table 1: Structural Parameters and Empirical Targets

Parameter	Value	Description	Target[a]
Preference Parameters			
β	0.95	Rate of Time Preference	$r = 0.04$
σ	1.50	Coefficient of Relative Risk Aversion	Literature
η	0.36	Relative Share of Leisure in Utility	$N = 0.3$
Production Parameters			
α	0.30	Output Elasticity of Private Capital	Literature
ϕ	0.14	Output Elasticity of Public Capital	Literature
δ_K	0.10	Depreciation Rate of Private Capital	Literature
δ_G	0.05	Depreciation Rate of Public Capital	$\frac{K_G}{Y} = 0.67$
Policy Parameters			
g_c	0.15	Government Consumption (Share of GDP)	$\frac{G_c}{Y} = 0.15$
g_I	0.04	Government Investment (Share of GDP)	$\frac{I_G}{Y} = 0.04$
g_{TR}	0.09	Government Transfers (Share of GDP)	$\frac{TR}{Y} = 0.09$
τ	0.38	Income Tax Rate	$TS = 0.31$

[a] Sources for empirical targets come from both the existing literature and aggregate U.S. data for the period 1990-2015. Specifically, data from the World Bank Development Indicators (WDI), the International Monetary Fund's Investment and Capital Stock database, and the U.S. BEA were used.

Table 2: Model Fit - Aggregate Variables

Variable	Description	Data[a]	Model
C/Y	Consumption-Output Ratio	0.66	0.59
K/Y	Private Capital-Output Ratio	2.20	2.15
B/Y	Public Debt-Output Ratio	0.70	0.72

[a] The data represents averages for the U.S. for the sample period 1990-2015. Sources: U.S. BEA and IMF.

Table 3: Model Fit - Endogenous Distributions

	Wealth Distribution		Income Distribution	
	Data[a]	Model	Data[a]	Model
Q1	-1.60	-2.04	-0.40	0.45
Q2	0.10	-0.15	3.19	6.59
Q3	1.64	2.94	12.49	14.80
Q4	8.29	7.36	23.33	20.29
Q5	91.57	91.88	61.39	57.87

[a] Data Source: U.S. Survey of Consumer Finances, 2007.

Figure 1: Adjustment in Aggregate Variables

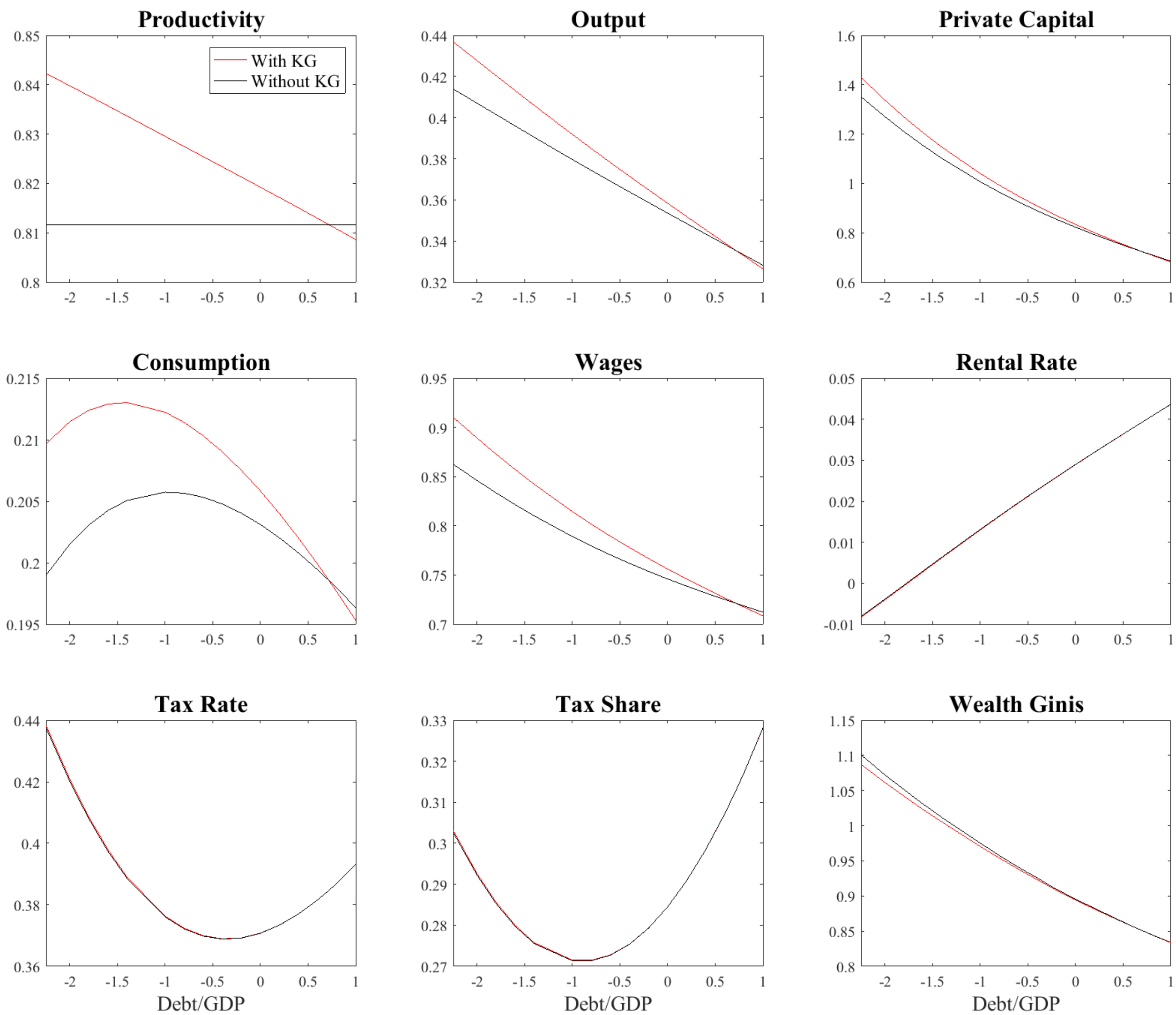


Figure 2: Welfare Profile

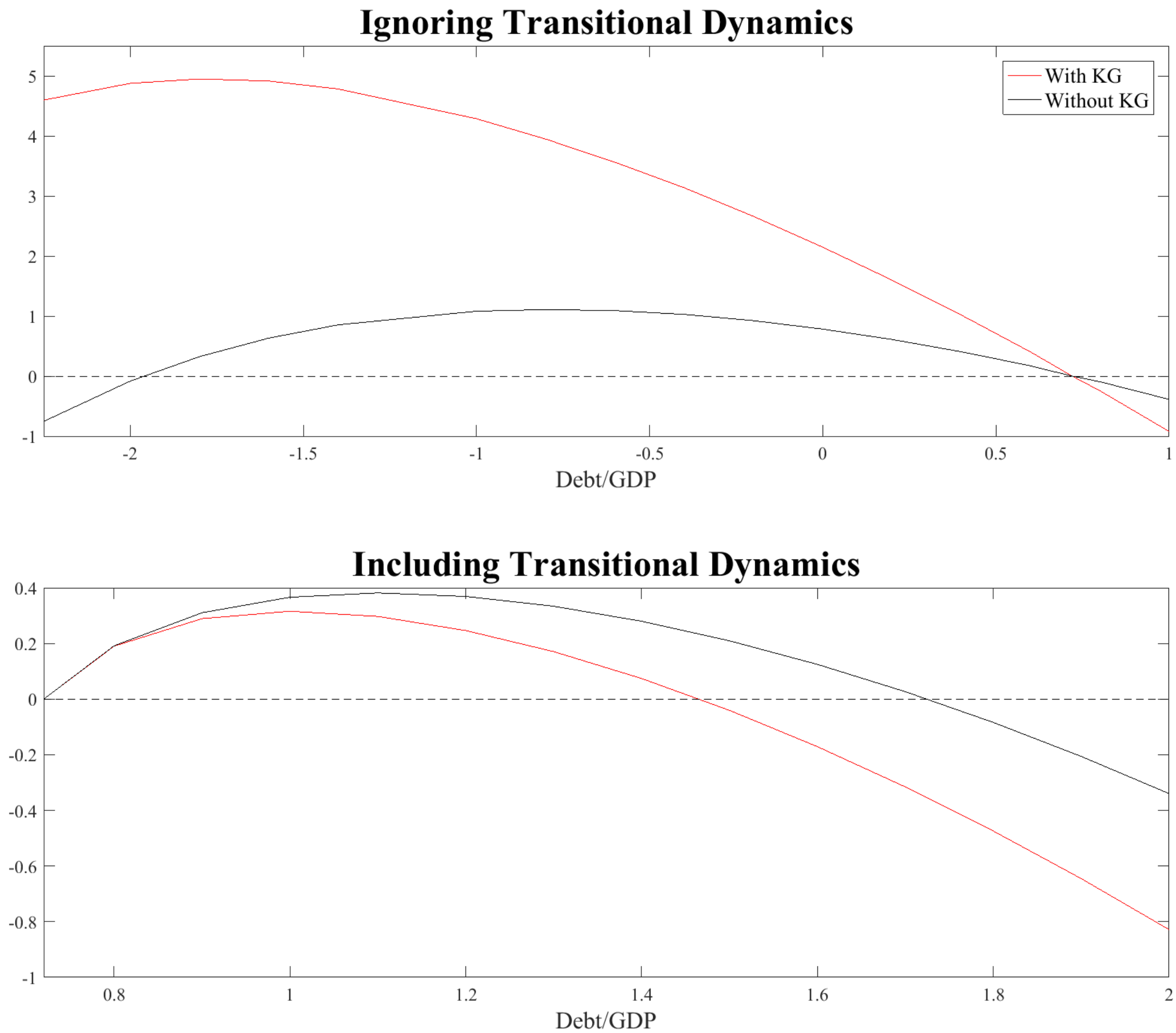
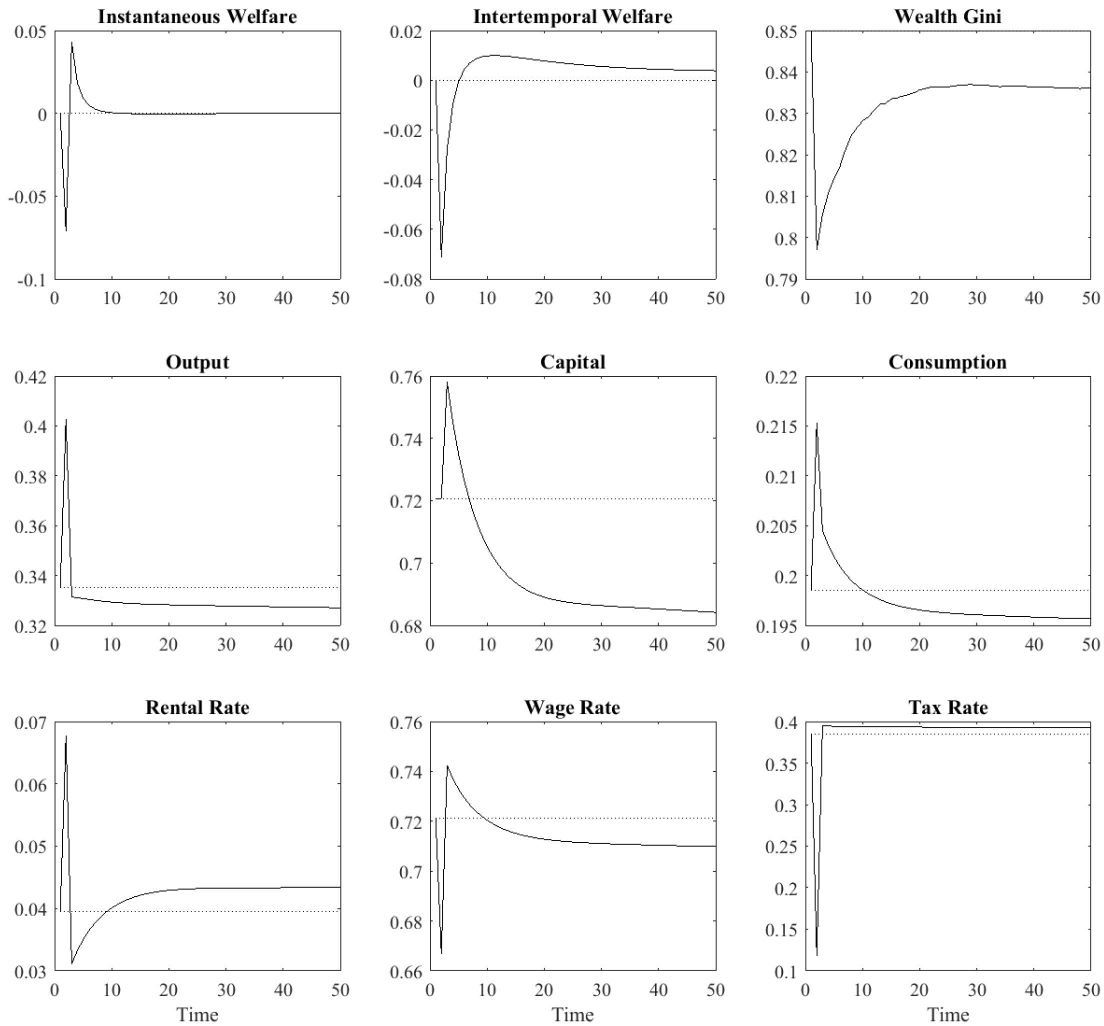


Figure 3: Transitions Paths



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