Filling potholes: macroeconomic effects of maintenance versus new investments in public infrastructure

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Abstract

The maintenance of existing public infrastructure has often been neglected in favor of building new infrastructure in developing countries. This paper develops a dynamic general equilibrium model that analyzes the reasons and effects of such neglect. First, the optimal level of maintenance is shown to depend on the size of new investments as a share of existing public infrastructure and on the productivity of infrastructure. Second, the model is parameterized and solved numerically for a sample of Latin American countries. Quantitative results show that reallocating funds from new infrastructure to maintenance can have positive effects on these countries’ GDP.

Keywords: Public infrastructure; Maintenance; Growth

JEL classification: E62; H50; O40

1. Introduction

Good roads, canals, and navigable rivers, by diminishing the expense of carriage, put the remote parts of the country more nearly upon a level with those in the neighboring town. They are upon that account the greatest of all improvements. Adam Smith, 1776

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A country’s roads and water systems are the foundations on which economic activity takes place. Most economists agree that public infrastructure is important for a country’s growth. In developing countries, much attention is usually devoted to building new public infrastructure projects. However, filling the potholes that invariably appear with time and usage receives much less attention. The maintenance of existing public infrastructure has been neglected in favor of starting new, highly visible projects. This paper both presents evidence and quantifies the macroeconomic effects of such neglect. In addition, the paper derives the optimal level of maintenance and quantitatively evaluates the effects of redistribution between the two expenditures.

In general, maintenance is defined as those activities that allow public infrastructure to efficiently deliver the outputs for which they were built (Gyamfi et al., 1992). A well maintained road should last 10–15 years before resurfacing is necessary. Lack of maintenance can cause severe deterioration requiring resurfacing in as soon as 5 years. The World Bank reports that an additional $12 billion spent on timely road maintenance in Africa during the last decade could have saved $45 billion spent in reconstruction. Similarly, in the case of power lines, spending $1 million to reduce line losses could have saved $12 million in generating capacity (World Development Report, 1994). Maintenance disregard leads to road deterioration, irrigation canal blockage, leaks, and power line breakdowns reducing the economy’s productive capacity.

Although the World Bank has conducted a number of country studies of the adverse effects of deficient infrastructure maintenance (these are discussed in the background section), this topic has not been studied extensively in the academic literature. An exception that does not directly concentrate on maintenance, but provides motivation for this paper, is Devarajan et al. (1996) (henceforth DSZ). These authors find empirical support in developing countries for current public expenditures (like maintenance) having positive effects on GDP, while capital expenditures (like new infrastructure) have negative effects on GDP. DSZ argue that much has already been spent on infrastructure (often at maintenance’s expense), so at the margin an extra dollar spent on maintenance may be more productive. However, most of the focus of the academic research has been on the effect of overall public investment on economic growth. Seminal empirical work in this area (using growth accounting and cross-country regression analysis) includes Easterly and Rebelo (1993), Canning and Fay (1993), Canning (1998, 1999) and Hulten (1996) (among others). Nevertheless, recent compelling empirical evidence (by DSZ and the World Bank) suggest that to fully understand the effects of public infrastructure, one should also study maintenance’s role. Given that motivation, in this paper I attempt to describe the determinants of

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1There are also a number of papers that study the effects of public expenditures in the US. These include: Aschauer (1989), Batina (1999), Fernald (1999), Holtz-Eakin and Schwartz (1995), Nadiri and Mamuneas (1994), Morrison and Schwartz (1996), and Munnell (1992). Studies of infrastructure in OECD countries include Demetriades and Mamuneas (2000).
optimal infrastructure policy, the reasons for maintenance neglect, and to quantify the tradeoffs associated with spending on new infrastructure versus repairing existing infrastructure given limited resources.

For that purpose, this paper constructs a dynamic general equilibrium model that incorporates both types of expenditures and can be used for policy analysis. Following the Lucas (1987) program, the model is internally consistent and micro-foundations based. As Lucas suggests, the model makes a quantitative connection between policies and their consequences. The model has three sectors: households, firms, and a government. The public infrastructure stock is a government provided input in firms’ production. As it is commonplace in developing countries, new public projects are mostly financed by international donors (typically by governments of industrial countries or international organizations). The maintenance of existing public infrastructure, conversely, is financed by taxation. In the model, the depreciation rate of infrastructure is endogenous: it depends on the level of maintenance and on usage. First, a simplified version of the model is solved obtaining closed form solutions. The Ramsey solution for the optimal level of maintenance shows that it depends on the size of new investments as a share of existing public infrastructure and on how productive infrastructure is. Second, the model is solved numerically and parameterized using historical data from seven Latin American countries. Quantitative results show that reallocating some of the donor aid away from new investments and towards maintenance can have positive effects on GDP.

The paper is organized as follows. Section 2 discusses the background of various findings; Section 3 presents the model. Section 4 derives closed form solutions to a simplified version of the model. Section 5 is the quantitative evaluation, and Section 6 concludes.

2. Background

The World Bank has conducted most of the research on the issue of maintenance of infrastructure. These studies have concentrated mostly on one particular type of

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3Previous related theoretical work includes: Barro (1990), Devarajan et al. (1998), Glomm and Ravikumar (1994), and Rioja (1999). However, these papers do not analyze the optimal mix and tradeoffs between maintenance and new public investments as done in the present paper.

4The choice of depreciation rate for private capital was studied by Haavelmo (1960). This concept was also used by Auerbach (1979) and Abel (1981) to study the ‘choice of asset life’ problem. These two papers examine how producers choose asset life when faced with inflation and a corporate income tax code (like in the US) that allows ‘historic’ depreciation rather than ‘replacement cost’ depreciation. Abel concludes that inflation may increase or decrease the durability of capital depending on the ratio of the nominal discount rate to depreciation rate being greater or less than a critical value, which depends on tax parameters.

5Other researchers have started studying the role of maintenance of private capital. For instance, McGrattan and Schmitz (1999) study its effect on business cycles using Canadian data.
infrastructure: roads. Their findings are surprising. For example, between 1979 and 1984, 6000 km of paved road were built in Brazil. During this same period, another 6000 km of roads went from ‘fair’ to ‘poor’ and 2000 km went from ‘good’ to ‘poor’ due to maintenance neglect (Harral, 1988). In Chile, the main north-to-south highway, which runs 3000 km, was paved in the 1960s. In the 1970s, half of this highway collapsed due to poor maintenance requiring its reconstruction at great expense.

While maintenance disregard can cause larger expenditures in the future, it can also impose an additional, immediate, cost to users. Consider the following example from Zambia reported in Roth (1996). In 1992, the Federation of Zambian Road Hauliers commissioned a study on the effects of bad road conditions on a vehicle’s operating costs.\(^5\) The study compared the extra costs of damaged shocks, springs, brake shoes, clutch, etc., due to continuous travel on a pothole-filled road in order to deliver products. These estimated costs are shown in Table 1.

In general, these trucks were delivering products to the market or bringing needed inputs for production. The increases in operating costs to the Zambian company due to an un-maintained road were large (about $14 000). The resulting increases in the costs of inputs should decrease the supply of their good. Infrastructure in poor condition, consequently, should affect a country’s output at the aggregate level.

The ways of financing maintenance and new infrastructure in developing countries should also be described briefly. Maintenance expenditures, which fall under the ‘current public expenditures’ classification, have been typically financed

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\(^5\)Two sets of vehicles were used for the experiment. Each set was composed of a truck and 22-wheel tractor trailer. For a whole year, one set of vehicles was assigned to deliver goods back and forth using the pothole-filled road, while the other set of vehicles used the road in good condition.
by these countries governments from general tax revenue (DSZ, 1996). New
public infrastructure construction, however, has been typically financed by foreign
official sources using: bilateral loans (from an industrial country’s government to
the developing country’s government); multilateral loans (from an international
organization like the World Bank or a regional development bank); and outright
grants from these sources. According to the World Development Report 1994,
bilateral and multilateral sources made up as much as 80% of public infrastructure
funding up to the early 1990s (p. 90). While private lending for infrastructure has
increased since the mid 1990s, the fact is that official sources from abroad have
been largely responsible for financing the construction of public infrastructure in
the developing world.

Table 2 further describes some of the characteristics of this financing for seven
Latin American countries and seven African countries. First, note that a good
portion of official debt in these countries is routinely forgiven. Between 1989 and
1999 about 22% of official debt in Latin America and 33% of official debt in
Africa was forgiven. Second, official debt has liberal grace periods that average 5
years. Finally, the average maturity of these ‘loans’ is about 20 years, which is
again quite generous compared to the average maturity of private loans of 7 years.

Table 2
Official debt characteristics in Latin America and Africa

<table>
<thead>
<tr>
<th>Country</th>
<th>% Debt forgiven&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Average grace period&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Average maturity&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Latin America</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina</td>
<td>30</td>
<td>4.5</td>
<td>15.7</td>
</tr>
<tr>
<td>Brazil</td>
<td>13</td>
<td>3.9</td>
<td>14.8</td>
</tr>
<tr>
<td>Chile</td>
<td>50</td>
<td>4.8</td>
<td>17.4</td>
</tr>
<tr>
<td>Colombia</td>
<td>n.a.</td>
<td>4.8</td>
<td>17.9</td>
</tr>
<tr>
<td>Mexico</td>
<td>20</td>
<td>4.0</td>
<td>30.1</td>
</tr>
<tr>
<td>Peru</td>
<td>8</td>
<td>4.5</td>
<td>16.6</td>
</tr>
<tr>
<td>Venezuela</td>
<td>14</td>
<td>4.2</td>
<td>15.0</td>
</tr>
<tr>
<td>Average</td>
<td>22.5</td>
<td>4.4</td>
<td>18.2</td>
</tr>
<tr>
<td><strong>Africa</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cameroon</td>
<td>10</td>
<td>7.1</td>
<td>25.1</td>
</tr>
<tr>
<td>Dem. Rep. Congo</td>
<td>2</td>
<td>5.4</td>
<td>22.0</td>
</tr>
<tr>
<td>Côte d’Ivoire</td>
<td>54</td>
<td>6.7</td>
<td>22.9</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>84</td>
<td>8.0</td>
<td>31.5</td>
</tr>
<tr>
<td>Nigeria</td>
<td>17</td>
<td>4.1</td>
<td>14.9</td>
</tr>
<tr>
<td>South Africa</td>
<td>n.a.</td>
<td>2.8</td>
<td>14.3</td>
</tr>
<tr>
<td>Sudan</td>
<td>n.a.</td>
<td>5.4</td>
<td>20.5</td>
</tr>
<tr>
<td>Average</td>
<td>33.4</td>
<td>5.6</td>
<td>21.6</td>
</tr>
</tbody>
</table>

<sup>a</sup> As percent of Total Debt 1989–1999.
<sup>b</sup> Average over 1975–1999.
Basically, developing countries finance infrastructure at extremely generous terms from industrial countries and international organizations, and a good portion of those ‘loans’ are in practice donations as they are later forgiven.

3. The model

The framework is grounded in the neoclassical growth model. In brief, the model has a representative household who earns income from renting its capital and effort to a private firm. The household allocates this income between consumption and investment to augment its capital stock so as to maximize utility. Firms produce a final good, so they decide on the amounts of labor and capital to hire for production. Public infrastructure is a government-provided input in production. Donations from foreign countries finance new public infrastructure investments. However, the government must raise revenues by taxing output at the firm level to pay for maintenance of public infrastructure. A formal description of each of these sectors follows.

3.1. Households

The representative household chooses a stream of consumption \( \{c_t\}_{t=0}^\infty \) so as to maximize its utility function,

\[
\sum_{t=0}^\infty \beta^t u(c_t),
\]

where \( 0 < \beta < 1 \). The household earns a wage rate, \( w_t \), for the labor, \( l_t \), it supplies to the firm. Labor is supplied inelastically, so \( l_t = 1 \), and population is fixed. Also, the household receives a return, \( r_t \), on the private physical capital, \( k_t \), it rents to the firm. Hence, each period the household must also choose a level of investment, \( i_t \), to augment capital stock. Private capital evolves according to,

\[
k_{t+1} = i_t + (1 - \delta)k_t,
\]

where \( \delta \) denotes the depreciation rate of private capital. Household total income is allocated between consumption, \( c_t \), and investment, \( i_t \). The household’s budget constraint is then,

\[
c_t + i_t \leq w_t l_t + r_t k_t,
\]

The model builds on previous theoretical work of Barro (1990) and Glomm and Ravikumar (1994) by introducing maintenance and exploring its role.
3.2. Firms

There are many identical competitive firms in the market. The representative firm decides on how much private capital and labor inputs to hire in order to produce the final good. In addition, the stock of public infrastructure, $K_{Gi}$, is a publicly provided input. Then the production technology of the representative firm can be characterized by $y_i = f(K_{Gi}, k_i, l_i)$ where $f$ exhibits constant returns to scale (CRS) to private inputs.\(^7,8\)

The government taxes the firm’s output at rate $\lambda_i \in [0, 1]$. Hence, the firm’s net-of-tax profit function is:

$$\Pi_i = (1 - \lambda_i)f(K_{Gi}, k_i, l_i) - w_i l_i - r_i k_i.$$  \hspace{1cm} (4)

The total tax revenue collect by the government from firms equals $\lambda_i y_i$.

3.3. Government

The government allocates tax revenues, $\lambda_i y_i$, to maintenance of existing public capital, $m_i$.\(^9\) Then the government’s budget constraint is:

$$\lambda_i y_i = m_i.$$ \hspace{1cm} (5)

In addition, international donors give $D_i$ aid to this country every period. This aid is earmarked for the construction of new public projects. This is very common in actuality in developing countries as new airports, highways, water systems, etc., are financed by international aid.\(^10\) Then the public capital evolution equation is:

$$K_{Gi+1} = D_i + (1 - \delta_{Gi}(m_i, k_i))K_{Gi}.$$ \hspace{1cm} (6)

That is, next period’s public capital stock ($K_{Gi+1}$) equals new investment this period, $D_i$, plus the surviving public capital stock this period, $(1 - \delta_{Gi}(m_i, k_i))K_{Gi}$. Note that the depreciation rate of public capital, $\delta_{Gi}(m_i, k_i)$, is endogenous, and it depends on maintenance expenditures and on ‘usage.’ More maintenance reduces

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\(^7\)Given the competitive structure and the CRS assumption, net-of-tax revenues will be exhausted by payments to the private factors of production.

\(^8\)Feehan (1998) states that a consensus in the public inputs literature has emerged to model productive public inputs (e.g., infrastructure) as factor augmenting. The production function here follows this literature.

\(^9\)Hence, $\lambda$ can be interpreted as the share of GDP that is devoted to maintaining and repairing infrastructure. For simplicity, the model abstracts from government consumption.

\(^10\)As documented in Section 2, even when part of the funds to finance new infrastructure are borrowed from industrial countries’ governments or international organizations, much of this debt is effectively later forgiven or never fully repaid. Hence, for simplicity and as a starting point to study the maintenance versus new infrastructure issue, new infrastructure financing is modeled as a gift in this paper.
this depreciation rate \((\partial \delta_{g,t}/\partial m < 0)\) as roads and water systems are kept in better condition. Conversely, higher accumulation of private capital (i.e., a proxy for higher ‘use’ of infrastructure) leads to higher infrastructure depreciation \((\partial \delta_{g,t}/\partial k > 0)\). For example, the larger the number of trucks traveling on the highway, the faster the highway will develop potholes.\(^{11}\)

The government seeks to maximize the well being of its constituents: \(\max \sum_{t=0}^\infty \beta^t u(c_t),\) subject to the above constraints (5) and (6). Hence, the government chooses \(\lambda\) optimally which implicitly determines the public infrastructure stock \(K_{g,t}\) given the level of donor aid as Eq. (6) describes.

### 3.4. Market clearing

The goods market clearing condition is:

\[
y_t = c_t + i_t + m_t. \tag{7}\]

The economy’s output \((y_t)\) must equal expenditures: consumption plus investment plus maintenance.

### 4. Equilibrium in a simplified version

In this section, a simplified version of the above model is solved. Much intuition can be gained from simplifying assumptions which allow for closed-form description of the private decision rules and optimal public policy.

Assume that preferences are characterized by a simple logarithmic function and that the production function has Cobb–Douglas form:

\[
u(c_t) = \log c_t, \quad y_t = K_{g,t}^\theta k_t^{a(1-\theta)} \tag{8}\]

with \(\theta + \alpha \leq 1\). Furthermore, assume that private capital depreciates completely after each period, hence \(\delta = 1\). Finally, assume that the depreciation rate of public capital only depends on maintenance expenditures, i.e., \(\delta_{g,t}(m_t)\).

\(^{11}\)McGrattan and Schmitz (1999) have used a similar specification for an endogenous depreciation rate. One possible alternative specification suggested by a referee would be \(\delta_{g}(m_{-}, c_{-})\), where consumption proxies for higher use of the infrastructure. Consumer driving to stores and shopping malls increase the wear on roads. As consumption will be a function of the state variable, \(k_{-}\), results should be similar. Hence, we use the simpler specification \(\delta_{g,t}(m_t, k_t)\) for tractability in the analysis. In any event, the alternative specification has also been used in the quantitative analysis of Section 5 for completeness.
4.1. The private sector’s problem

The household’s problem is to choose $c_t$ and $k_{t+1}$ so as to maximize utility subject to their budget constraint.

$$
\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \log(c_t)
$$

subject to

$$
c_t + k_{t+1} = w_i l_t + r k_t,
$$
given $k_0$, $w_i$, $r$, and $\lambda_t$, $\forall t$

The firm’s problem is to choose $k_t$ and $l_t$ each period so as to maximize profit according to:

$$
\max_{\{k_t, l_t\}} (1 - \lambda_t)K_G^a k_t^a l_t^{1-\alpha} - w_i l_t - r k_t,
$$
subject to $k_t \geq 0$ and given $\lambda_t$ and $K_G$.

To define a competitive equilibrium for the economy, denote an arbitrary government policy $\pi = \{\lambda_t\}_{t=0}^{\infty}$. Then a $\pi$-competitive equilibrium is a set of allocations $\{c_t, k_t, l_t\}_{t=0}^{\infty}$ and a set of prices $\{w_t, r_t\}_{t=0}^{\infty}$ that solve the households problem; the firm’s problem; and satisfy the market clearing conditions, $c_t + k_{t+1} = (1 - \lambda_t)\gamma_t$ and $l_t = 1$.

Given the public good nature of infrastructure and the distortionary taxation, the second welfare theorem does not hold. However, it is possible to compute a competitive equilibrium by solving an artificial planning problem as in Glomm and Ravikumar (1994). The artificial planning problem is first written recursively using a value function. Then a linear guess for the value function depending on the two state variables $k$ and $K_G$ is hypothesized. This guess is substituted into the problem’s standard Euler equation. Finally, the undetermined coefficients are computed and checked for consistency. Applying this method, the decision rules for private capital and consumption are

$$
k_{t+1} = \alpha \beta (1 - \lambda_t) K_G^a k_t^a
$$

$$
c_t = (1 - \alpha \beta) (1 - \lambda_t) K_G^a k_t^a
$$

It is straightforward to show that these decision rules are a unique solution to the private sector’s problem (see Glomm and Ravikumar (1994), Proposition 2).

4.2. The public sector’s problem

At the same time, the government seeks to determine its optimal policy with respect to maintenance. International donations for new infrastructure are taken as given, but the government must decide on $\lambda_t \in [0, 1]$, the share of GDP devoted to maintaining the existing infrastructure. Assume that the government’s objective
function is to maximize the well-being of the population. Hence the government’s optimal policy will be a sequence \( \{ \lambda_t \}_{t=0}^{\infty} \) that solves

\[
\max_{\{\lambda_t\}} \sum_{t=0}^{\infty} \beta^t \log(c_t(\pi))
\]

subject to

\[
K_{t+1} = D_t + (1 - \delta_{t+1}(m_t))K_t,
\]

\[
m_t = \lambda_t K_t^{\theta} k_t^\alpha (\pi)^{1-\alpha}(\pi)
\]

given \( k_0 \) and \( K_{G0} \).

The private decision rules computed in Section 4.1 above can be used to solve the government’s optimal policy problem, which can be re-written as

\[
\max_{\{\lambda_t\}} \sum_{t=0}^{\infty} \beta^t \log((1 - \alpha \beta)(1 - \lambda_t)K_t^{\theta} k_t^\alpha)
\]

st.

\[
K_{t+1} = D_t + (1 - \delta_{t+1}(m_t))K_t,
\]

\[
k_{t+1} = \alpha \beta (1 - \lambda_t)K_t^{\theta} k_t^\alpha
\]

\[
m_t = \lambda_t K_t^{\theta} k_t^\alpha
\]

\[
0 \leq \lambda_t \leq 1,
\]

given \( k_0 \) and \( K_{G0} \). Furthermore, assume a simple functional form for the depreciation rate of public capital, \( \delta_{t+1} = 1 - \psi m_t \), with \( \psi \geq 0 \). This function is convenient since if nothing is spent on maintenance \( (m_t = 0) \), public infrastructure depreciates completely \( (\delta_{t+1} = 1) \). The solution to the government’s problem is:

\[
\lambda_t^* = \frac{\beta \theta}{1 + \beta(\theta + \alpha)} - \frac{(1 + \alpha \beta)(D_t)}{(1 + \beta(\theta + \alpha))\psi K_t^{\theta} k_t^\alpha (K_t)}
\]

(9)

Hence the optimal share of GDP devoted to maintenance, \( \lambda_t^* \), depends on parameters, the ratio of donations to existing infrastructure \( (D_t/K_{Gt}) \), and other state variables.\(^3\) Given \( \alpha > 0 \), \( \alpha + \theta \leq 1 \), and that the sequence \( \{K_{Gt}\}_{t=0}^{\infty} \) is bounded above, the above solution will be the unique solution to the public policy problem. The first condition, \( \alpha > 0 \), implies that in equilibrium the marginal product of private capital is positive. The second condition, \( \alpha + \theta \leq 1 \), implies that there are non-increasing returns to scale to augmentable factors. Finally, the third condition ensures that \( \Sigma_{t=0}^{\infty} \beta^t \log(c_t) \) is bounded above. Intuition for this

\(^3\)Further assume that the sequence \( \{m_t\}_{t=0}^{\infty} \) is bounded above so that \( \delta_{t+1} \geq 0 \) for all \( t \).

\(^4\)Devarajan et al. (1998) and Glomm and Ravikumar (1994) compute the optimal share of GDP that should be devoted to overall infrastructure investment. Their measure is not directly comparable to \( \lambda_t^* \), since \( \lambda_t^* \) is the optimal share of GDP that should be devoted to maintenance of infrastructure.
solution is described below, while the specifics of the derivation are left for Appendix A.

4.3. The determinants of optimal maintenance

The determinants of the optimal share of GDP devoted to maintenance, $\lambda^*_t$, have some intuitive appeal and can be analyzed by studying Eq. (9). First, consider how the ratio of international donations to existing public capital stock $(D_t/K_{Gt})$ affects optimal maintenance $\lambda^*_t$. This relationship is clearly negative as can be easily determined by looking at Eq. (9):

$$\frac{\partial \lambda^*_t}{\partial (D_t/K_{Gt})} = -\frac{(1 + \alpha \beta)}{(1 + \beta(\theta + \alpha))\psi K_{Gt}^\theta K_t^\alpha} < 0$$

Intuitively, if new infrastructure investment as a share of existing infrastructure stock rises (let’s say because donors raise aid), then the optimal maintenance expenditure should decrease. That is, if the country is to receive increased funding to build a few more bridges, it is optimal for the country’s policymakers to cut maintenance expenditures. Policymakers in this model care about how large the overall public capital stock is (which affects output and consumption and hence their objective function). As donors fund more new (and often highly visible) projects adding to the infrastructure stock, it is optimal to decrease maintenance, which has to be financed by taxes. This is supported by the Brazilian experience between 1979 and 1984 described in Section 2. During this period, according to Harral (1988), 6000 km of new paved roads were built, while 2000 km went from ‘good’ to ‘poor’ condition and 6000 km went from ‘fair’ to ‘poor’ due to maintenance neglect.

It would be erroneous, however, to imply that infrastructure aid causes a disservice to developing economies. Perhaps the correct policy implication would be to re-allocate portions of this aid to maintenance. More on this in the quantitative evaluation section below.

In order to gain yet more intuition to this solution, assume that $D_t = 0$. That is, assume foreign donors stop financing new infrastructure entirely. The optimal maintenance share of Eq. (9) simplifies to:

$$\lambda^*_t = \frac{\beta \theta}{1 + \beta(\theta + \alpha)}.$$  \hspace{1cm} (10)

Recall that the parameter $\theta$, infrastructure’s coefficient in the production function, describes how productive infrastructure is. Consider a country where infrastructure is very productive (i.e., $\theta$ is high). Should policymakers choose to spend more or less on maintenance of their infrastructure network in this case? The answer is obtained by computing
This simply says that the more productive the infrastructure, the more it should be maintained, which is intuitively appealing.

5. Quantitative evaluation

The simplified version of the model yields a closed form solution for the optimal maintenance expenditure policy which provides valuable intuition. This section attempts to extend that analysis by quantifying the macroeconomic effects of changes in public expenditures. The starting point of the quantitative evaluation, however, is the actual average allocation (called benchmark allocation) between maintenance and new investments according to the data of seven Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela). Hence, in this section we do not assume the government is choosing maintenance spending optimally; it simply balances its budget with the allocation given by the historical data. Rather, households and firms optimize taking government expenditure policy as given by the data.

The questions addressed in this section are: starting from the benchmark allocation,

- How would the economy be affected if a fraction of donor aid (earmarked to build new roads and bridges) was redistributed towards maintenance of existing infrastructure?
- Conversely, how would the economy be affected if maintenance was reduced devoting those resources to new infrastructure construction instead?

These are questions that policymakers in developing countries often face. The direction and magnitude of the effects of such redistributions are not readily apparent. This section shows that these effects depend on the initial relative shares of GDP devoted to each type of expenditure and on parameters.

Functional forms and parameter values are needed to numerically solve the complete version of the model. The functional forms for utility and production functions are given as in Eq. (8). The function used for the depreciation rate of public capital is \( \delta_{G} = 1 - (\psi m_{t}/k_{t})^{\mu} \), where \( \psi, \mu > 0 \). This function is decreasing in maintenance and increasing in private capital (i.e., the more private economic activity, the higher the wear on infrastructure).

The numerical solution proceeds as follows. First, the first-order conditions
(FOCs) for the household and firms are derived. Second, the model is parameterized using estimates from the literature for Latin American countries. Third, the system of equations composed of the FOCs, the government budget constraint (Eq. (5)), the infrastructure accumulation equation (6), and the goods market clearing condition (7) are solved numerically. Fourth, the effects of policy experiments varying the mix of public expenditures are computed.

5.1. Parameterization

An average of parameter estimates of seven Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela) are used for the quantitative evaluation. When parameters specific to these Latin American countries are not available, estimates from averages of developing countries are used. Some key parameters come from Victor Elias’s (1992) detailed growth accounting study of these seven countries. As such detailed growth accounting study of African countries was not available, we do not attempt a quantitative evaluation for that continent. Table 3 lists the parameters used, and the parameter choices are explained in the text below.

First, technology parameters are determined as follows. Capital’s share of income is on average 54% in these seven countries according to Elias (1992); hence, $\alpha$ is set to 0.54. Next, the coefficient on public infrastructure, $\theta$, has been estimated by Hulten (1996), Canning and Fay (1993), and Easterly and Rebelo (1993) with an average value of 0.10. The estimate is for an average of developing countries including the seven countries being studied. Unfortunately, there are no available estimates of this parameter specifically for these Latin American countries.

A standard depreciation rate for physical capital of about 10% per year is used in the benchmark (hence $\delta = 0.10$). This figure falls in Elias’s (1992) estimated

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark parameters</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$\psi$</td>
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<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
</tbody>
</table>

$^{14}$As this is standard, the FOCs are reported in part B of Appendix A.
range for depreciation in Latin America. Note, however, that this figure is an average for structures and equipment across the economy. Public infrastructure capital, conversely, is mostly composed of structures. According to Greenwood et al. (2000), the depreciation rate of structures is about 6% per year in the US. In developing countries, however, the depreciation rate of infrastructure is on average twice as much according to the *World Development Report 1994*; hence $\delta_c = 0.12$ in the benchmark. To be consistent with this, the parameters $\mu = 0.0112$ and $\psi = 100$ are calibrated so that $\delta_c$ is 0.12 in the benchmark. Of course, this public capital depreciation rate will vary in various policy experiments (e.g., larger maintenance expenditures will reduce $\delta_c$).

Easterly and Rebelo (1993) estimate that overall public investment is about 6% of GDP in these seven Latin American countries. According to Gyamfi et al. (1992) and Fuy (2001), only about 1% of GDP is spent on maintenance of existing infrastructure (hence $\lambda = m/y = 0.01$). The remainder 5% of GDP is spent on new infrastructure construction (hence, $d = D/y = 0.05$). Note that as described in Section 4, maintenance is financed by taxes, while new investments are financed by international donors. This allocation is used as benchmark in the solution. Finally, a standard discount rate of $E_T = 0.99$ is used as in, for example, Rebelo and Végh (1995).

### 5.2. Quantitative results of public expenditure reallocation

This section describes the quantitative consequences of reallocating resources between maintenance and new public investment. Table 4 presents the long-run (steady state) effects of various reallocations. The starting point is denoted Benchmark 1: where maintenance ($my$) is 1% of GDP and new investment ($dy$) is

<table>
<thead>
<tr>
<th>$my$</th>
<th>$dy$</th>
<th>$\delta_c$</th>
<th>$\Delta K_c$</th>
<th>$\Delta y$</th>
<th>$\Delta c$</th>
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<td>-3.48</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>1.50</td>
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<td>1.87</td>
<td>1.87</td>
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<td>-10.2</td>
<td>-2.30</td>
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<tr>
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<td>-5.73</td>
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<td>-10.7</td>
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<td>-18.2</td>
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<td>0.04</td>
<td>-81.7</td>
<td>-30.8</td>
<td>-30.8</td>
</tr>
</tbody>
</table>

*Denotes benchmark. $\Delta$ in percent change.
5% of GDP (these appear starred in Table 4). Note also that in this benchmark the depreciation rate of public capital, \( \delta_{CAP} \), is 12% for reasons discussed above.

5.2.1. Reducing maintenance to build new infrastructure

What would be the effects of policymakers reducing maintenance expenditures, for instance by 0.5% of GDP, and allocating those resources to build new infrastructure? The first row in Table 4 (i.e., \( my = 0.5; dy = 5.5 \)) describes the effects. First, as less is spent on maintenance, the depreciation rate of public infrastructure would rise to 0.15 per year. This would in turn reduce the stock of public infrastructure (\( \Delta K_{CAP} \)) by 15% in the long-run from its benchmark level. As the surviving stock of public capital (an essential input in production) has diminished, output (\( \Delta y \)) and consumption (\( \Delta c \)) are adversely affected falling by \(-3.48\%\). Hence, given the present levels of expenditure in Latin American countries, reallocating resources away from, for example, filling potholes in favor of building new bridges would affect these economies adversely. This result agrees with Devarajan et al. (1996), who find empirically that raising capital expenditures may have adverse effects on output as described in Section 1.

5.2.2. Reducing construction of new infrastructure to raise maintenance

What would be the effects of policymakers instead raising maintenance by 0.5% of GDP by reducing new investment expenditures? The third row in Table 4 (i.e., \( my = 1.5; dy = 4.5 \)) describes the effects. As more is spent on maintenance, this reduces the depreciation rate of public capital \( \delta_{CAP} \) from 0.12 to 0.10. With a lower \( \delta_{CAP} \), more public capital survives in the long run; the public infrastructure stock is 7.09% larger. Under this policy, even though less is invested in new infrastructure, 7.09% more of the public capital stock survives in the long run thanks to more maintenance. This affects output and consumption positively (\( \Delta y = \Delta c = 1.50\% \)) as public capital is an essential input in production. Once again, this result agrees with empirical findings of Devarajan et al. (1996) that higher ‘current expenditures’ (like maintenance) have positive effects on GDP in developing countries.

5.2.3. How much is too much and what is the optimal mix?

Higher maintenance expenditures appear beneficial to the economy, but how much is too much? Consider, for example, raising \( my \) from 1.0 to 4.0 (by redistributing donor aid worth about 3% of output). The effects of such redistribution are presented in the column \( my = 4.0; dy = 2.0 \) on Table 4. The depreciation rate of public capital is significantly reduced to 0.06. Nevertheless,

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\(^{15}\)This notation is adopted to make public expenditure reallocation clear. Suppose \( z \) is the percentage of GDP to be redistributed from one use to the other. Then, the share of GDP devoted to maintenance is denoted \( my = \lambda + z \), while the share of GDP devoted to new public investment is denoted \( dy = d - z \). Naturally, \( z = 0 \) in the benchmark as there is no redistribution.
the public infrastructure stock would be 23.8% smaller in the long run. The reason is that so much has been taken away from new construction that less of the infrastructure will survive in the long-run even though more is spent on maintaining it. This reallocation would have adverse effects on output and consumption of $-5.73\%$. Therefore, too much maintenance can be deleterious.

What is the optimal mix under the Benchmark 1 specification? Results on Table 4 show that the highest gains in output are obtained when maintenance receives about 2% of GDP and new public investment about 4.0% of GDP (i.e., $m_y = 2.0$; $d_y = 4.0$). Given that these Latin American governments presently spend about 1% of GDP on maintenance, doubling these expenditures by redistributing donor aid of about 1% of GDP away from new projects yields the highest long-run gains in output. Fig. 1 graphically illustrates the effects on output described on Table 4. The horizontal axis measures maintenance (as a percent of GDP), while the vertical axis measures the long run effects on GDP of various reallocations. The vertical line at $m_y = 1.0$ simply marks the starting point for all the Benchmark 1 experiments. As described above, lowering $m_y$ below its benchmark reduces GDP,

![Graph showing output effects of redistributions: Benchmark 1.](image-url)
while raising $my$ has positive effects over some range. The curve peaks around $my = 2.0\%$ of GDP where the positive effect on long-run GDP are largest.\footnote{The quantitative evaluation was also performed using the specification where depreciation of public capital depends on consumption rather than private capital: $\delta_t(m, c)$. The results obtained are virtually identical and are available from the author on request.}

### 5.2.4. Effects of parameter changes

This subsection studies how the results are affected by changing some of the benchmark parameters. First, the initial expenditure position is changed from $my = 1.0$, $dy = 5.0$ to $my = 0.5$, $dy = 5.5$. This case is labeled Benchmark 2 where the government initially only spends 0.5\% of GDP on infrastructure maintenance. In fact, this allocation is not far from reality for some of the poorer Latin American countries, which spend very little on maintenance. Table 5 reports results of shifting expenditures for this Benchmark 2 (Fig. 2 describes these effects graphically). As in Benchmark 1, there are gains to re-allocating towards maintenance. Raising $my$ from 0.5 to 2.0 helps reduce the public capital depreciation rate from 0.12 to 0.07 per year. Hence, the infrastructure stock will be 28.3\% larger in the long run. This results in an output and consumption gain of 5.56\%. The potential gains in Benchmark 2 are larger than in Benchmark 1. This can be explained since in Benchmark 2 countries initially spend only 0.5\% of GDP on maintenance versus 1\% of GDP in Benchmark 1. Because so little is spent on maintenance under Benchmark 2, there are larger potential gains at the margin.

The second change involves raising the coefficient of public infrastructure, $\theta$, from 0.10 to 0.15 keeping everything else as in the original benchmark. This higher $\theta$ is reasonable as it is about what Easterly and Rebelo (1993) estimate for developing countries. This case is referred to as Benchmark 3. As Table 5 shows, a 1\% raise in maintenance has a larger positive effect on output than in the original Benchmark 1 ($\Delta y$ is 3.28\% here versus 1.87\% in the original). This result is expected since, in countries where infrastructure is more productive, a larger surviving stock of infrastructure (due to more maintenance spending) would raise output by more. For visual comparison with the original, Fig. 3 displays the two cases.

Finally, the third change involves calibrating the model more specifically to Brazil as it is often useful to look at one country in detail rather than at a composite of countries. Key Brazilian parameter estimates available are used to calibrate the model: $\alpha = 0.45$ (Elias, 1992); $\lambda = 1.22$; $d = 6.08$ (Easterly and Rebelo, 1993). This means that Brazilian expenditure on maintenance and on new infrastructure has been somewhat higher than the Latin American average. Results of reallocations for this Benchmark 4 are presented on Table 5. A 1\% raise in maintenance from 1.22 to 2.22 would raise output by 2.52\%. Compare this to the average Latin American country (Benchmark 1) where an equal-size raise in
Table 5
Long run effects of public expenditures reallocation

<table>
<thead>
<tr>
<th>my</th>
<th>dy</th>
<th>δ_ε</th>
<th>ΔK_ε</th>
<th>Δy</th>
<th>Δε</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark 2</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>0.025</td>
<td>5.75</td>
<td>0.14</td>
<td>−16.0</td>
<td>−3.73</td>
<td>−3.73</td>
</tr>
<tr>
<td>0.5*</td>
<td>5.5*</td>
<td>0.12*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
<td>5.0</td>
<td>0.10</td>
<td>17.8</td>
<td>3.62</td>
<td>3.62</td>
</tr>
<tr>
<td>1.5</td>
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<td>0.08</td>
<td>26.1</td>
<td>5.18</td>
<td>5.18</td>
</tr>
<tr>
<td>2.0</td>
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<td>0.07</td>
<td>28.3</td>
<td>5.56</td>
<td>5.56</td>
</tr>
<tr>
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<td>3.5</td>
<td>0.06</td>
<td>25.3</td>
<td>5.02</td>
<td>5.02</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
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<td>17.7</td>
<td>3.60</td>
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</tr>
<tr>
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<td>0.05</td>
<td>5.76</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>4.0</td>
<td>2.0</td>
<td>0.05</td>
<td>−10.3</td>
<td>−2.33</td>
<td>−2.33</td>
</tr>
<tr>
<td>4.5</td>
<td>1.5</td>
<td>0.04</td>
<td>−30.2</td>
<td>−7.52</td>
<td>−7.52</td>
</tr>
<tr>
<td>5.0</td>
<td>1.0</td>
<td>0.04</td>
<td>−53.4</td>
<td>−15.3</td>
<td>−15.3</td>
</tr>
<tr>
<td>5.5</td>
<td>0.5</td>
<td>0.04</td>
<td>−78.5</td>
<td>−28.4</td>
<td>−28.4</td>
</tr>
</tbody>
</table>

| Benchmark 3 (u = 0.15) |     |     |      |     |     |
| 0.5 | 5.5 | 0.15 | −17.2 | −5.99 | −5.99 |
| 1.0* | 5.0* | 0.12* | 0 | 0 | 0 |
| 1.5 | 4.5 | 0.10 | 8.28 | 2.63 | 2.63 |
| 2.0 | 4.0 | 0.09 | 10.4 | 3.28 | 3.28 |
| 2.5 | 3.5 | 0.08 | 7.44 | 2.37 | 2.37 |
| 3.0 | 3.0 | 0.07 | −0.06 | −0.02 | −0.02 |
| 3.5 | 2.5 | 0.06 | −11.7 | −3.98 | −3.98 |
| 4.0 | 2.0 | 0.06 | −27.0 | −9.78 | −9.78 |
| 4.5 | 1.5 | 0.05 | −45.5 | −17.9 | −17.9 |
| 5.0 | 1.0 | 0.05 | −66.2 | −32.0 | −32.0 |
| 5.5 | 0.5 | 0.04 | −86.1 | −47.4 | −47.4 |

| Benchmark 4 (Brazil) |     |     |      |     |     |
| 0.72 | 6.58 | 0.15 | −13.7 | −2.65 | −2.65 |
| 1.22* | 6.08* | 0.12* | 0 | 0 | 0 |
| 1.72 | 5.58 | 0.10 | 9.03 | 1.58 | 1.58 |
| 2.22 | 5.08 | 0.09 | 14.7 | 2.52 | 2.52 |
| 2.72 | 4.58 | 0.08 | 17.4 | 2.95 | 2.95 |
| 3.22 | 4.08 | 0.07 | 17.3 | 2.94 | 2.94 |
| 3.72 | 3.58 | 0.06 | 14.4 | 2.48 | 2.48 |
| 4.22 | 3.08 | 0.06 | 8.62 | 1.51 | 1.51 |
| 4.72 | 2.58 | 0.05 | −0.33 | −0.06 | −0.06 |

Δ in percent change. *Denotes benchmark.

Maintenance would raise output by a lower 1.87%. This result can be explained because Brazil has spent more (as percent of GDP) that the average Latin American country on infrastructure. Consequently, the infrastructure network in Brazil is larger than in the average Latin American country. Hence, the same raise in maintenance (as percent of GDP) would raise Brazilian GDP by more than in the average country case. Overall the Brazilian calibration is qualitatively similar.
to the one for the average of seven Latin American countries as there are gains to increasing maintenance in both cases.

6. Conclusions

The construction of new, highly visible public infrastructure projects has received great attention in many developing countries. Repairing and maintaining existing infrastructure has generally been neglected. However, the empirical evidence from developing countries (i.e., Devarajan et al., 1996), shows that current public expenditures like maintenance have a positive effect on output, while public capital expenditures have a negative effect on output. More new infrastructure at maintenance’s expense may not be beneficial to these countries. Hence, policymakers may have overemphasized building new infrastructure as a recipe for development.

This paper contributes to the literature by developing a dynamic general equilibrium model from which the optimal Ramsey expenditure on maintenance is
derived. Its key determinant is the size of donor-financed new investments as a share of existing infrastructure, which is negatively related to the optimal maintenance expenditure. That is, policymakers may find it optimal to cut maintenance if donors raise funding for new construction. This can partially explain the overemphasis on new infrastructure. In addition, donors often tie their aid to highly visible projects. According to the World Development Report (1994), “like ministries of public works, donor agencies find it easier to measure their achievements in new project approvals” (p. 91). A policy recommendation for industrial countries is to not earmark all infrastructure aid for new construction. Rather, part of this aid could go to an escrow account committed to maintaining the infrastructure for a number of years.

Another contribution of the paper is to quantitatively evaluate the tradeoffs of various redistributions of funds between the two expenditures using parameters from seven Latin American countries. Reallocations, without additional donor aid or taxation, can have positive effects on a country’s GDP. Results show that an optimal allocation in these countries implies maintenance should receive about 2%
of GDP. Presently, maintenance expenditures in Latin America appear below this optimal allocation.

Potential extensions of the model include allowing the government a choice not only on maintenance, but also on the amount of infrastructure investment and on government consumption. This could be done in a Ramsey framework in which the government’s problem would now have two or three decision variables. Alternatively, this could be done in a political economy model where, for instance, the median voter was allowed to choose the levels of expenditures. In any event, filling potholes appears to deserve much more attention than it has received.

Acknowledgements

I am grateful to Jim Alm, Shanta Devarajan, Ellen McGrattan, Shannon Mudd, Gyan Pradhan, and seminar participants at the 1999 Econometric Society Latin American meetings and the Federal Reserve Banks of Atlanta and Kansas City for useful comments.

Appendix A

A.1. Derivation of optimal maintenance expenditure

After substituting the private decision rules in, the public policy problem can be written as a dynamic program:

\[ V(k, K_G) = \ln((1 - \alpha \beta)(1 - \lambda)K_G^\theta k^\alpha) + \beta V(k', K_G') \]

\[ \text{st.} \]

\[ K_G' = D + (1 - \delta_G(m))K_G \]
\[ k' = \alpha \beta (1 - \lambda)K_G^\theta k^\alpha \]
\[ m = \lambda K_G^\theta k^\alpha \]

Notice that following Bellman’s notation, time subscripts are removed from the variables (e.g., \( k_t \) is simply \( k \)). According to this notation, all prime variables denote next period’s value (e.g., \( k_{t+1} \) is \( k' \)).

The first-order condition (FOC) for \( \lambda \) is given by (after the envelope conditions are substituted),

\[ -\left( \frac{1}{(1 - \lambda)} \right) - \frac{\beta \theta}{K_G'} \left( \frac{\partial \delta_G}{\partial m} \cdot \frac{\partial m}{\partial \lambda} \cdot K_G \right) - \frac{\alpha \beta}{k'}(\alpha \beta K_G^\theta k^\alpha) = 0 \]
Substituting the private decision rules for \(k'\) and \(K'\), then rearranging the equation,

\[
\frac{1}{1-\lambda} = -\left( \hat{\beta} \theta \cdot \frac{\partial \delta_g}{\partial m} \cdot \frac{\delta m}{\partial \lambda} \cdot \frac{K_G}{D + (1 - \delta_g(m))K_G} \right) - \left( \frac{\alpha \beta}{1-\lambda} \right)
\]

or by combining terms,

\[
\frac{(1 + \alpha \beta)}{(1 - \lambda)} = -\hat{\beta} \theta \cdot \frac{\partial \delta_g}{\partial m} \cdot \frac{\delta m}{\partial \lambda} \cdot \frac{K_G}{D + (1 - \delta_g(m))K_G}.
\]

Given:

\[
\delta_{Gt} = 1 - \psi m_t,
\]

\[
m_t = \lambda K^a_{Gt} k^a_t,
\]

then \(\partial \delta_{Gt}/\partial m = -\psi\) and \(\partial m/\partial \lambda = K^a_{Gt} k^a_t\). Substituting these two derivatives into the equation,

\[
\frac{(1 + \alpha \beta)}{(1 - \lambda)} = \hat{\beta} \theta \psi K^a_{Gt} k^a_t \cdot \frac{K_G}{D + (1 - \delta_g(m))K_G}.
\]

Inverting and rearranging this equation,

\[
(1 - \lambda) = \frac{(1 + \alpha \beta)}{(\hat{\beta} \theta \psi)} \cdot \frac{D + (1 - \delta_g(m))K_G}{K^a_{Gt} k^a_t}.
\]

Substituting \(\delta_g(m)\)'s functional form,

\[
(1 - \lambda) = \frac{(1 + \alpha \beta)}{(\hat{\beta} \theta \psi)} \cdot \frac{D + [1 - \psi \lambda K^a_{Gt} k^a_t]}{K^a_{Gt} k^a_t}.
\]

Finally solving for \(\lambda\) and re-inserting time subscripts so that all variables are clearly distinguished from parameters,

\[
\lambda_t = \frac{\beta \theta}{1 + \beta (\theta + \alpha)} - \frac{(1 + \alpha \beta)}{(1 + \beta (\theta + \alpha)) \psi K^a_{Gt} k^a_t} \left( \frac{D_t}{K_t} \right).
\]

**B.1. First-order conditions for the complete version of the model**

The first-order conditions (FOCs) for the complete version of the model are standard. That is, they are obtained by solving the household’s problem (i.e., maximizing utility subject to their budget constraint); and the firm’s problem (i.e., maximizing profit). They are described by:

\[
u_t(c) = \beta (r' + (1 - \delta)) u_t(c')
\]

\[r = (1 - \lambda) f_k(k, n)\]
These equations plus the government budget constraint (Eq. (5)), the infrastructure accumulation equation (6), and the goods market clearing condition (7) determine the equilibrium in the model.

References

Hulten, C., 1996. Infrastructure capital and economic growth: how well you use it may be more important than how much you have. NBER Working Paper No. 5847.