A Disinflation Tradeoff: Speed versus Final Destination

by John A. Carlson and Neven T. Valev*

Purdue University and Georgia State University

Abstract

Does a central bank, when introducing a new monetary regime designed to reduce inflation, prefer more or fewer economic agents who form informed forecasts of inflation? The relevance of the question arises because the central bank can make a decision about how much information to disseminate about the nature of the new regime. We find that the central bank will prefer a higher proportion of agents who form rational expectations if it disinflates from a high level of inflation but not so if it disinflates from a moderate or low inflation level.

JEL Classification: E58 - Central Banks and Their Policies

Carlson
    phone: 765-494-4450
    Fax:    765-494-9658
    Email: carlson@mgmt.purdue.edu

Valev
    Phone: 404-651-0418
    Fax:  404-651-4985
    Email: nvalev@gsu.edu

* The authors would like to thank John Barron, Kenneth Matheny, and Cabrielle Camera for their suggestions and comments. We would also like to thank the Center for International Business Education and Research at Purdue University for their financial support.
A Disinflation Tradeoff: Speed versus Final Destination

1. Introduction.

Whether or not expectations of inflation are rational is an open question. Rational forecasts require knowledge and information that some agents may not find worthwhile acquiring. Instead, since past inflation is a cheap and potentially informative signal about the policies of the central bank, those agents with less information may resort to extrapolation from past inflation to a greater extent than those with more information. In other words, for all agents, expectations have a rational (forward-looking) and an adaptive (backward-looking) component but differences across agents in terms of information will lead to a separation between those who form more rational and those who form more adaptive expectations.¹

A simpler heterogeneity -- agents with purely rational or with purely adaptive expectations -- has been adopted in some models.² In that context, when a central bank is the major source of information about monetary policy, it could potentially influence the proportion of agents in each group. Therefore, a natural question is what are the central bank’s preferences regarding the distribution. More specifically, does the central bank prefer many or few agents with rational expectations when introducing a monetary regime designed to reduce inflation?

We will analyze the properties of a Barro-Gordon (1983a) model of monetary policy where some agents form rational and some adaptive expectations. In that setting, a higher

---

¹ The notion of “economically rational” expectations introduced by Feige and Pearce (1976) was further examined theoretically by, for example, Sethi and Franke (1995), Crettez and Michel (1992), and empirically by Baghestani (1992).
² See for example Haltiwanger and Waldman (1989).
proportion of agents with adaptive expectations generally slows down the disinflation process but it also allows for a lower long-run inflation rate. An implication of this is that the central bank will prefer a higher proportion of agents who form rational expectations if it disinflates from a high level of inflation but not so if it disinflates from a moderate or low inflation level.\(^3\)

The information issues that arise with a change in a monetary regime designed for disinflation have received much attention in the economic literature. How fast agents, including the central bank, learn to navigate in the new conditions is an important problem. Since the parameters of the new monetary environment have uncertain values, economic agents, including the central bank, use observations on the unfolding macro developments to form (least squares) estimates of the parameters (Lewis [1989], Cosimano and Van Huyck [1993], Wieland [2000], Sargent [???], Mankew [???]). A number of interesting questions are addressed in that set-up. For example, is equilibrium possible where the learning process stops short? Can the central bank exploit its control over monetary aggregates to generate observations useful in the learning process, i.e. can learning be an active process?

The structure of our model is simpler in the sense that, aside from a random shock to output, the central bank has direct control over inflation and the output response of changes in prices is clear. In addition, a segment of the population shares the perfect information of the central bank while the rest are assumed to form adaptive expectations. The gradual convergence to steady state produced by the presence of naïve agents is a feature shared with

\(^3\) A central bank’s preferences for ambiguity are studied by Cukierman and Meltzer (1996). The policymaker prefers some ambiguity because it allows monetary flexibility even if it comes at the cost of lower credibility.
models where learning about a new regime is also a gradual process. The simpler set up of the model make it analytically tractable and allow us to isolate some clearer insights compared to more complicated models.

We also ask whether the central bank would prefer a certain proportion of agents with rational expectations and find that a large portion of rational agents is preferred when inflation is high. That result is similar to Balvers and Cosimano [1994] although in their framework, a rapid reduction is money growth is warranted in order to facilitate learning. In our framework, rational agents are preferred since the cost of disinflation is lower. If disinflation from lower levels of inflation, however, the central bank prefers some naïve agents as the presence of those agents contribute to lower steady state inflation.

Unlike other papers, where learning is modeled as least squares estimation and Bayesian updates, the process of learning is not explicitly modeled here. While that feature contributes substantially to the analytical tractability of the paper, it may leave our assumption of the ability of the central bank to influence the formation of expectations somewhat unclear. A brief discussion of the motivation for the paper may clarify our interpretation.

The motivation for the paper comes in part from observations of the behavior of the central bank in Bulgaria, where a currency board was introduced on July 1st 1997.4 Orthodox currency boards are fixed exchange rate regimes that operate like a gold standard except that central bank reserves are kept in foreign currency rather than gold. Domestic money is fully convertible and has complete coverage in foreign exchange reserves. The central bank

---

4 In this paper, less information may be preferred because naïve forecasts partially resolve the time consistency problem and allow for lower long-term inflation.
has no discretionary authority regarding money supply. The features make a currency board a powerful disinflation device since the rules credibly bind the authorities to a low inflation policy. At the time it is introduced, the central bank with therefore have the advantage of advertising these features.

For a number of reasons, mostly to provide temporary relief in extreme circumstances, most currency boards are not orthodox and have lender of last resort or other features that allow some monetary discretion. It may not be to the advantage of the central bank to advertise these features very aggressively leaving the possibility for monetary discretion not immediately apparent to most people. Such policy may contribute to less concerns on the part of the public and to lower expected inflation.

While a lot of news was disseminated in Bulgaria about the currency board at the time of its introduction, there has not been as much public information since then about how the currency board operates. The policymaker maintained instead that people should pay attention to the bank’s track record with low inflation.\(^5\) In other words, similar to models of central bank secrecy such as Cosimano and Van Huyck (1993) and Cukierman and Meltzer (1996) there is asymmetry between the knowledge of the central bank and other agents and the central bank could influence the amount and type of information available to the public.

The paper is structured as follows. In section 2, we develop the model with two types of agents – those with adaptive expectations and those with rational expectations. Section 3 discusses influences on the dynamics of the inflation process, with emphasis on the effect of

\(^4\) For a discussion of the history and operation of currency boards, see Williamson (???) and Schwartz (???).
having more or fewer naïve agents. Section 4 addresses the issue of what proportion of naïve agents the central bank will choose, if its actions can influence that proportion. Section 5 concludes.

2. **A model of monetary policy with heterogeneous agents.**

Output $y_t$ (all variables in logarithms) differs from its natural level by an amount determined by the real wage $(w_t - p_t)$:

$$y_t = \bar{y}_t - (w_t - p_t) - u_t,$$

where $u_t$ is an i.i.d. supply shock with mean zero and variance $\sigma^2$. A positive value of $u_t$ represents a negative output shock.

Inflation $\pi_t$ is defined by:

$$\pi_t = p_t - p_{t-1}$$

Agents are heterogeneous in the way they form expectations of inflation. A proportion $\Theta$ of all agents form adaptive expectations:

$$E_{t-1}^{A} \pi_t = \pi_{t-1}$$

which yields:

$$E_{t-1}^{A} p_t = p_{t-1} + \pi_{t-1}$$

The remaining $(1-\Theta)$ agents form rational expectations. Denote their expectations of the price level by $E_{t-1}^{R} p_t$.

Before the shock $u_t$ has been observed, the nominal wage is set at the average expected price:

$$w_t = \Theta(p_{t-1} + \pi_{t-1}) + (1-\Theta)E_{t-1}^{R} p_t$$
Define the monetary authorities’ loss function \( L_t \) as:

\[
      L_t = \left( y_t - \bar{y}_t \right)^2 + \alpha \pi_t^2
\]

where \( \bar{y}_t \) is the level of output targeted by the policymaker. By substituting (5) into (1) and (1) into (6), we write the loss function as:

\[
    L_t = \left[ \pi_t - (1 - \Theta)E_{t+i} \pi_t - \Theta \pi_{t-i} - k_i - u_i \right]^2 + \alpha \pi_t^2
\]

where \( E_{t+i} \pi_t = E_{t+i} \pi_t - p_{t+i} \) is expected inflation by agents with rational expectations and \( k_t = \bar{y}_t - \bar{y}_t > 0 \) is the difference between the target and the natural level of output.

After observing the nominal wage and the shock \( u_i \), the government chooses inflation \( \pi_t \) to minimize:

\[
    V_t = \sum_{i=0}^{\infty} \beta^i E_t L_{t+i}
\]

where \( \beta \in [0, 1] \). Substitute from (7) into (8) and assume \( k_{t+i} = k_i \) all \( i \). The objective can then be written:

\[
    \min_{\pi_t} V_t = \min_{\pi_t} \left[ \left( \pi_t - (1 - \Theta)E_{t+1} \pi_t - \Theta \pi_{t-1} - k - u_t \right)^2 + \alpha \pi_t^2 + 
    \sum_{i=1}^{\infty} \beta^i E_t \left[ \pi_{t+i} - (1 - \Theta)E_{t+i+1} \pi_{t+i} - \Theta \pi_{t+i-1} - k - u_{t+i} \right]^2 + \alpha \sum_{i=1}^{\infty} \beta^i E_t \pi_{t+i}^2 \right]
\]

Provided that some agents form adaptive expectations \( (\Theta > 0) \) inflation in period \( t \) is built into expectations of inflation for period \( t+1 \) and beyond. Monetary authorities choose inflation to balance their current and future inflation and output objectives.

The first-order condition with respect to \( \pi_t \), using certainty equivalence, yields:

\[
    \beta \Theta^2 E_t \pi_{t+1} - (1 + \alpha + \beta \Theta^2) \pi_t + (1 - \Theta)E_{t-1} \pi_t + \Theta \pi_{t-1} = -(1 - \beta \Theta)k - u_t
\]
Then taking expectations of both sides of (9) as of time \( t-1 \) and collecting terms yields the following difference equation:

\[
E_{t-1}p_{t+1} - \left( \frac{\alpha + \theta + \beta \theta^2}{\beta \theta^2} \right) E_{t-1} \pi_t + \frac{1}{\beta \theta} \pi_{t-1} = -(1-\beta \theta) \frac{k}{\beta \theta^2}
\]

As shown in the Appendix, the solution for (10) can be written:

\[
E_{t-1} \pi_t = \lambda \pi_{t-1} + (1- \lambda) \bar{\pi}
\]

where \( \lambda (0 < \lambda < 1) \) is the smaller root of the characteristic equation and

\[
\bar{\pi} = (1-\beta \theta) \frac{k}{\alpha}
\]

3. **Inflation Dynamics**

In the absence of future shocks, inflation is expected to gradually approach a long-run equilibrium level of \( \bar{\pi} \). Note that \( \bar{\pi} \) will be lower the higher the proportion (\( \theta \)) of naïve agents. The effect is magnified if the central bank places more weight on the expected value of its future losses as indicated by greater values for \( \beta \). Intuitively, naïve agents have a “disciplining” effect on the central bank when it considers raising inflation to realize short-run output gains. Higher current inflation is built into the expectations of naïve agents and, thus, implies greater future expected losses to the central bank.\(^6\) Also, from (12), long-run inflation decreases in the resolve of the monetary authorities to fight inflation (higher \( \alpha \)) and increases in the magnitude of their output objectives (higher \( k \)).

The \( \lambda \) parameter is shown in the Appendix to be:

---

\(^6\)Note that without naïve agents (\( \theta=0 \)) or with a myopic policymaker (\( \beta=0 \)), long run inflation is \( k/\alpha \), the solution to a one-period Barro-Gordon problem. This result is similar to earlier papers where the existence of
The anticipated speed of adjustment toward the long run inflation rate is given by $(1 - \lambda)$ or, put differently, the degree of persistence in inflation is given by $\lambda$. We are interested in the effect of the resolve of the central bank to fight inflation ($\alpha$), the discount factor ($\beta$), and the proportion of naïve agents ($\theta$) on this inflation persistence.

First, note that $\lambda$ is a decreasing function of $\alpha$. The greater the relative weight that the central bank puts on inflation in its objective function, the more rapidly it will try to bring down inflation to the long-run level, as well as having a lower long-run inflation target.

A similar intuition applies to the fact that $\lambda$ is a decreasing function of $\beta$. If the monetary authority puts relatively more weight on future losses, it wants to get high inflation out of the system more quickly.

We also find that $\lambda$ is generally an increasing function of $\theta$. A higher proportion of naïve agents slows down the speed of adjustment and adds to the persistence of inflation. This result, coupled with the effect of $\theta$ on the long-run inflation rate, gives rise to a tradeoff for the central bank between rapid disinflation and lower long-run inflation. This is discussed more fully below.\(^7\)

Next we consider briefly how responsive the monetary authority in this framework is to a supply shock. As shown in the Appendix, current inflation is:

\begin{equation}
\lambda = \frac{\alpha + \theta + \beta \theta^2 - \sqrt{\left(\alpha + \theta + \beta \theta^2\right)^2 - 4 \beta \theta^3}}{2 \beta \theta^2}
\end{equation}

\(^{7}\) Numerical analysis indicates that when $\beta$ is high, $\lambda$ as a function of $\theta$ may reach a peak at high values of $\theta$ and then decrease slightly. Intuitively, with high $\beta$, the costs in future periods from not reducing inflation now may outweigh the persistence effect of marginally higher $\theta$.\(^{\dagger}\)
The coefficient on $u_t$ is unambiguously positive, so that a negative shock to output will call for an increase in inflation. How large that response will be depends on the parameters $\alpha$, $\beta$, and $\Theta$. A greater proportion of naïve agents will generally decrease the inflation response to a supply shock as high current inflation is built into future expectations.

4. Preferences of the central bank over the distribution of agents.

We now address the question about whether the central bank with a mandate to generate disinflation has any preference regarding the distribution of agents in the two groups -- with rational and with adaptive expectations at the time it introduces the new policy. In particular, the bank may choose to engage in more or less dissemination of information about the features and implications of the new regime. Formally, we write the objective function as (see Appendix):

$$W_t = \left[ \pi_t - \left( \lambda_t - \lambda_t \Theta + \Theta \right) \pi_{t-1} - (1 - \lambda_t)(1 - \Theta)\pi_t - k \right]^2 + \alpha \sigma_t^2 +$$

$$+ \sum_{j=1}^{\infty} \beta^j \left\{ \Theta (\lambda - 1) \lambda_{t-j} (\pi_t - \bar{\pi}) - k \right\}^2 + \alpha \left[ \pi_t + (1 - \lambda_t) \pi_t \right]^2 \}

We are interested in what value of $\Theta$ minimizes (12) given the bank’s preferences, reflected in the $\alpha$, $\beta$ and $k$ parameters, and the prior level of inflation $\pi_{t-1}$.

Figures 1, 2 and 3 show three examples in which the objective function (12) is calculated for $\Theta$ in the interval between 0 and 1. In all examples $\beta = 0.9$, $\alpha = 0.2$ and $k = 1$, so
that new long term-inflation in the absence of naïve agents (with \( \theta = 0 \)) would be 5.\(^8\) We calculated the objective function with three different values for \( \pi_{t-1} \) chosen to proxy for high, moderate and low inflation. In Figure 1, \( \pi_{t-1} = 30 \) (high compared to equilibrium inflation with no naïve agents), in Figure 2, \( \pi_{t-1} = 15 \) (“moderate” but still higher than equilibrium inflation with no naïve agents), and in Figure 3, \( \pi_{t-1} = 5 \) (equal to equilibrium inflation with no naïve agents).

When authorities want to minimize the objective function (12), the figures deliver an ambiguous message that it depends on where the economy starts. Figure 1 indicates that if the economy starts at a very high inflation rate, the objective function is minimized by having very few naive agents. This is because with very few naive agents substantial progress can be made early in bringing down inflation and those early declines outweigh the costs of higher long-run inflation. In this case, there should be a lot of information about the new regime so that more agents can more rationally take into account how the regime will achieve a disinflation.

At the other extreme if initial inflation is already fairly low, the objective function is minimized by having a high proportion of naive agents as depicted in Figure 3. In that case, the gains in reducing long-run inflation outweigh the loss in bringing inflation down less rapidly, and the authorities may want to withhold information hoping that most agents will form their expectations as a simple extrapolation of what has been most recently observed. The intermediate case, as in Figure 2, suggests that with moderate initial inflation the optimal solution is to have a mix of both naive and rational agents.

\(^8\) Steady state inflation with rational agents equal to 5 is the same as the Nash equilibrium inflation in Sargent (1999, page 84). Replacing rational with adaptive expectations and with a discount factor of .97, Sargent (1999) reports equilibrium inflation of 1.57. In out calibrations, with \( \theta = 1 \), i.e. only adaptive expectations, \( \alpha = 0.2, k = 1 \), and with a discount factor of 0.9, steady state inflation is 0.5. If we use Sargent’s discount factor of .97, steady state inflation is 0.15.
5. Discussion.

A stylized fact about inflation stabilization has been a pattern of rapid declines in inflation from high to moderate levels but slow convergence from moderate to low levels. In fact, the episodes of rapid disinflation documented by Sargent (1982) are explained by rational expectations while the episodes of real exchange rate appreciation summarized by Calvo and Vegh (1994) are often explained by persistence in expectations. In our variation of the Barro-Gordon model, when the central bank initiates a new disinflation policy, more informed (rational) agents are desirable because they increase the rate of disinflation but are undesirable to the extent that they contribute later to a stubborn persistence of lower inflation when output targets exceed the natural rate of output.

These implications suggest that a central bank faced with the task of disinflating from a high rate of inflation would want to provide substantial information about the new disinflation policy at its inception and then gradually withdraw from public discussion as inflation declines. This appears to be what has been happening in Bulgaria. At the introduction of the Bulgarian currency board, policymakers were engaged significantly in explaining how a currency board works and what it has done for other countries. Once inflation was lower, policymakers began instead referring to the track record with low inflation rather than explaining how low inflation comes about or what policies it has at its disposal. As we pointed out earlier, a possible reason is that the design of the currency board in Bulgaria, as that of most other currency boards, allows discretion over monetary policy. One example is the facilities for liquidity to the banking system. Understanding of the balance sheet of the central bank may raise concerns and, respectively, expected inflation on the part of rational agents.
In terms of the model, one could argue that, if the distribution of agents is a choice variable for the central bank in each period, such information should be incorporated in the expectations of rational agents. A switch in the extent of information from more to less over time, if anything however, helps the disinflation policy. This is because rational agents who anticipate that there will be relatively more naïve agents in the future will expect inflation to fall even more than if there were no change in the proportions of naïve and rational agents.
References


Figure 1. Value function, high inflation

Figure 2. Value function, moderate inflation

Figure 3. Value function, low inflation

Proportion of naive agents
Appendix

In the absence of any shocks, the first-order condition can be written as the following second-
order difference equation:

\[(A.1) \quad \pi_{t+1} - \frac{\alpha + \theta + \beta \theta^2}{\beta \theta^2} \pi_t + \frac{1}{\beta \theta} \pi_{t-1} = -(1- \beta \theta) \frac{k}{\beta \theta^2} \]

or in lag-operator notation:

\[(A.2) \quad (1- \lambda_1 L)(1- \lambda_2 L)\pi_{t+1} = -(1- \beta \theta) \frac{k}{\beta \theta^2} \]

where \(\lambda_1\) and \(\lambda_2\) are the roots of the characteristic equation

\[(A.3) \quad f(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) = \lambda^2 - \frac{\alpha + \theta + \beta \theta^2}{\beta \theta^2} \lambda + \frac{1}{\beta \theta} = 0 \]

Note that

\[(A.4) \quad f(0) = \frac{1}{\beta \theta} > 0 \]

\[(A.5) \quad f(1) = (1 - \lambda_1)(1 - \lambda_2) = -\frac{\alpha}{\beta \theta^2} < 0 \]

These imply that the smaller root lies between 0 and 1 and the larger root is greater than 1. The
smaller root can be written explicitly as

\[(A.6) \quad \lambda_1 = \frac{\alpha + \theta + \beta \theta^2 - \sqrt{(\alpha + \theta + \beta \theta^2)^2 - 4\beta \theta^3}}{2 \beta \theta^2} \]

One can use (A.4) and (A.5) to show that an increase in \(\alpha\), with \(0 < \theta < 1\), lowers \(\lambda_1\). A
numerical analysis establishes that an increase in \(\beta\) also lowers \(\lambda_1\).

If one multiplies (A.2) through by \((1 - \lambda_2 L)^{-1}\), the result assuming no bubbles is:

\[(1- \lambda_1 L) \pi_{t+1} = \frac{1 - \beta \theta}{\lambda_2 - 1} \frac{k}{\beta \theta^2} \]

Then after substituting for \((\lambda_2 - 1)\) from (A.5):

\[\]
\[(A.7) \quad \pi_{t+1} = \lambda \pi_t + (1-\lambda)\pi \]

where \( \lambda = \lambda_1 \) and

\[(A.8) \quad \pi = (1-\beta \theta)^{-k} \frac{\lambda}{\alpha} \]

\[(A.7) \] also implies that

\[(A.9) \quad E_t \pi_t = \lambda \pi_{t-1} + (1-\lambda)\pi \]

\[(A.10) \quad E_t \pi_{t+n} = \pi + \lambda^n (\pi_t - \pi) \]

Given the quadratic objective function, we can use certainty equivalence and rewrite the objective function as:

\[
\min_{\pi_t} W_t = \left[ \pi_t - (1-\Theta)E_{t-1}\pi_t - \Theta\pi_{t-1} - k - u_t \right]^2 + \alpha \pi_t^2 \\
+ \sum_{j=1}^{\infty} \beta^j \left[ \Theta E_j \pi_{t+j} - \Theta E_j \pi_{t+j-1} - k \right]^2 + \alpha \sum_{j=1}^{\infty} \beta^j E_j \pi_{t+j}^2 
\]

Substituting from (A.9) and (A.10) into (A.11), we have:

\[
W_t = \left[ \pi_t - (\lambda_t - \lambda_t \Theta + \Theta)\pi_{t-1} - (1-\lambda_t)(1-\Theta)\pi_t - k \right]^2 + \alpha \pi_t^2 \\
+ \sum_{j=1}^{\infty} \beta^j \left[ \Theta (\lambda_{t+1} - 1) \lambda_{j-1} (\pi_t - \bar{\pi}) - k \right]^2 + \alpha \sum_{j=1}^{\infty} \beta^j E_j \pi_{t+j}^2 
\]

Take the derivative of \( W_t \) with respect to \( \pi_t \) and set it equal to zero:

\[
\frac{\partial W_t}{\partial \pi_t} = 2[\pi_t - (\lambda_t - \lambda_t \Theta + \Theta)\pi_{t-1} - (1-\lambda_t)(1-\Theta)\pi_t - k] + 2\alpha \pi_t + \\
+ \sum_{j=1}^{\infty} \beta^j \left\{ \Theta (\lambda_{j} - 1) (\lambda_{j-1} - \lambda_t) (\pi_t - \bar{\pi}) - k \right\} + \alpha \sum_{j=1}^{\infty} \beta^j E_j \pi_{t+j} = 0 
\]

Differentiate (A.13) totally with respect to \( \pi_t \) and \( u_t \) to see how the optimal \( \pi_t \) varies in response to \( u_t \). As a result we have:

\[
\frac{\partial \pi_t}{\partial u_t} = \frac{1 - \beta \lambda^2}{(1 - \beta \lambda^2)(1 + \alpha) + \alpha \beta \lambda^2 + \beta \theta^2 (1-\lambda)^2} 
\]

Hence,
(A.15) \[
\pi_t = \lambda_t \pi_{t-1} + (1 - \lambda_t)\pi + \frac{1 - \beta \lambda^2}{1 + \alpha - \beta \lambda^2 + \beta \theta^2 (1 - \lambda)^3} \cdot \pi, 
\]