Semiparametric Competing Risks Analysis

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Appendix A – Web Tables
(Not for Publication)

Table A1. Monte Carlo Experiment: Mean Absolute Bias of parameters in % (5000 Obs.).

	RISK 1					RISK 2				
	δ_{11}	δ_{21}	δ_{31}	β_1	Mean	δ_{12}	δ_{22}	δ_{32}	β_2	Mean
Gamma DGP										
Model 1	.0058	.0397	.1326	.1679	.0865	.0092	.0448	.1271	.1825	.0909
Model 2	.0060	.0111	.0249	.0495	.0229	.0083	.0162	.0257	.0608	.0278
Model 3	.0080	.0156	.0314	.0515	.0267	.0246	.0375	.0469	.0662	.0438
Model 4	.0068	.0145	.0311	.0518	.0261	.0094	.0173	.0268	.0662	.0299
H-S BIC	.0063	.0211	.0621	.0909	.0451	.0126	.0305	.0644	.1048	.0531
H-S HQIC	.0063	.0118	.0268	.0512	.0240	.0156	.0256	.0360	.0628	.0350
Model 5	.0061	.0122	.0268	.0531	.0246	.0088	.0140	.0348	.0760	.0334
Model 6	.0060	.0101	.0258	.0518	.0234	.0087	.0157	.0268	.0669	.0295
Model 7	.0060	.0105	.0273	.0524	.0240	.0083	.0124	.0233	.0639	.0270
Model 8	.0062	.0125	.0299	.0527	.0253	.0086	.0151	.0257	.0690	.0296
Pol BIC	.0061	.0122	.0268	.0531	.0246	.0088	.0140	.0348	.0760	.0334
Pol HQIC	.0061	.0123	.0274	.0531	.0247	.0088	.0142	.0343	.0751	.0331
Heckman-Singer DGP										
Model 1	.0059	.0695	.3099	.4010	.1965	.0154	.0610	.2073	.2702	.1385
Model 2	.0061	.0081	.0238	.0610	.0247	.0084	.0119	.0252	.0666	.0280
Model 3	.0102	.0149	.0325	.0625	.0300	.0164	.0241	.0370	.0709	.0371
Model 4	.0079	.0120	.0284	.0639	.0281	.0142	.0225	.0340	.0732	.0360
H-S BIC	.0067	.0089	.0249	.0610	.0253	.0092	.0129	.0263	.0667	.0288
H-S HQIC	.0089	.0121	.0290	.0614	.0278	.0121	.0168	.0303	.0671	.0316
Model 5	.0079	.0350	.0460	.0733	.0405	.0089	.0172	.0421	.1106	.0447
Model 6	.0063	.0134	.0323	.0690	.0303	.0086	.0124	.0274	.0785	.0317
Model 7	.0062	.0131	.0289	.0662	.0286	.0086	.0129	.0263	.0769	.0312
Model 8	.0062	.0119	.0292	.0662	.0283	.0084	.0114	.0248	.0694	.0285
Pol BIC	.0065	.0213	.0369	.0714	.0340	.0085	.0137	.0324	.0897	.0361
Pol HQIC	.0060	.0132	.0310	.0690	.0298	.0084	.0120	.0268	.0769	.0310
Lognormal DGP										
Model 1	.0059	.0695	.3099	.4010	.1965	.0154	.0610	.2073	.2702	.1385
Model 2	.0061	.0080	.0236	.0607	.0246	.0082	.0114	.0246	.0650	.0273
Model 3	.0068	.0101	.0264	.0621	.0263	.0085	.0120	.0256	.0658	.0280
Model 4	.0085	.0148	.0321	.0648	.0300	.0197	.0299	.0414	.0739	.0412
H-S BIC	.0061	.0080	.0236	.0607	.0246	.0082	.0114	.0246	.0650	.0273
H-S HQIC	.0061	.0080	.0236	.0607	.0246	.0082	.0114	.0246	.0650	.0273
Model 5	.0079	.0350	.0460	.0733	.0405	.0089	.0172	.0421	.1106	.0447
Model 6	.0063	.0134	.0324	.0689	.0302	.0086	.0124	.0274	.0784	.0317
Model 7	.0062	.0131	.0289	.0663	.0286	.0086	.0129	.0264	.0771	.0312
Model 8	.0062	.0119	.0295	.0666	.0285	.0084	.0116	.0250	.0698	.0287
Pol BIC	.0073	.0269	.0413	.0728	.0371	.0088	.0152	.0362	.0970	.0393
Pol HQIC	.0063	.0147	.0333	.0694	.0310	.0086	.0127	.0281	.0800	.0323

Note: Results from 1000 repetitions of the Monte Carlo experiment. Computed using equation (22).

Table A2. Monte Carlo Experiment: Mean Absolute Bias of Parameters (Percent)
Based on the Choice of Best Model According to the Akaike Information Criterion

	Risk 1					Risk 2				
Model	**d11**	**d21**	**d31**	**beta1**	**mean**	**d12**	**d22**	**d32**	**beta2**	**mean**
1000 Obs.:										
					Gamma DGP					
H-S	.0352	.0619	.1257	.1351	.0895	.0510	.0794	.1234	.1610	.1037
Polynomial	.0137	.0224	.0623	.1262	.0562	.0190	.0305	.0682	.1631	.0702
					Heckman-Singer DGP					
H-S	.0244	.0335	.0762	.1349	.0672	.0299	.0391	.0662	.1576	.0732
Polynomial	.0142	.0307	.0707	.1537	.0673	.0212	.0304	.0645	.1921	.0770
					Lognormal DGP					
H-S	.0192	.0262	.0674	.1333	.0615	.0258	.0336	.0608	.1540	.0686
Polynomial	.0142	.0307	.0706	.1538	.0673	.0212	.0305	.0644	.1926	.0772
2500 Obs.:										
					Gamma DGP					
H-S	.0132	.0222	.0455	.0754	.0391	.0184	.0306	.0472	.0895	.0464
Polynomial	.0088	.0152	.0374	.0773	.0346	.0119	.0192	.0428	.0981	.0430
					Heckman-Singer DGP					
H-S	.0312	.0438	.0768	.0890	.0602	.0468	.0619	.0807	.0907	.0700
Polynomial	.0090	.0175	.0433	.0996	.0424	.0124	.0182	.0365	.1082	.0439
					Lognormal DGP					
H-S	.0182	.0255	.0527	.0886	.0462	.0374	.0494	.0669	.0890	.0607
Polynomial	.0090	.0175	.0433	.0996	.0424	.0124	.0183	.0365	.1087	.0440

Note: Results from 1000 repetitions of Monte Carlo experiment computed using equation 22. Compare with the results reported in tables 3 (1000 obs.) and 4 (2500 obs.) of the paper.

Table A3. Monte Carlo Experiment: Mean Absolute Bias in Percent for the Estimated Versus Actual Values of q_1 and q_2 Based on the Choice of Best Model According to the Akaike Information Criterion

	q_1				q_2			
Model	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
1000 Obs.:				**Gamma DGP**				
H-S	.1807	.0375	.0687	.3085	.1709	.0730	.0238	.5494
Polynomial	.1692	.0370	.0817	.3249	.1894	.0683	.0407	.4974
				Heckman-Singer DGP				
H-S	.0716	.0291	.0104	.1808	.1352	.0615	.0244	.4136
Polynomial	.0770	.0283	.0119	.1911	.1560	.0645	.0166	.4236
				Lognormal DGP				
H-S	.0714	.0290	.0097	.1787	.1284	.0557	.0227	.3548
Polynomial	.0769	.0283	.0119	.1911	.1561	.0644	.0175	.4236
2500 Obs.:				**Gamma DGP**				
H-S	.1608	.0233	.0960	.2467	.1274	.0492	.0207	.3466
Polynomial	.1525	.0228	.0867	.2367	.1580	.0477	.0218	.3517
				Heckman-Singer DGP				
H-S	.0457	.0190	.0042	.1114	.0853	.0369	.0118	.2310
Polynomial	.0484	.0194	.0090	.1232	.0960	.0398	.0173	.2622
				Lognormal DGP				
H-S	.0454	.0192	.0042	.1156	.0816	.0353	.0132	.2399
Polynomial	.0485	.0194	.0079	.1232	.0964	.0399	.0182	.2577

Note: $q_j(t)$ j=1,2 represent the probability associated with each risk at time t as defined in in equations 4 and 5 and bias computed for estimated versus actual function values. Computed using equation 23. Compare with the results in Table 5 of the paper.

Table A4. Monte Carlo Experiment: Mean Absolute Bias in % for the Estimated vs. Actual Values of q_1 and q_2 (Sample Size 5000)

	Gamma DGP				Heckman-Singer DGP				Lognormal DGP			
q_1	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
Model 1	.1766	.0139	.1347	.2241	.1225	.0139	.1016	.1772	.1225	.0139	.1016	.1772
Model 2	.1534	.0165	.1087	.2141	.0323	.0135	.0058	.0745	.0320	.0134	.0052	.0737
Model 3	.1546	.0165	.1058	.2131	.0331	.0136	.0062	.0745	.0327	.0135	.0056	.0757
Model 4	.1541	.0164	.1014	.2134	.0333	.0140	.0052	.1286	.0341	.0138	.0076	.0804
H-S BIC	.1609	.0198	.1087	.2214	.0323	.0134	.0058	.0745	.0320	.0134	.0052	.0737
H-S HQIC	.1538	.0168	.1087	.2131	.0324	.0138	.0058	.1286	.0320	.0134	.0052	.0737
Model 5	.1446	.0160	.0896	.2062	.0571	.0108	.0374	.1007	.0571	.0108	.0374	.1006
Model 6	.1496	.0163	.0910	.2119	.0352	.0130	.0075	.0801	.0352	.0131	.0076	.0801
Model 7	.1508	.0162	.0962	.2117	.0344	.0131	.0064	.0802	.0344	.0131	.0068	.0804
Model 8	.1515	.0161	.0955	.2140	.0337	.0133	.0044	.0764	.0336	.0133	.0048	.0777
Pol BIC	.1446	.0160	.0896	.2062	.0433	.0166	.0085	.1007	.0488	.0158	.0132	.1006
Pol HQIC	.1451	.0162	.0896	.2119	.0350	.0134	.0083	.0912	.0364	.0140	.0076	.0912
q_2	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
Model 1	.1663	.0379	.0890	.3398	.1796	.0334	.1042	.2830	.1796	.0334	.1042	.2830
Model 2	.1106	.0369	.0225	.2807	.0597	.0264	.0111	.1684	.0561	.0245	.0104	.1431
Model 3	.1101	.0377	.0236	.2924	.0642	.0275	.0129	.1808	.0584	.0254	.0119	.1586
Model 4	.1098	.0367	.0234	.2895	.0648	.0267	.0100	.1585	.0644	.0268	.0136	.1855
H-S BIC	.1289	.0456	.0225	.2849	.0597	.0266	.0111	.1684	.0561	.0245	.0104	.1431
H-S HQIC	.1114	.0374	.0225	.2807	.0598	.0265	.0111	.1684	.0598	.0265	.0111	.1684
Model 5	.1566	.0336	.0649	.3126	.1239	.0277	.0612	.2208	.1239	.0277	.0612	.2208
Model 6	.1233	.0344	.0422	.3034	.0714	.0275	.0169	.1749	.0714	.0275	.0165	.1749
Model 7	.1194	.0354	.0324	.2853	.0659	.0274	.0118	.1825	.0660	.0274	.0110	.1796
Model 8	.1153	.0359	.0276	.2799	.0622	.0270	.0107	.1758	.0624	.0270	.0091	.1755
Pol BIC	.1566	.0336	.0649	.3126	.0867	.0401	.0128	.2182	.1028	.0387	.0165	.2182
Pol HQIC	.1531	.0348	.0433	.3126	.0674	.0299	.0128	.1975	.0741	.0311	.0165	.1975

Note: $q_j(t)$ j=1,2 represents the probability associated with each risk at time t as defined in equations 4 and 5 and bias computed for the estimated vs. actual function values. Computed using equation (23).

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Appendix B: Web Figures
(Not for Publication)

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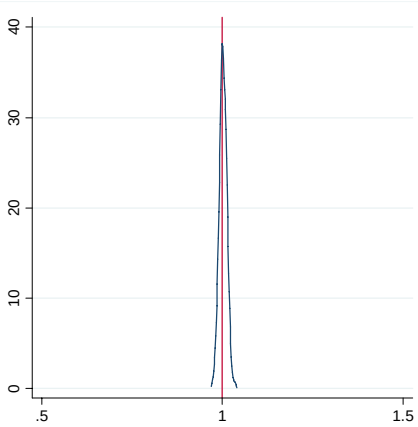
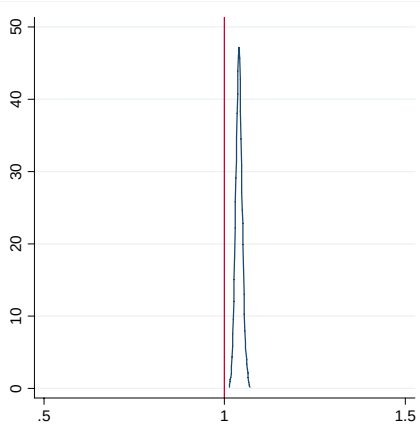
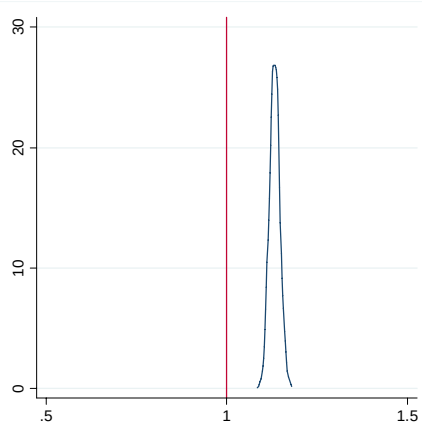
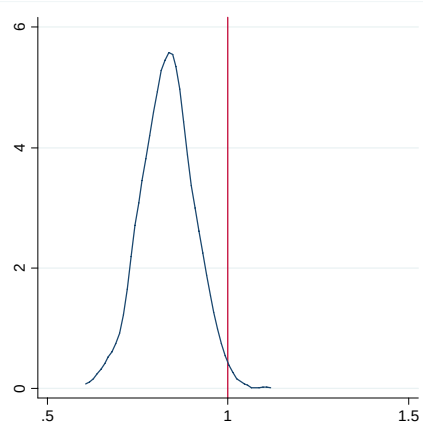
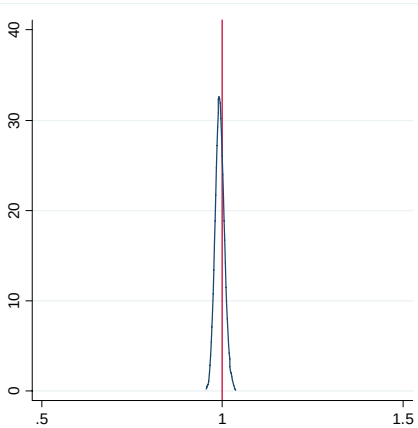
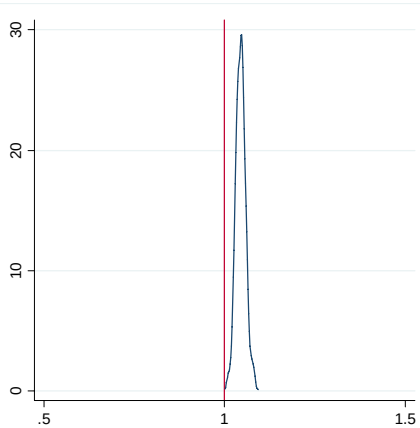
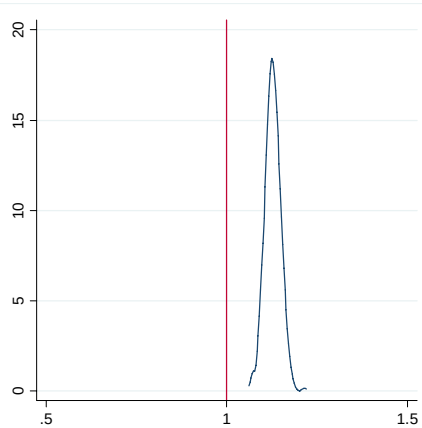
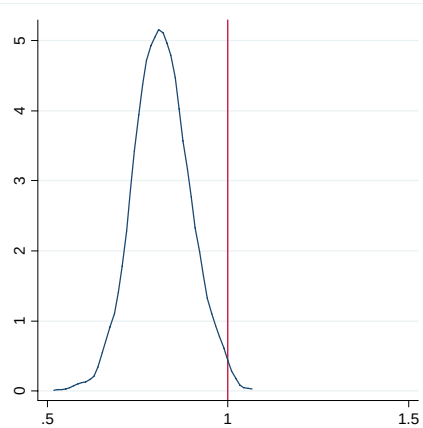
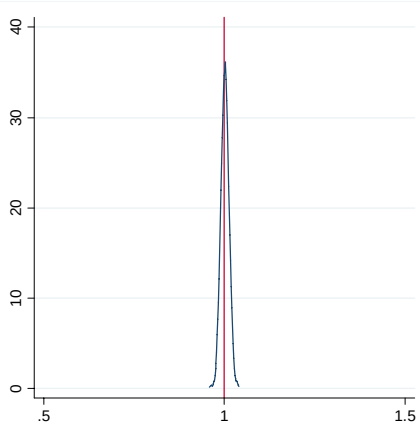
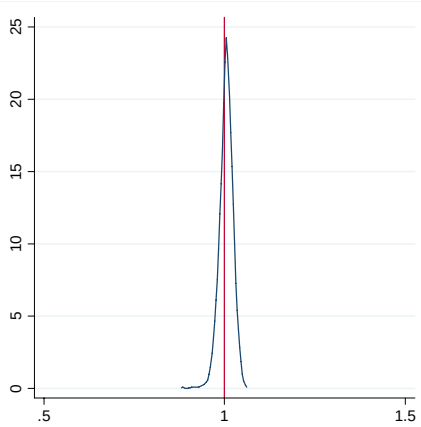
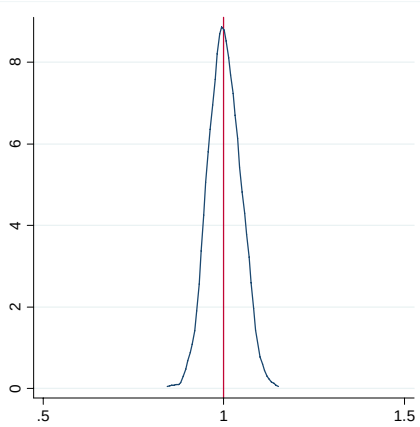
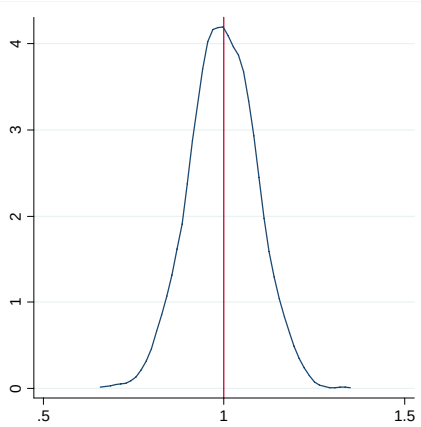
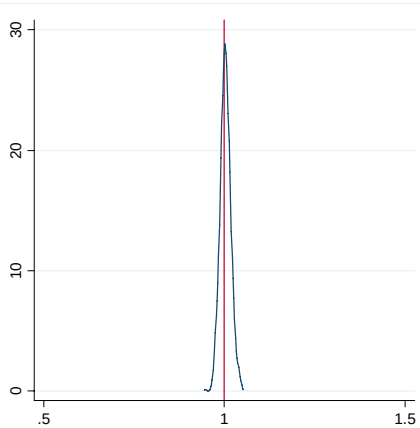
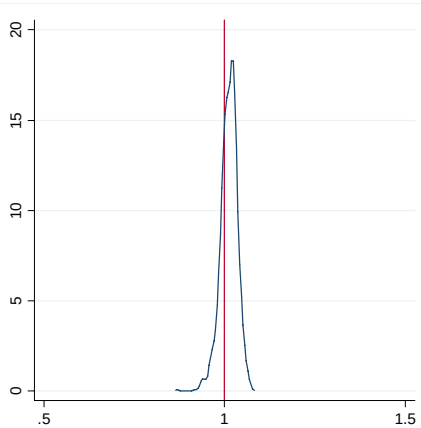
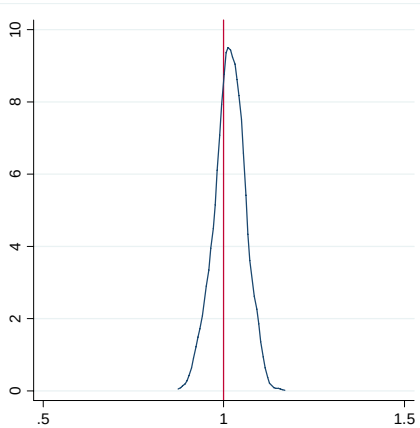
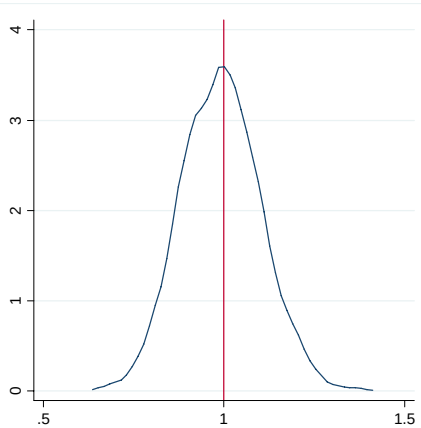
Figure B1. Kernel Density Estimates of the Sampling Distributions of the Estimates of Parameters (DGP: Gamma; 2500 simulated data points; R1 and R2 denotes risks 1 and 2)

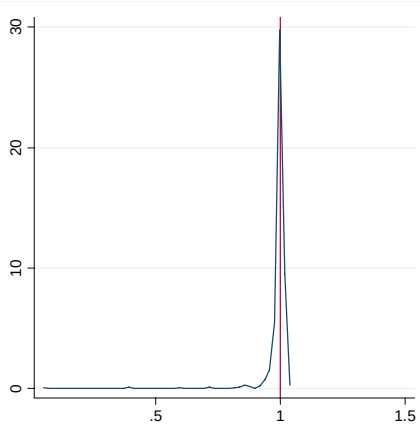
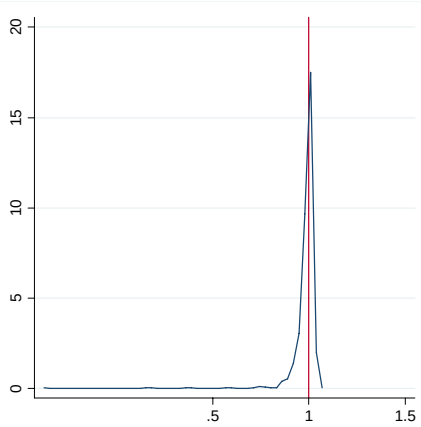
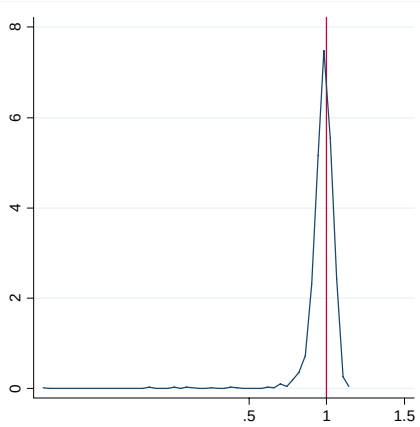
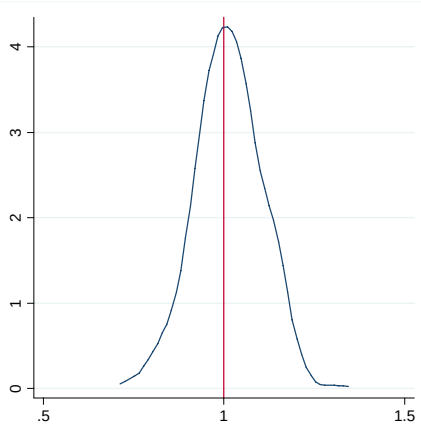
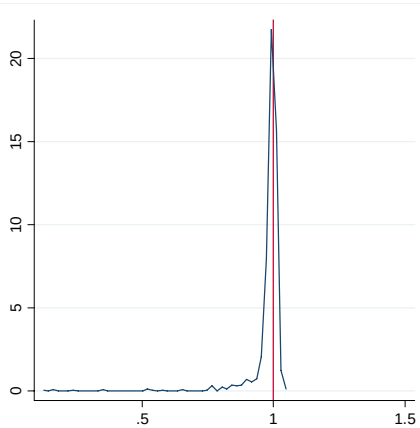
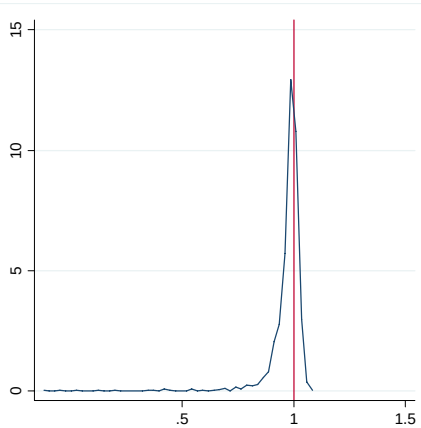
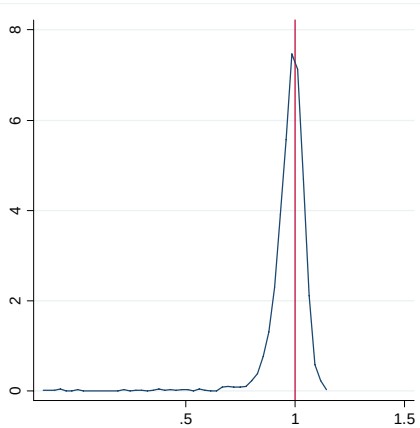
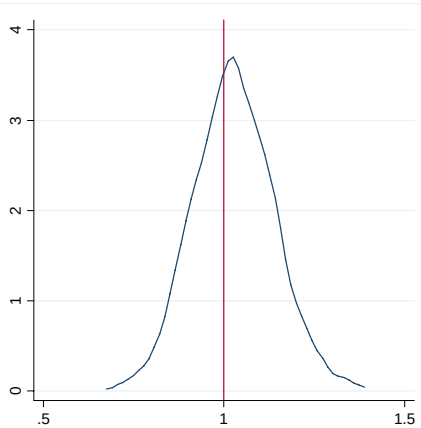
B1.1 Model: Heckman-Singer, No Unobserved Heterogeneity

	δ_1	δ_2	δ_3	β_2
R1				
R2				

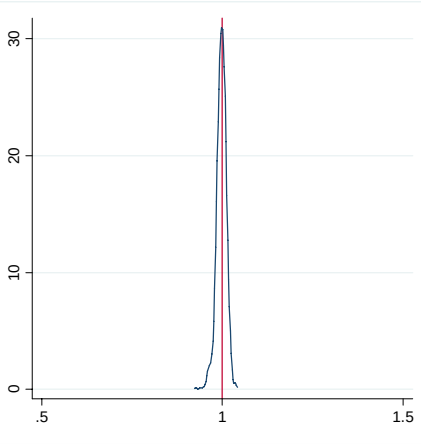
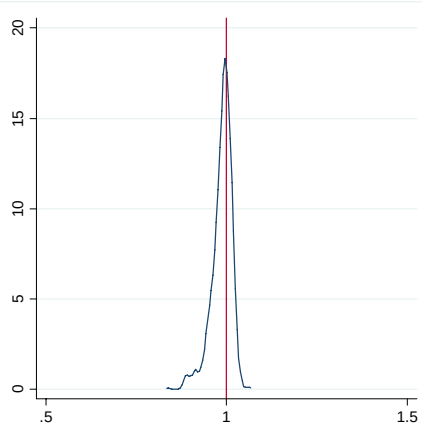
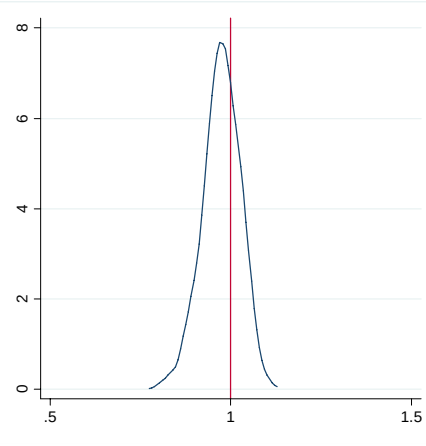
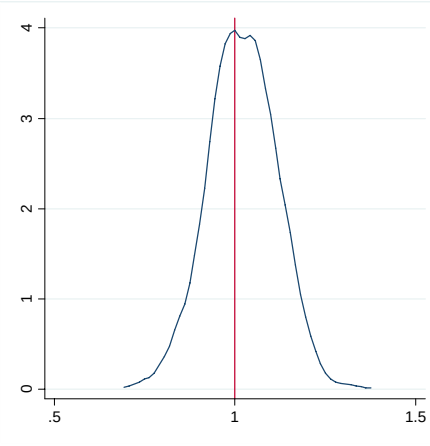
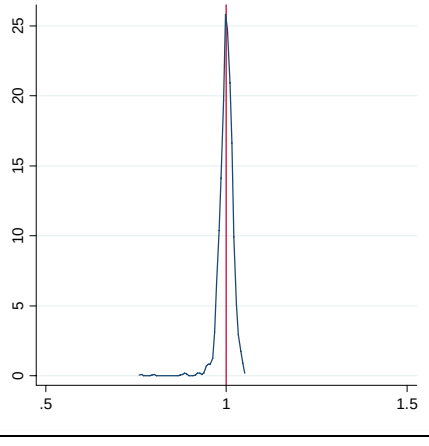
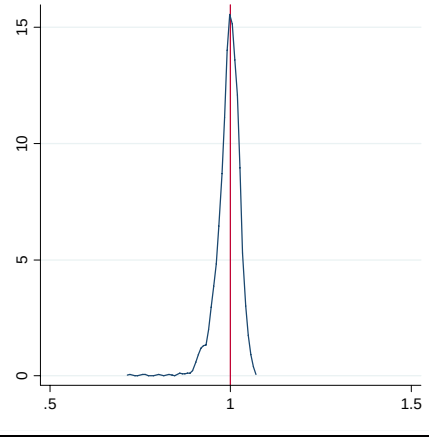
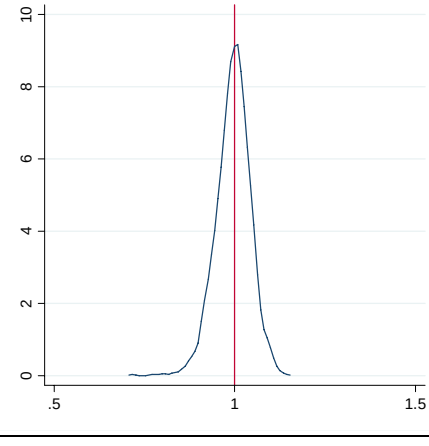
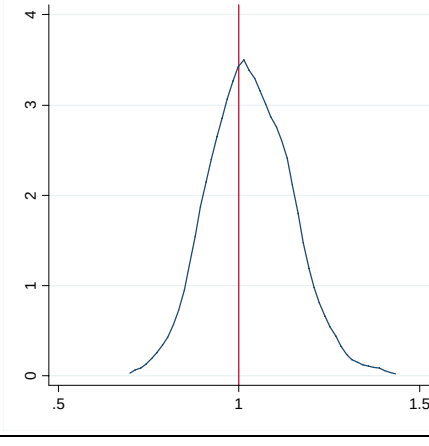
B1.2 Model 2: Heckman -Singer 2 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

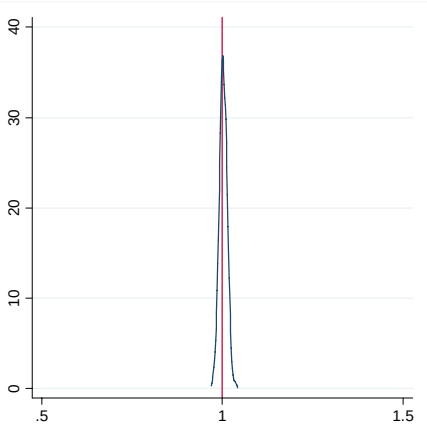
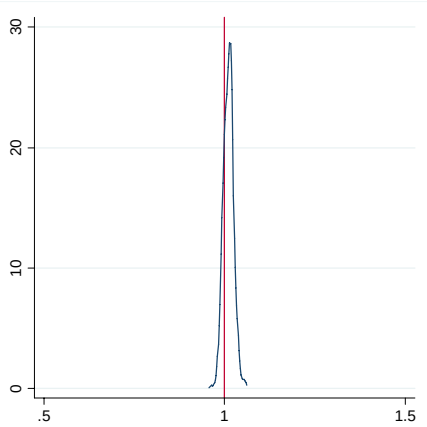
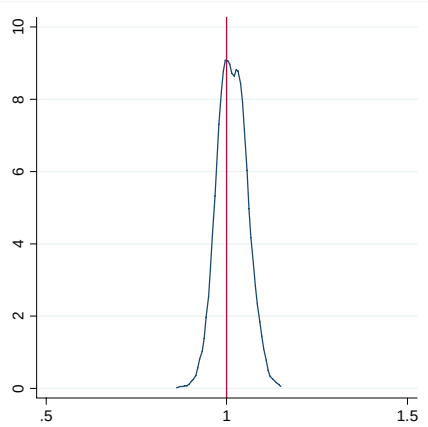
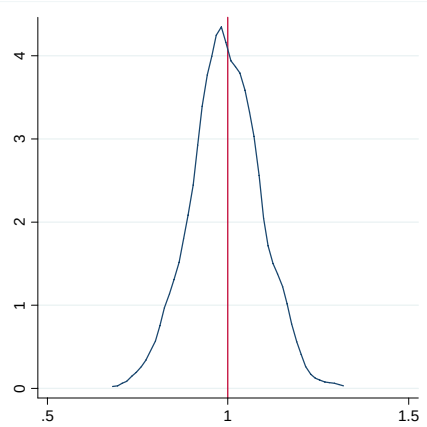
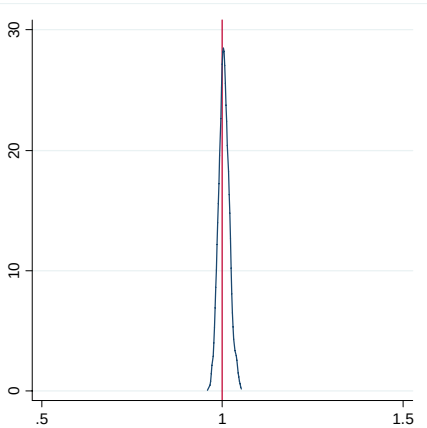
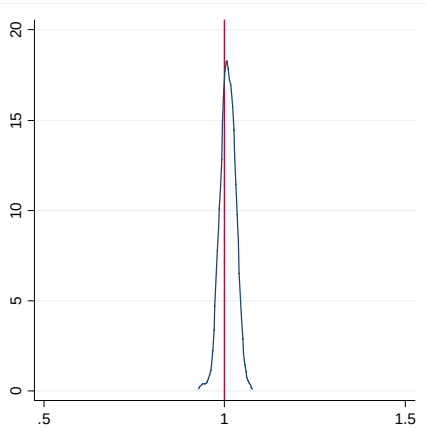
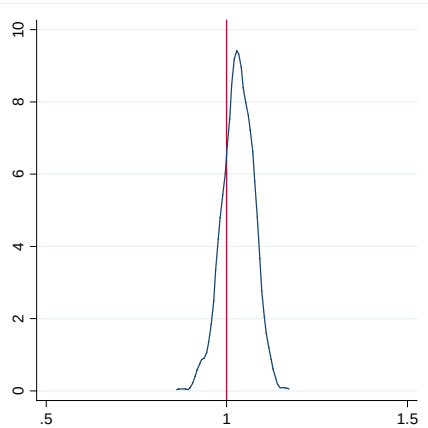
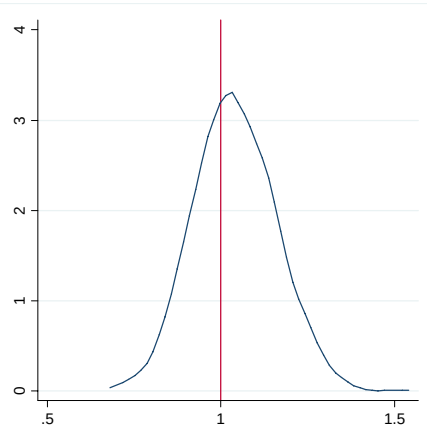
B1.3 Model 3: Heckman -Singer 3 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

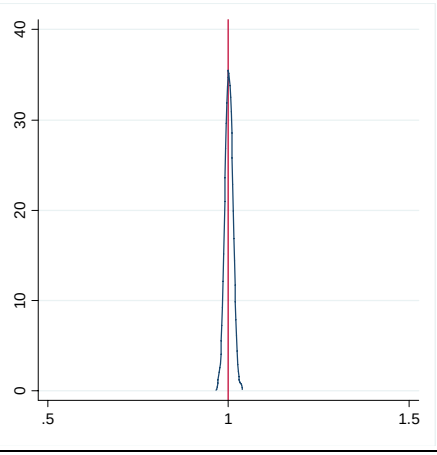
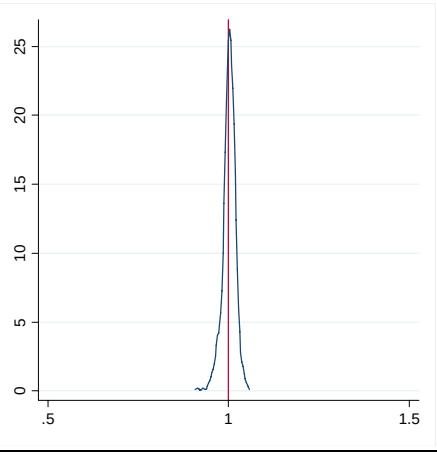
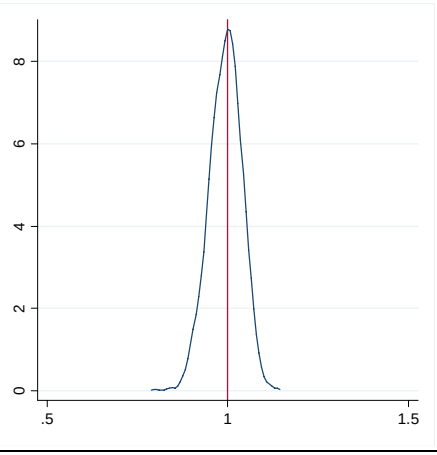
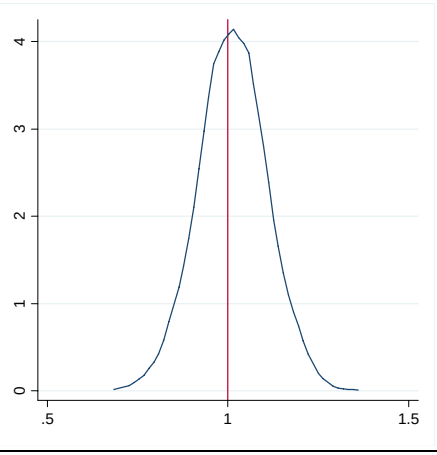
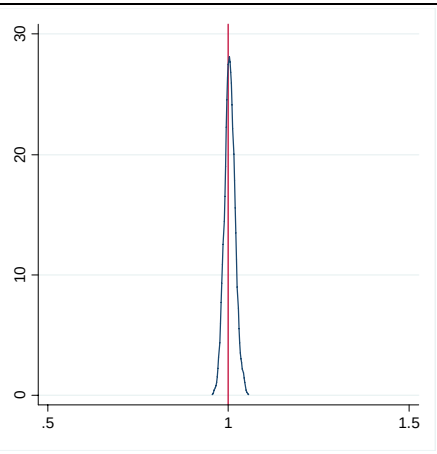
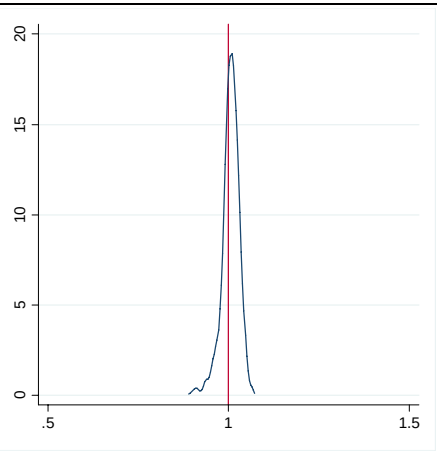
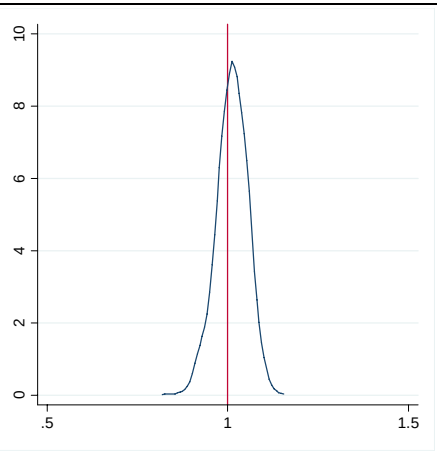
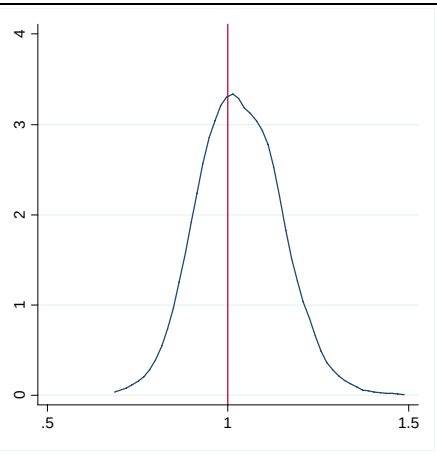
B1.4 Model 4: Heckman -Singer 4 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

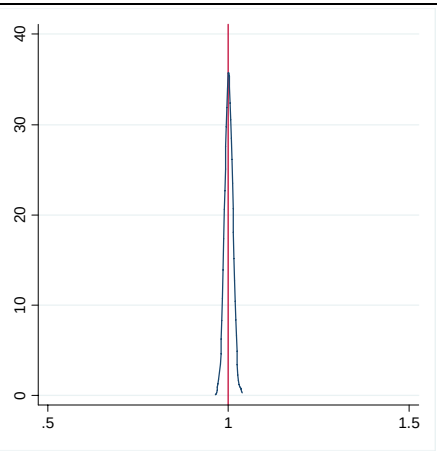
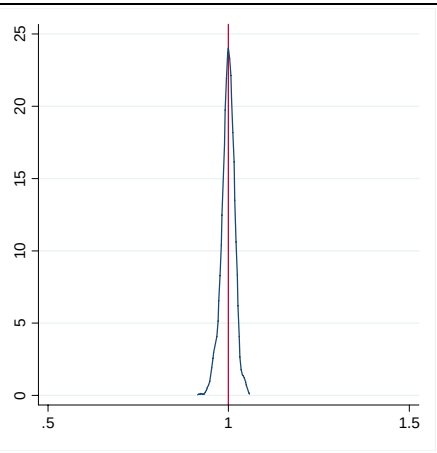
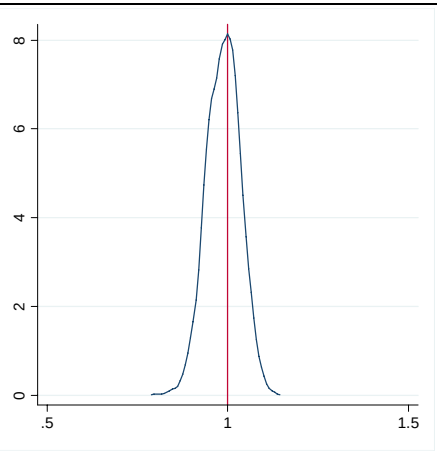
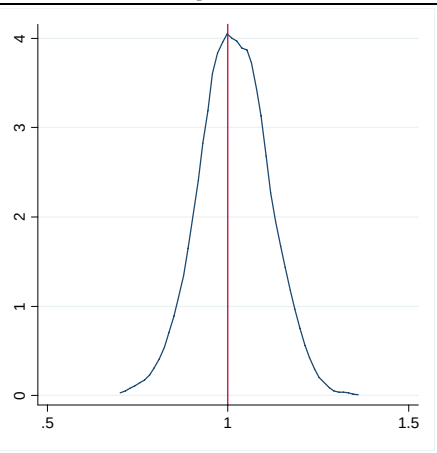
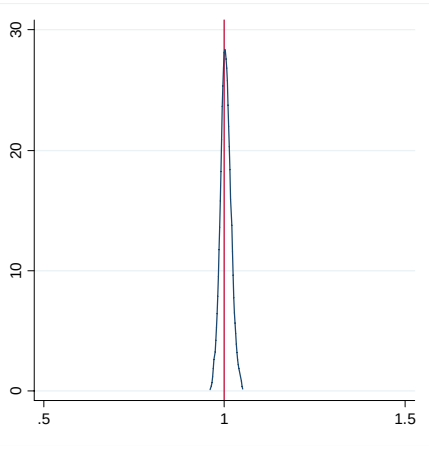
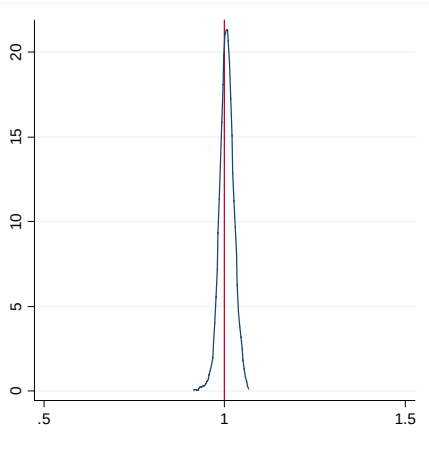
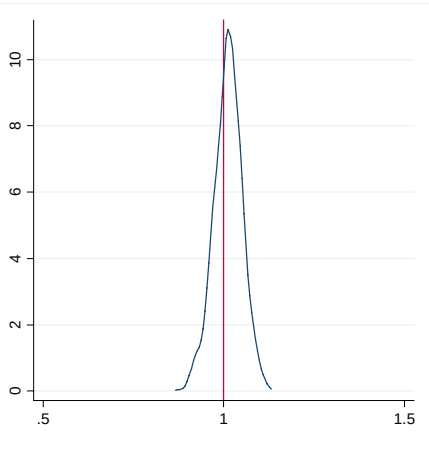
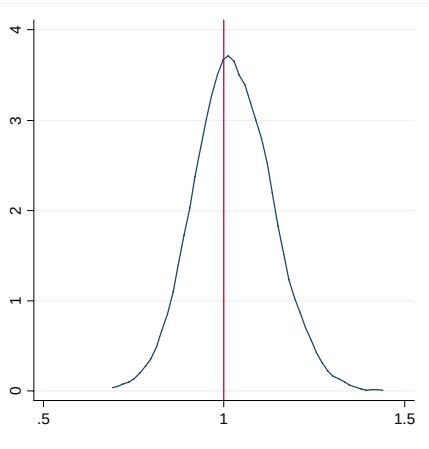
B1.5 Model 5: Polynomial Degree 0

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B1.6 Model 6: Polynomial Degree 1

	δ_1	δ_2	δ_3	β
R1				
R2				

B1.7 Model 7: Polynomial Degree 2

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B1.8 Model 8: Polynomial Degree 3

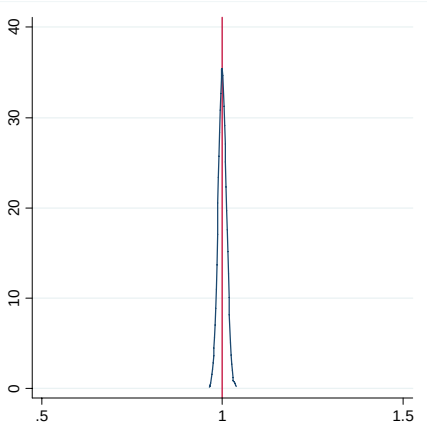
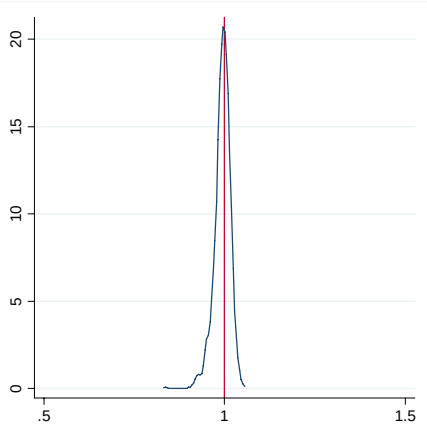
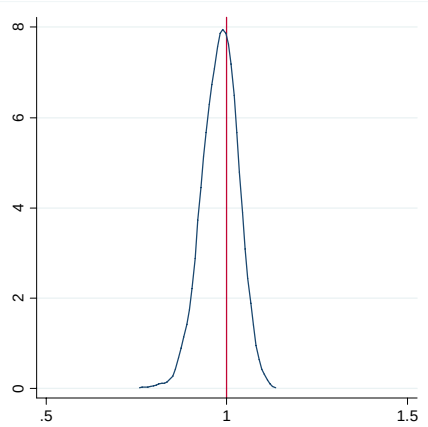
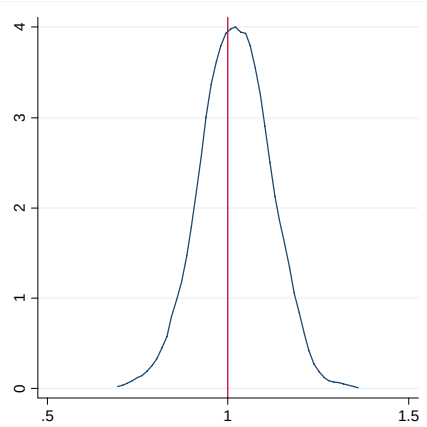
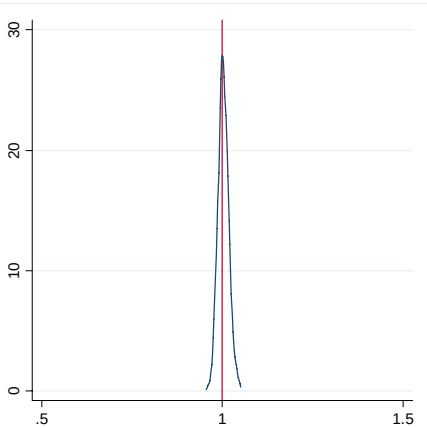
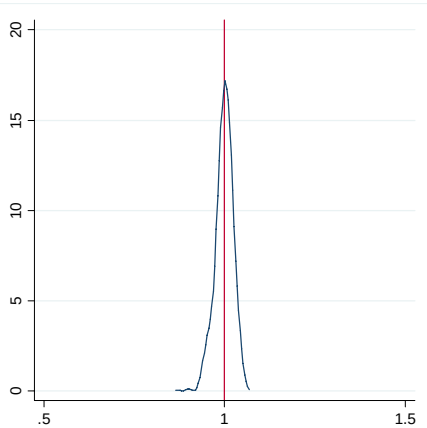
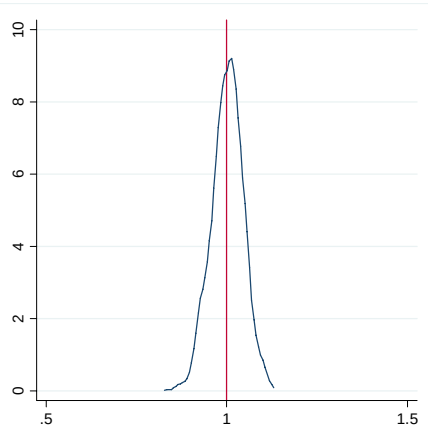
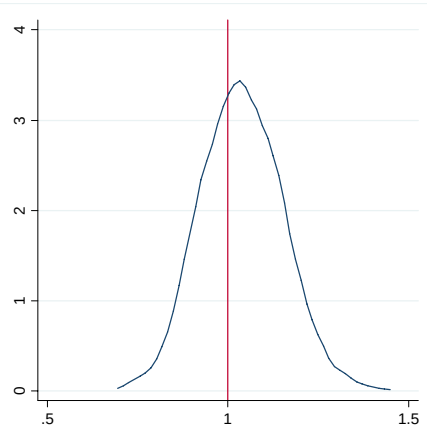
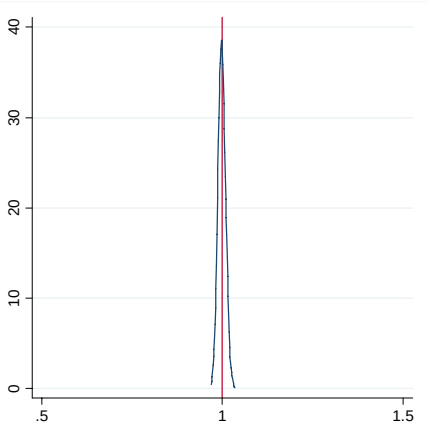
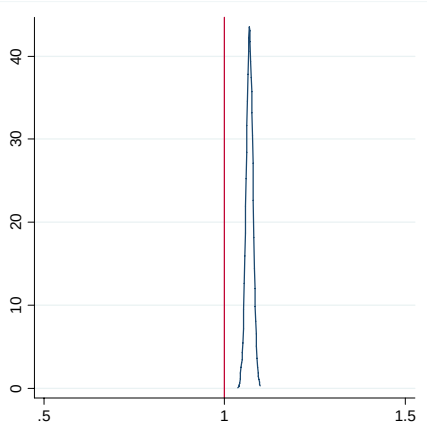
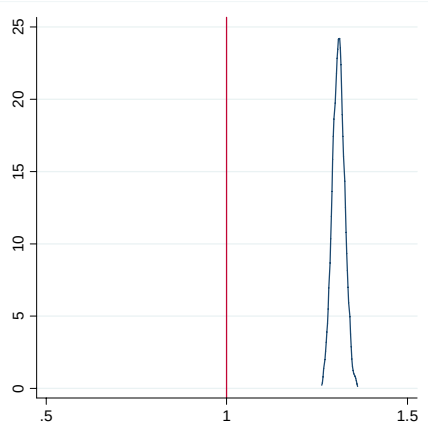
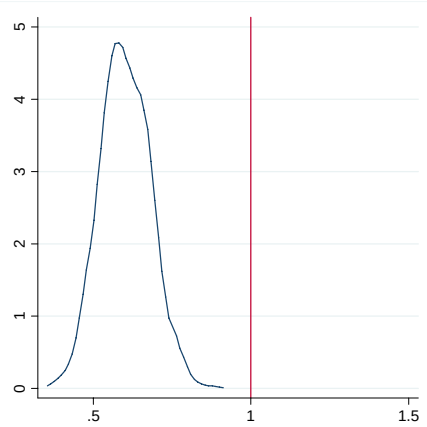
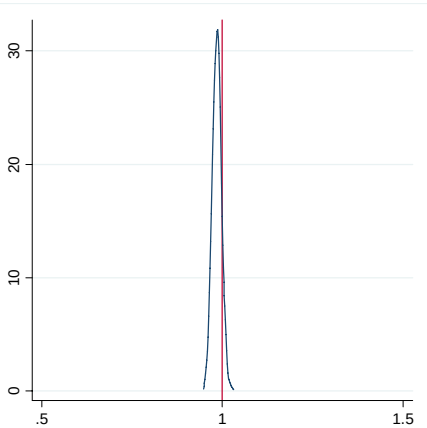
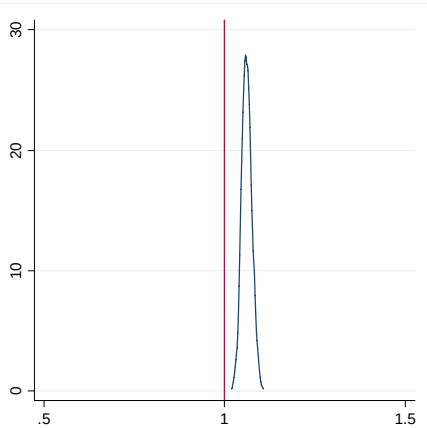
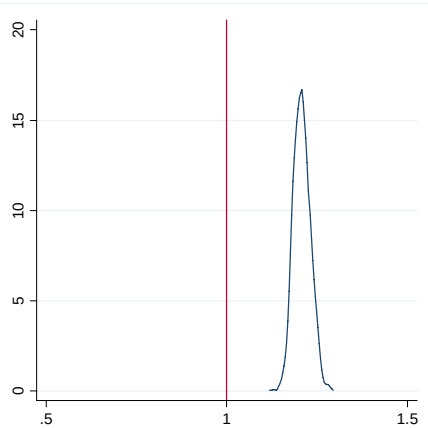
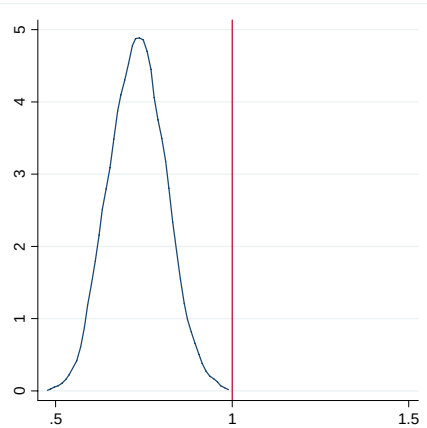
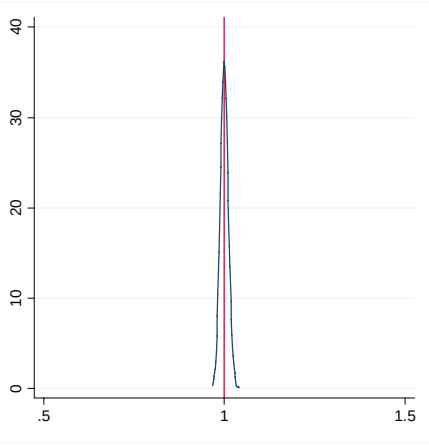
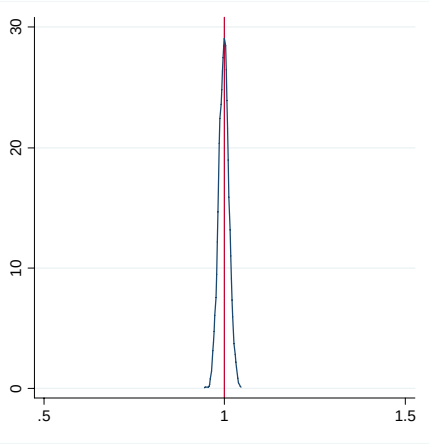
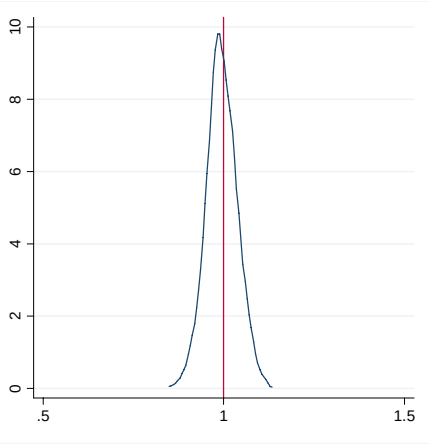
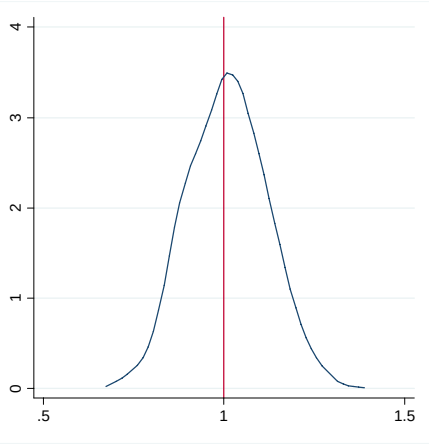
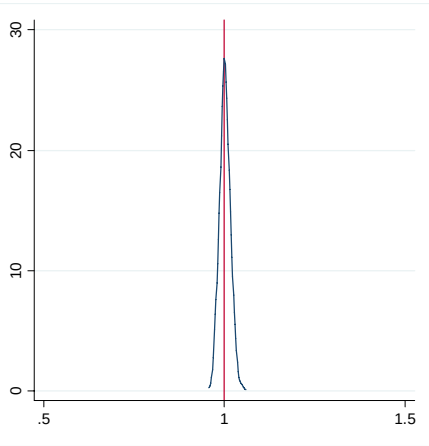
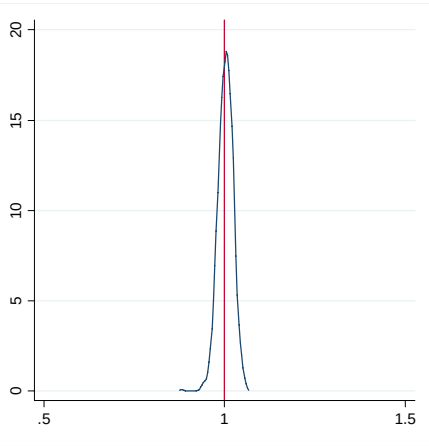
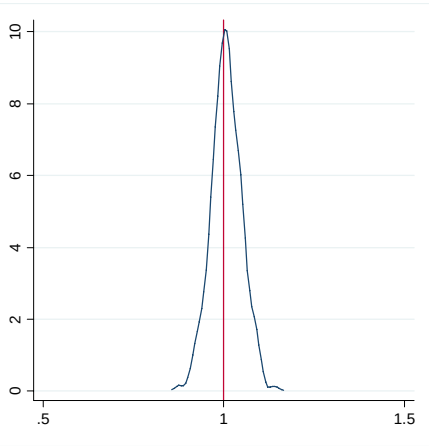
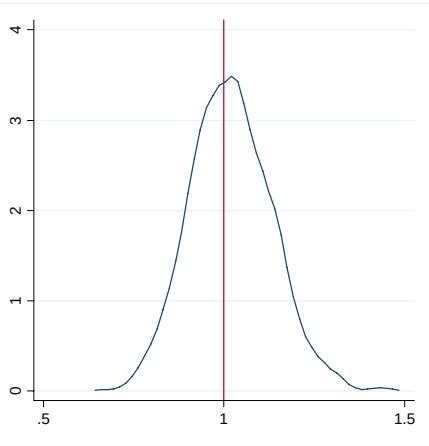
	δ_1	δ_2	δ_3	β_2
R1				
R2				

Figure B2. Kernel Density Estimates of the Sampling Distributions of the Estimates of Parameters (DGP: Heckman-Singer; 2500 simulated data points; R1 and R2 denotes risks 1 and 2)

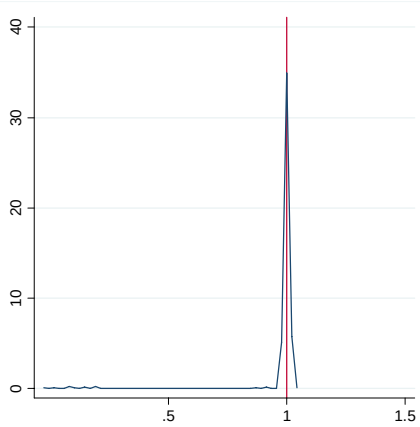
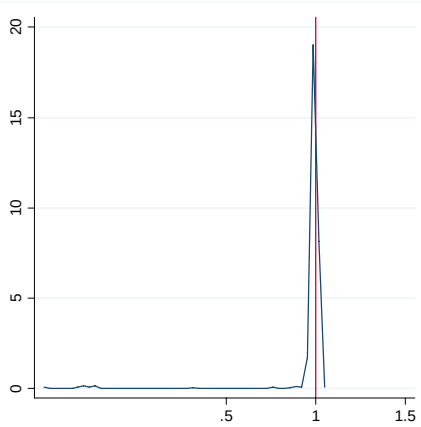
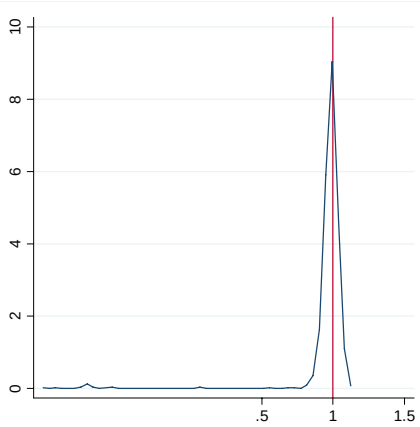
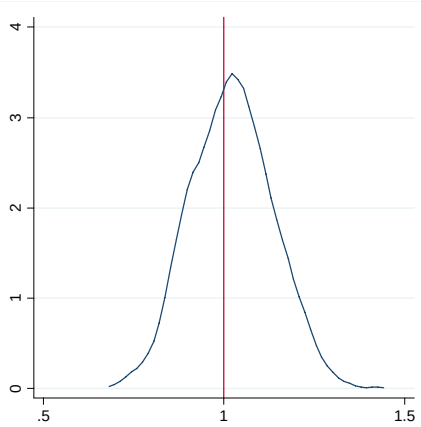
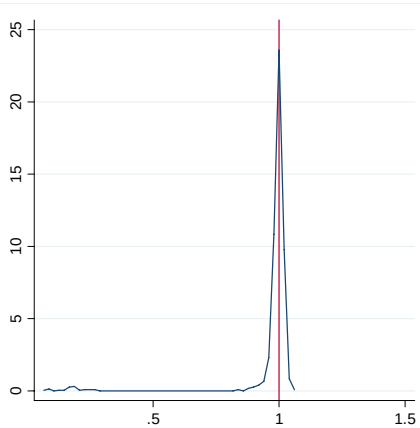
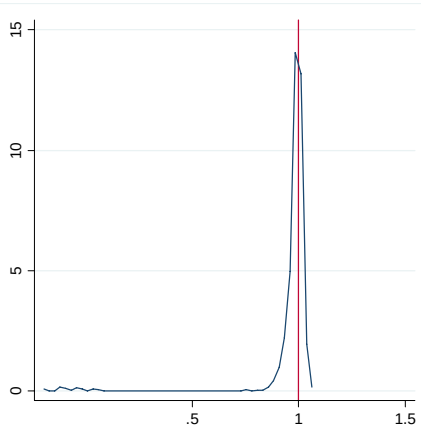
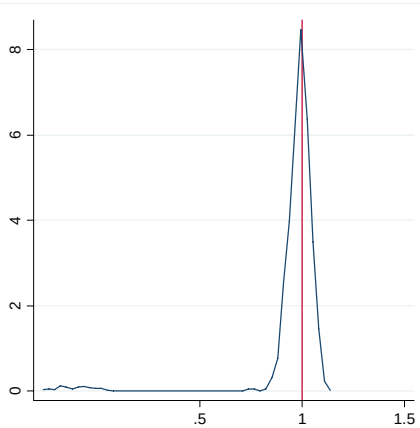
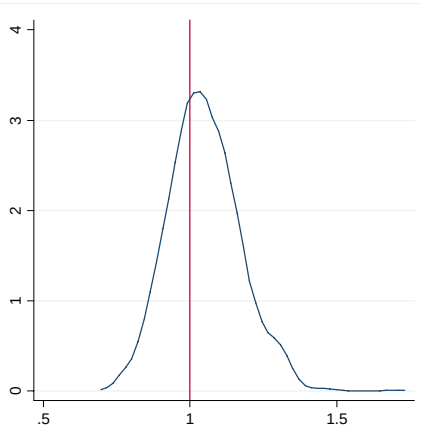
B2.1 Model 1: Heckman-Singer, No Unobserved Heterogeneity

	δ_1	δ_2	δ_3	β_2
R1				
R2				

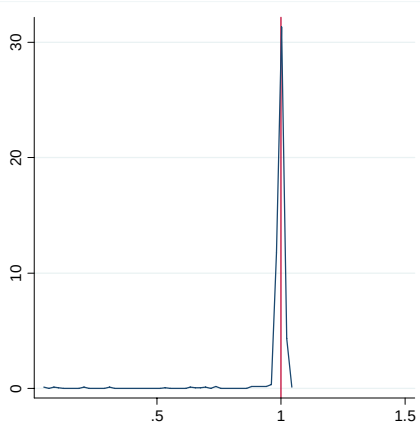
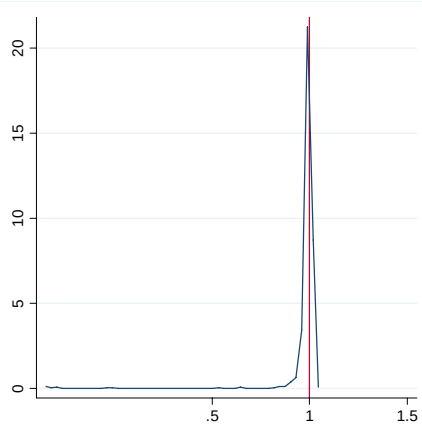
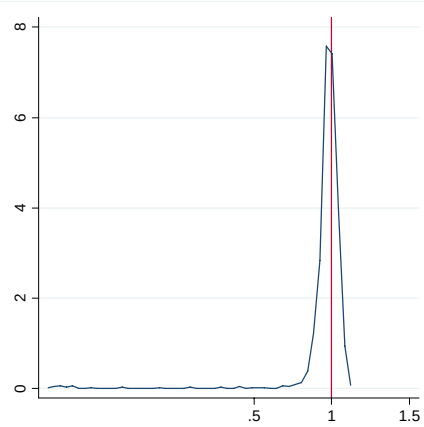
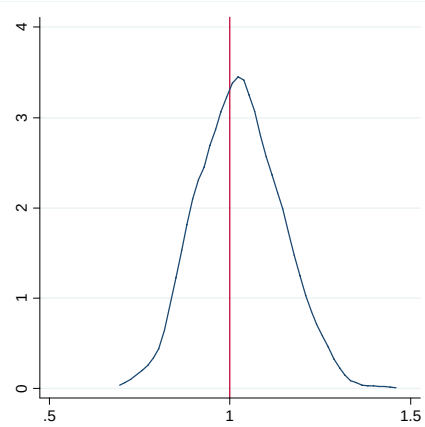
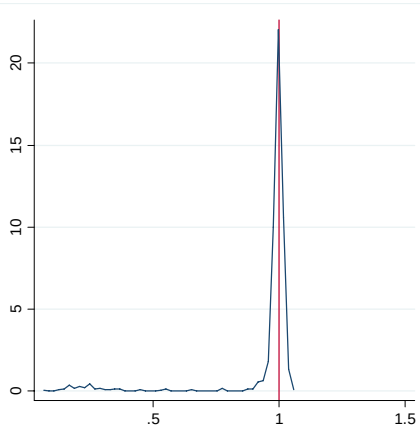
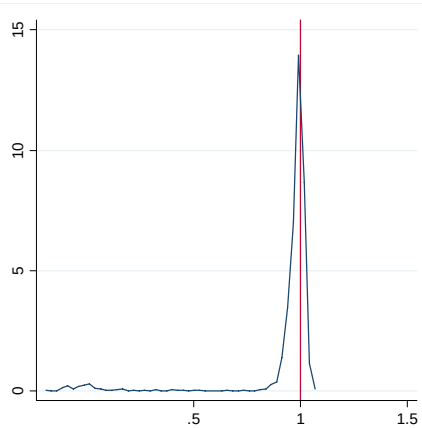
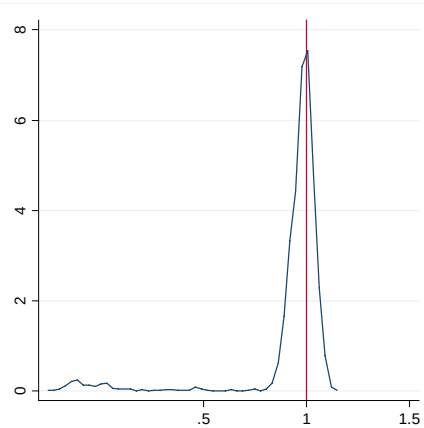
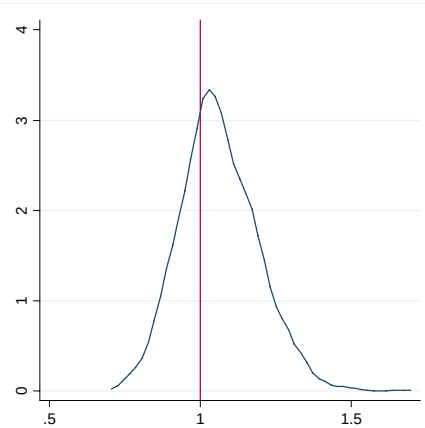
B2.2 Model 2: Heckman-Singer 2 points.

	δ_1	δ_2	δ_3	β_2
R1				
R2				

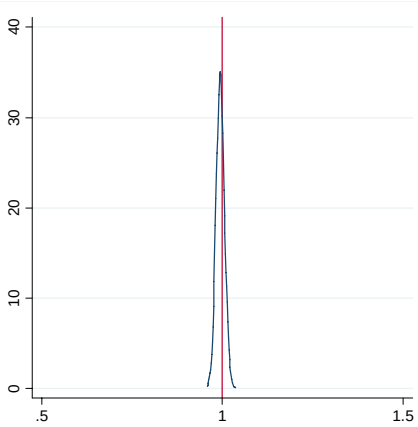
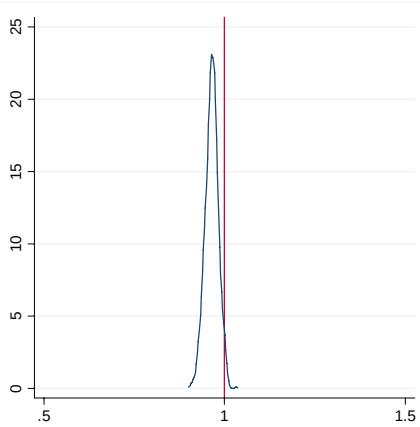
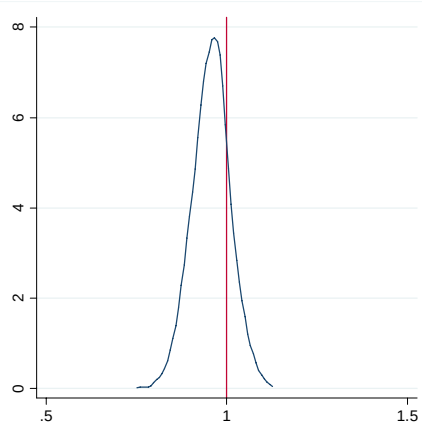
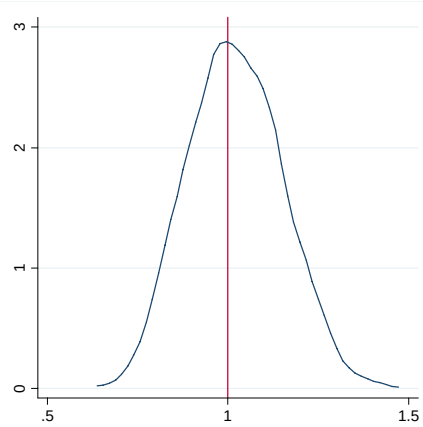
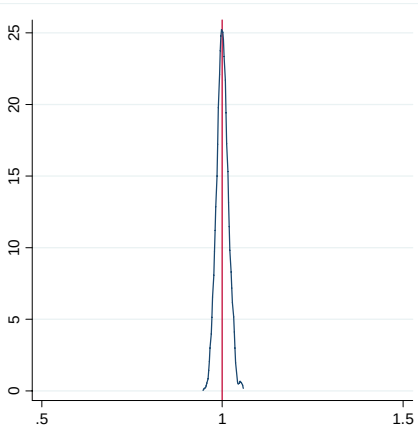
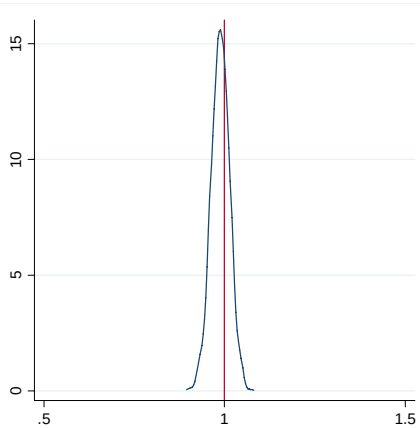
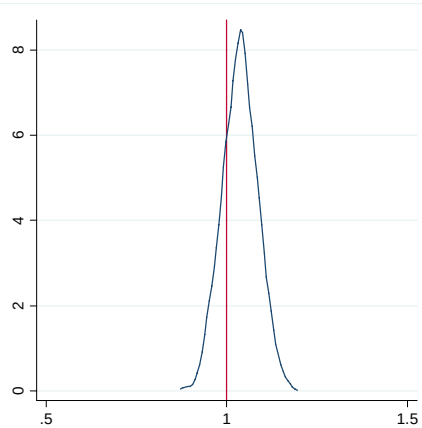
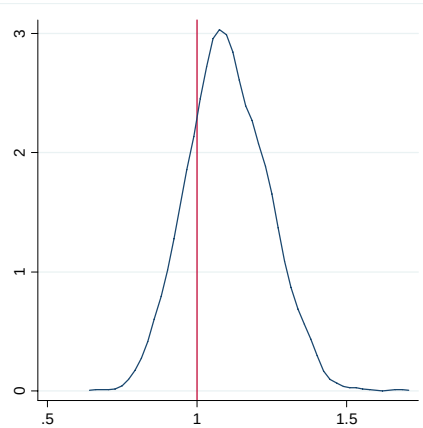
B2.3 Model 3: Heckman-Singer 3 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

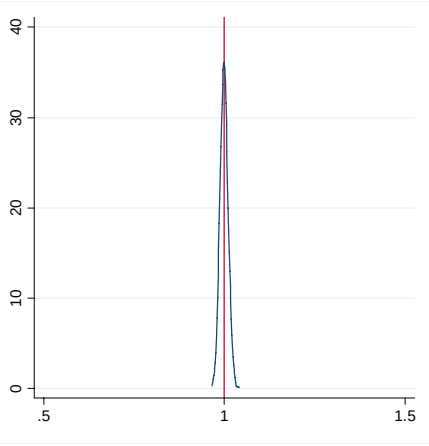
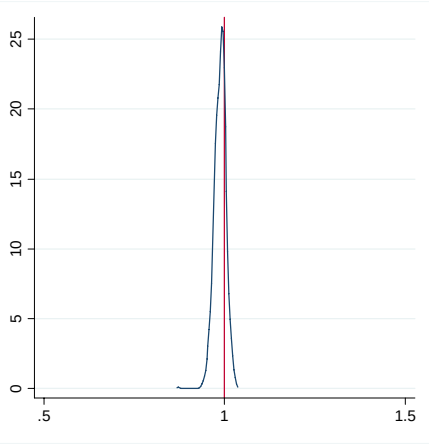
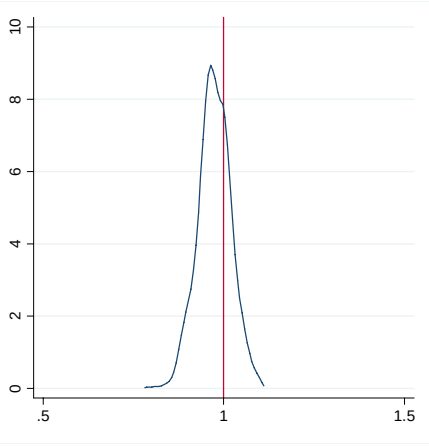
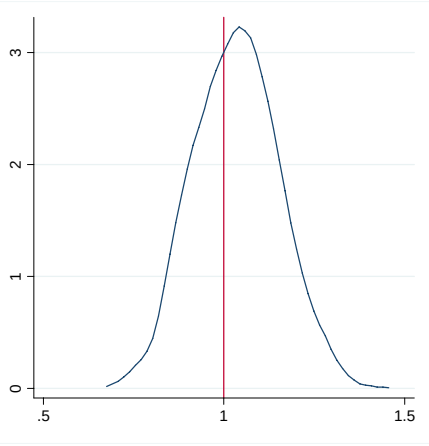
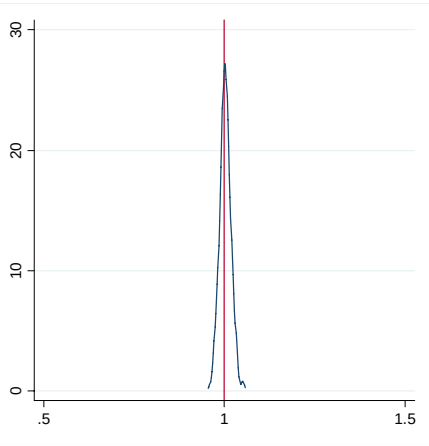
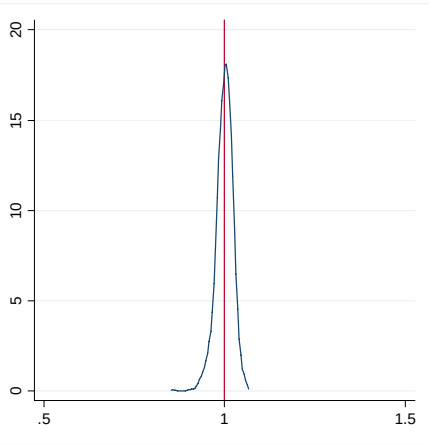
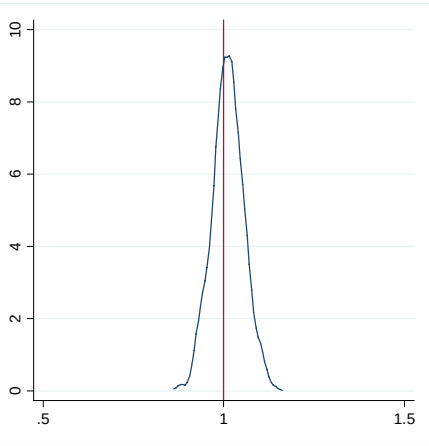
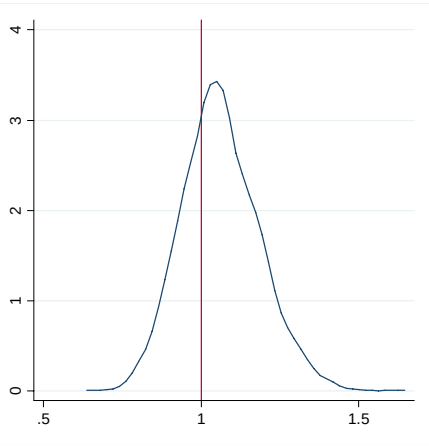
B2.4 Model 4: Heckman-Singer 4 Points.

	δ_1	δ_2	δ_3	β_2
R1				
R2				

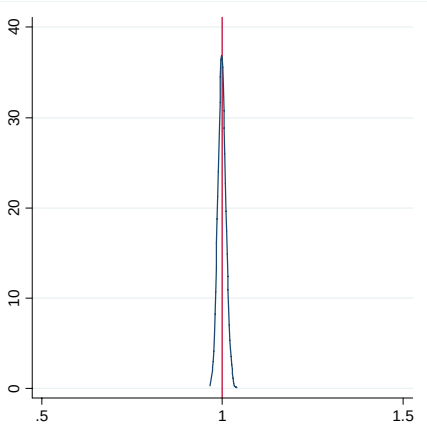
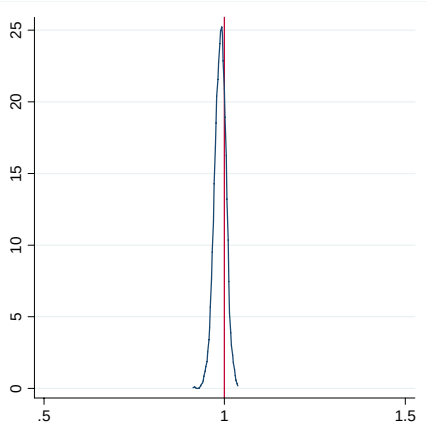
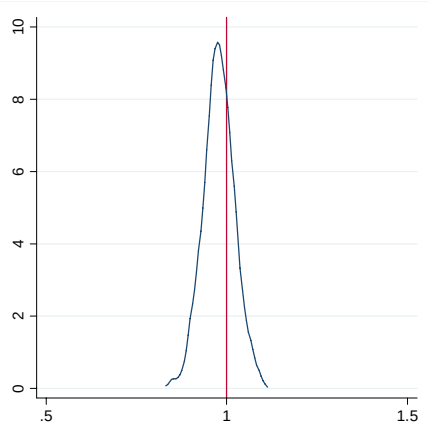
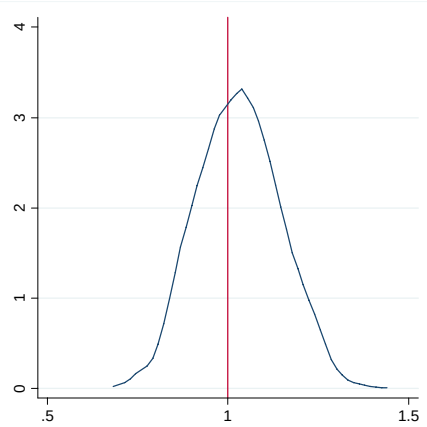
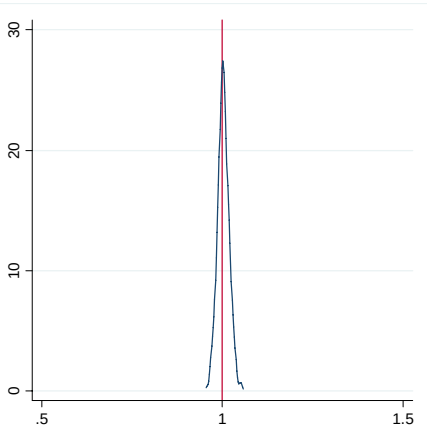
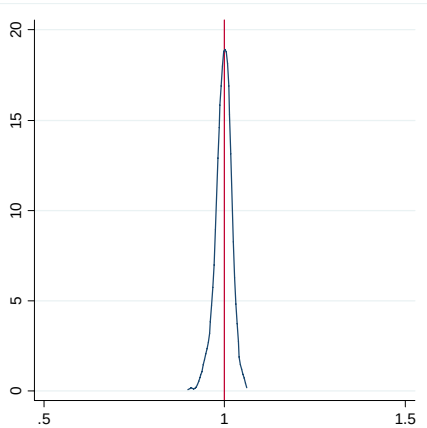
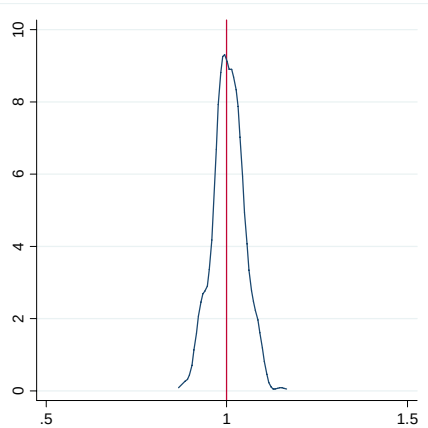
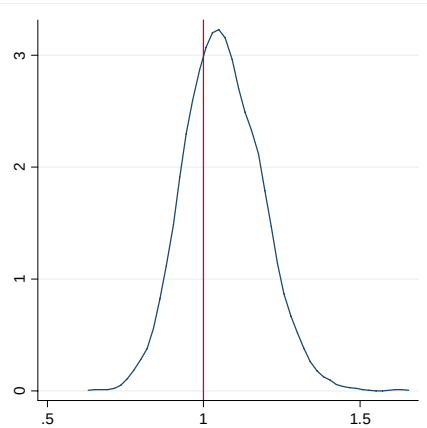
B2.5 Model 5: Polynomial Degree 0

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B2.6 Model 6: Polynomial Degree 1

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B2.7 Model 7: Polynomial Degree 2

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B2.8 Model 8: Polynomial Degree 3

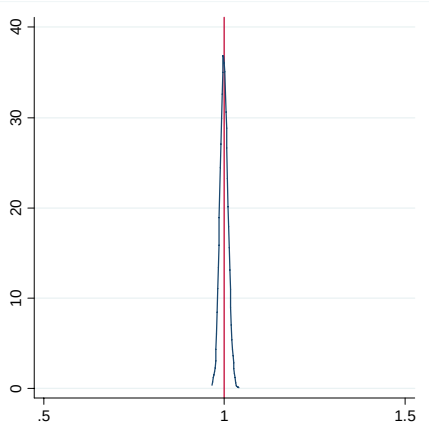
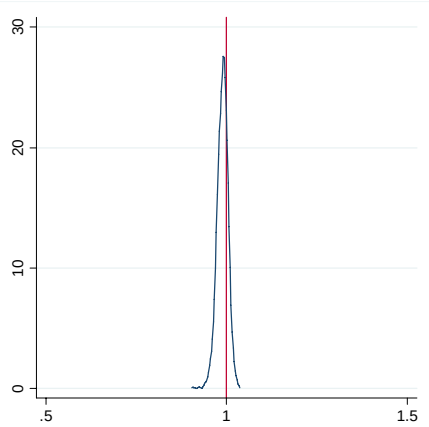
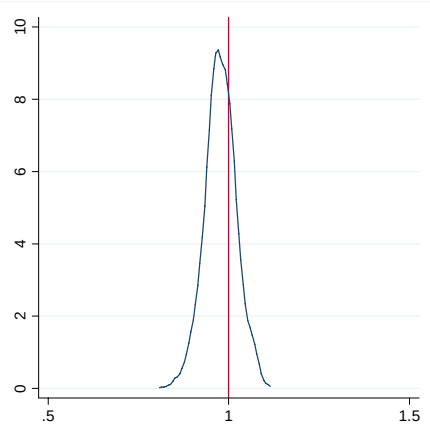
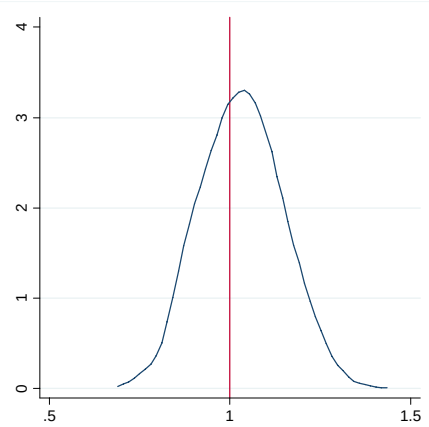
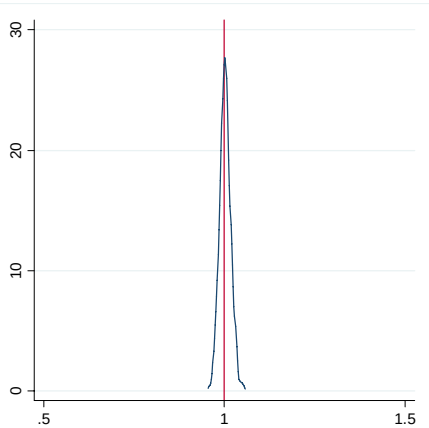
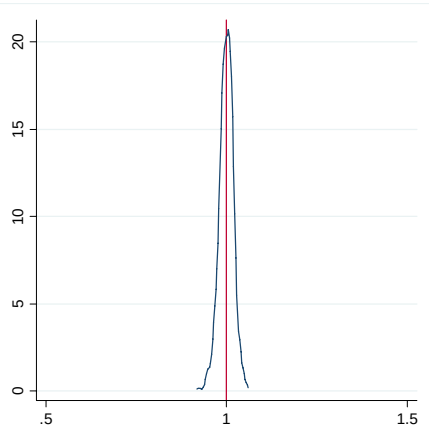
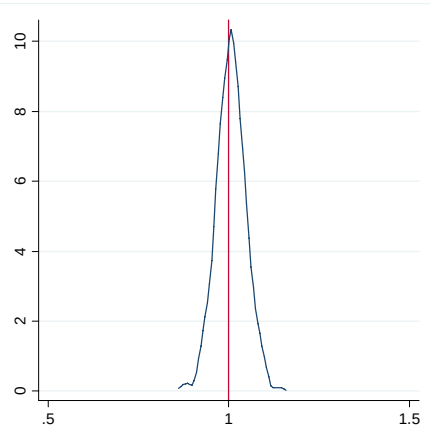
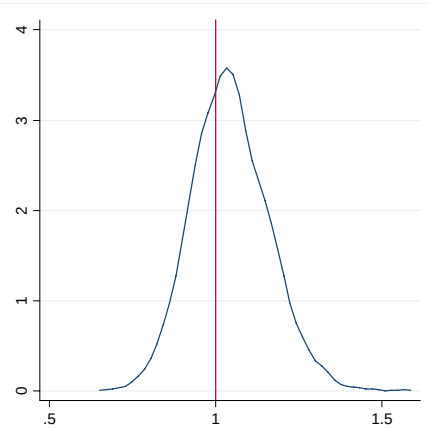
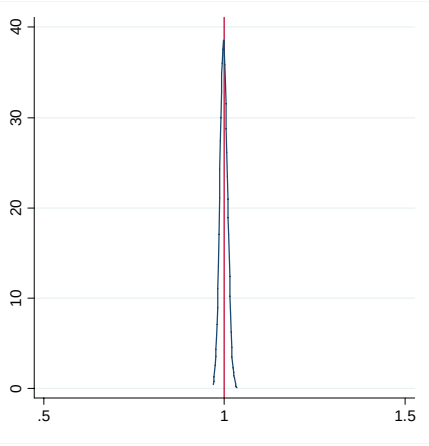
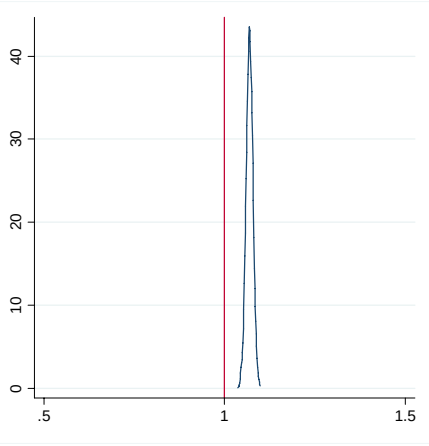
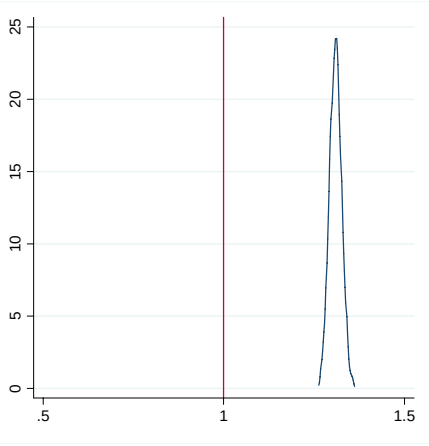
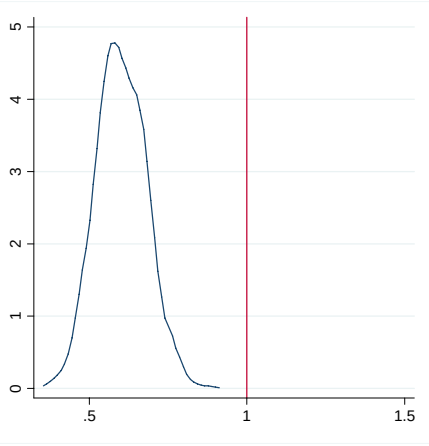
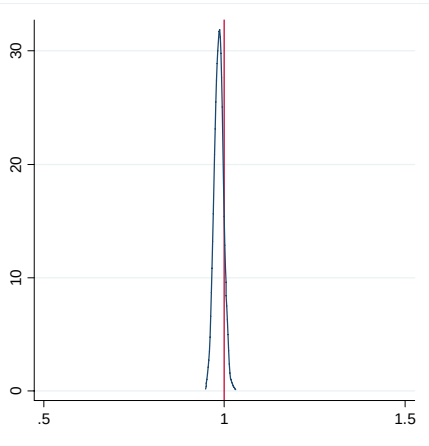
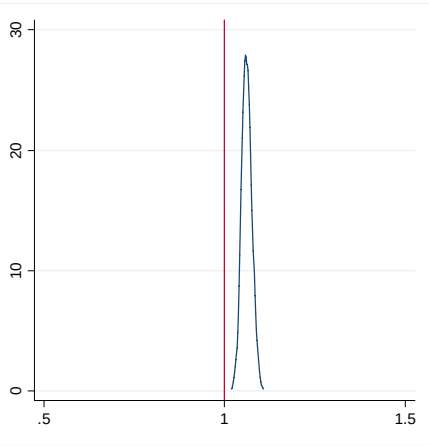
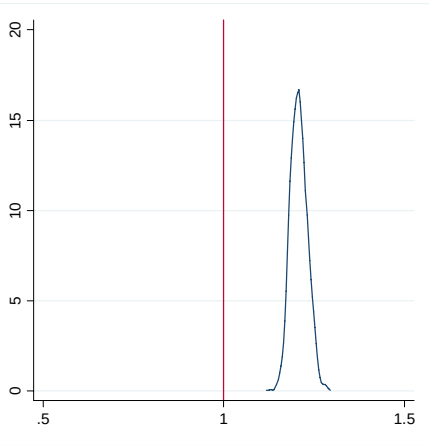
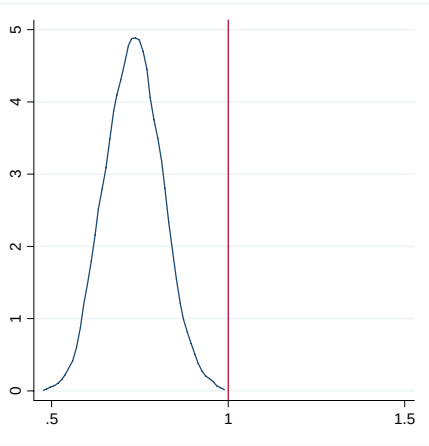
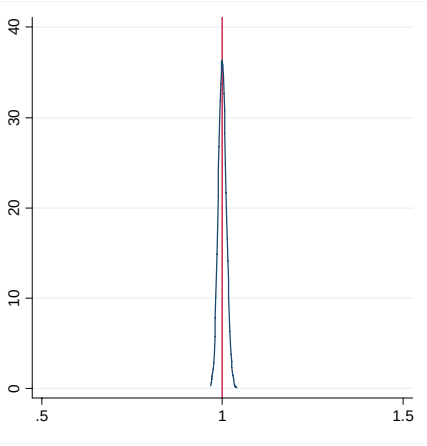
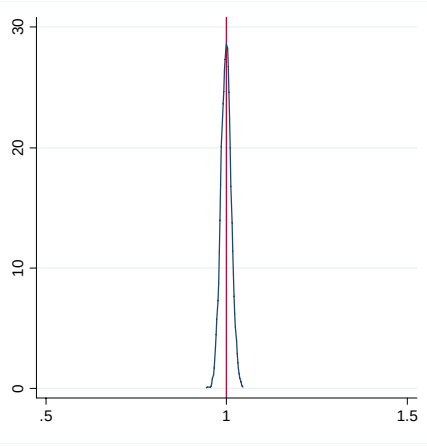
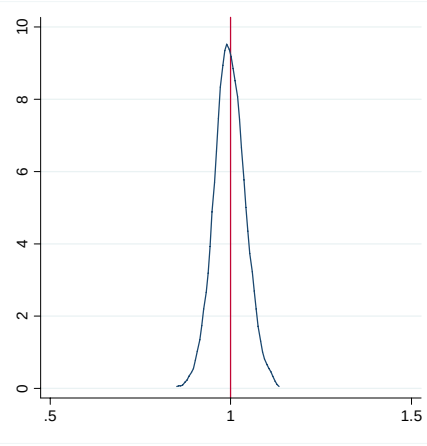
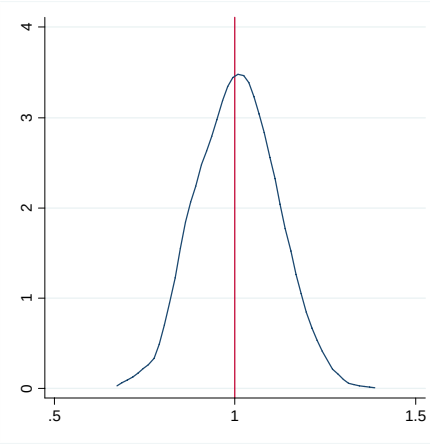
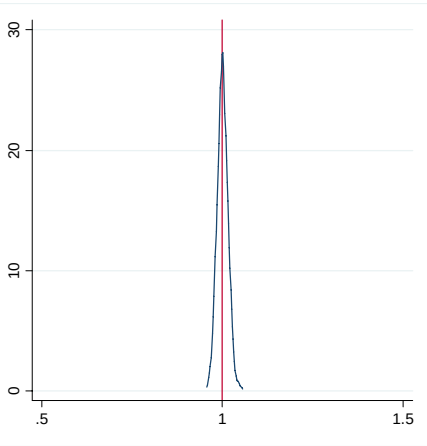
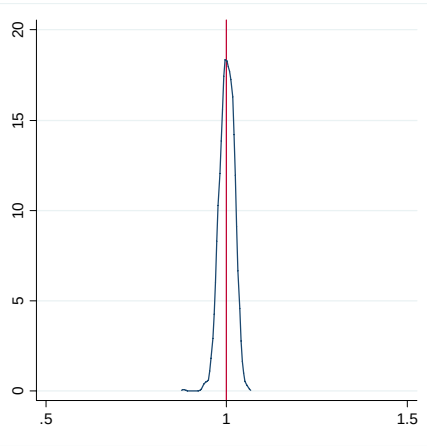
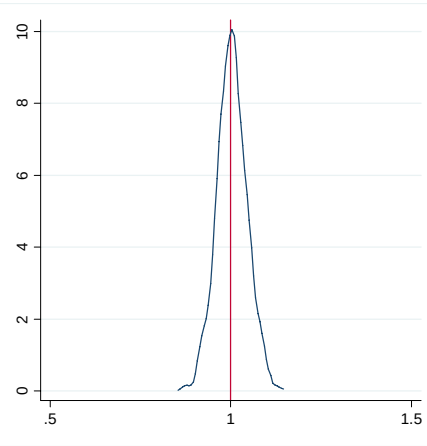
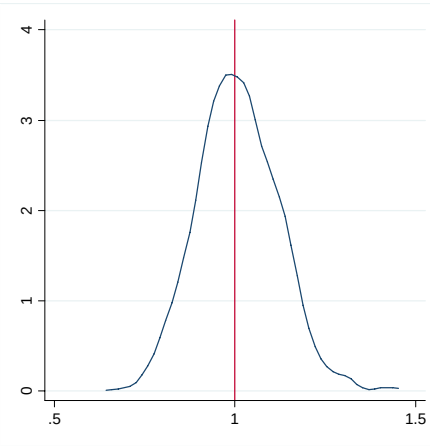
	δ_1	δ_2	δ_3	β_2
R1				
R2				

Figure B3. Kernel Density Estimates of the Sampling Distributions of the Estimates of Parameters (DGP: Lognormal; 2500 simulated data points; R1 and R2 denotes risks 1 and 2)

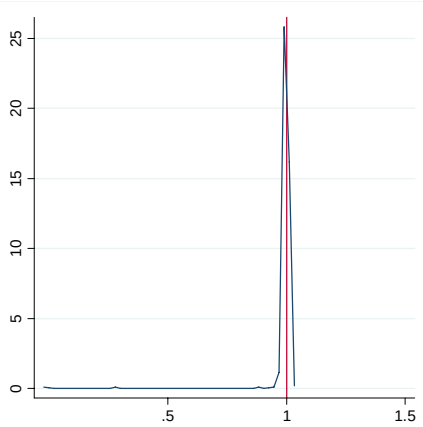
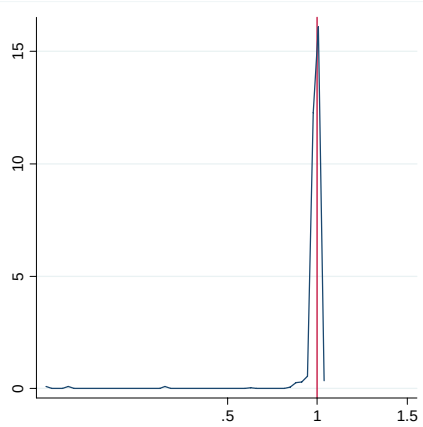
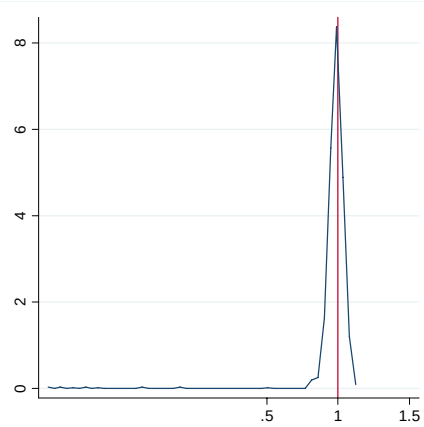
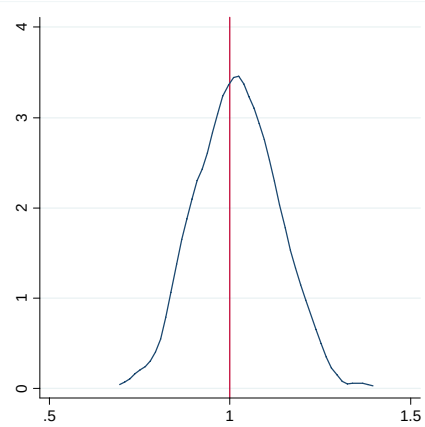
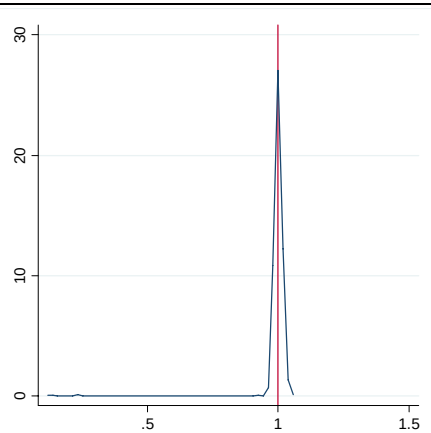
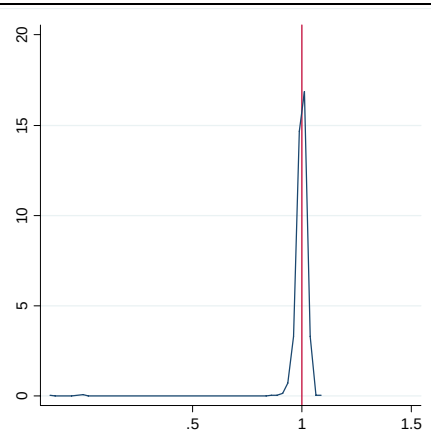
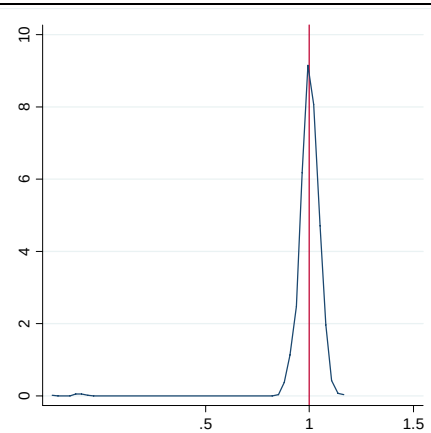
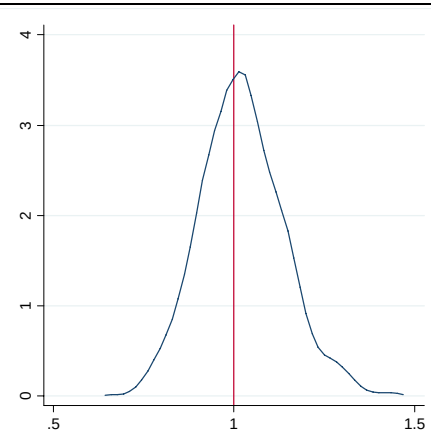
B3.1 Model 1: Heckman-Singer, No Unobserved Heterogeneity

	δ_1	δ_2	δ_3	β_2
R1				
R2				

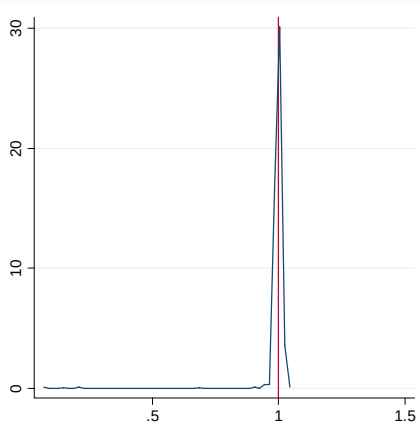
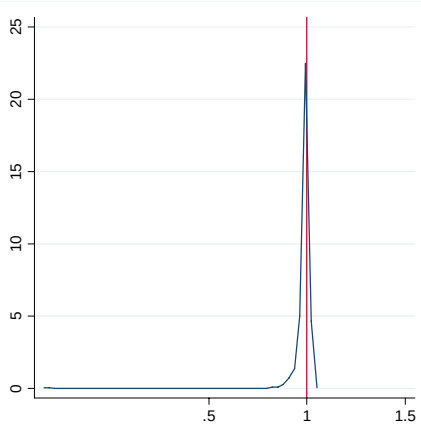
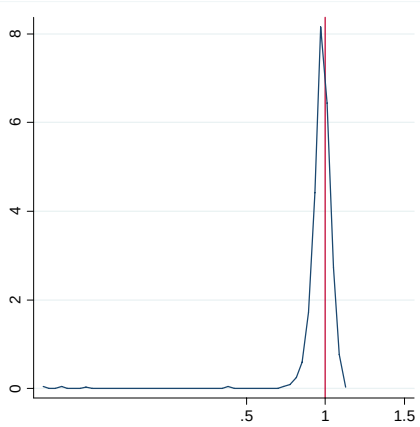
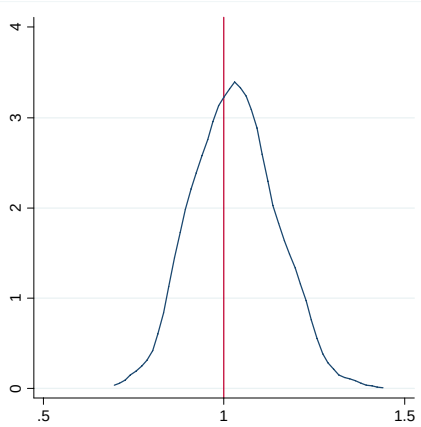
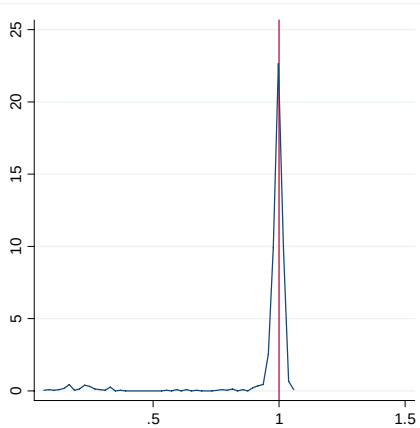
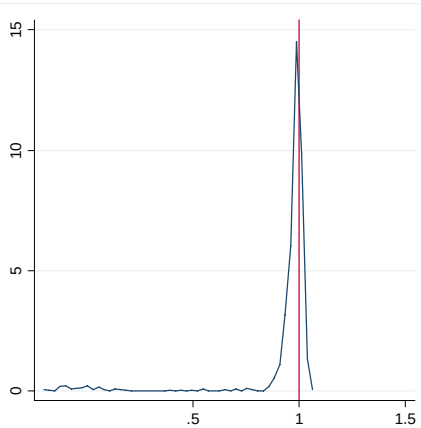
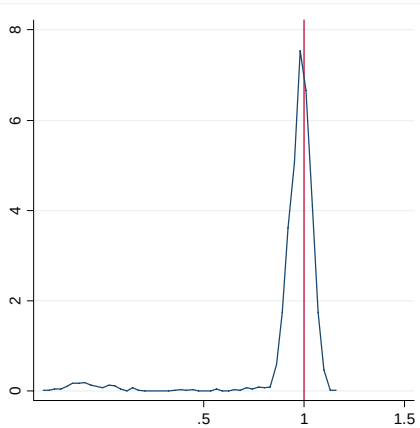
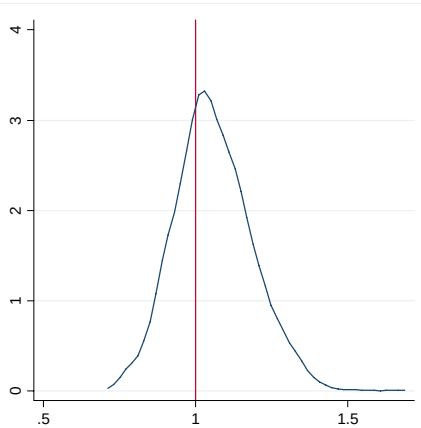
B3.2 Model 2: Heckman-Singer 2 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

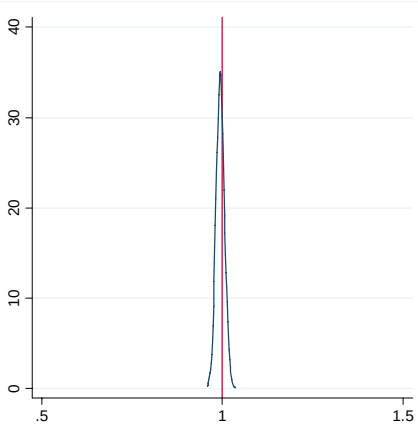
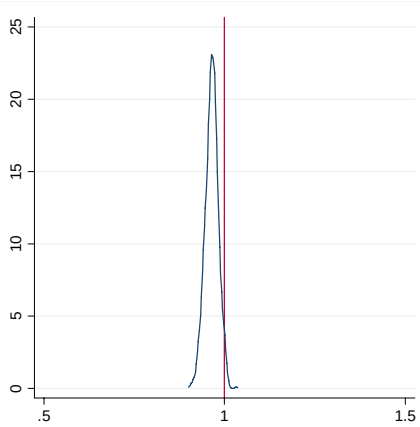
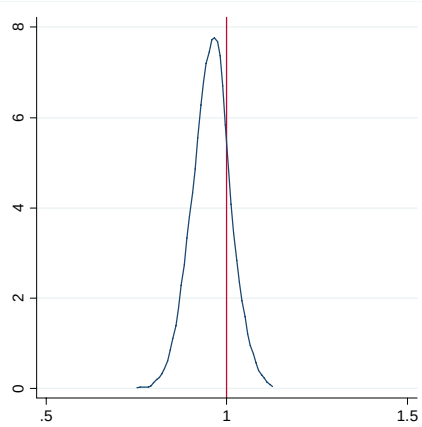
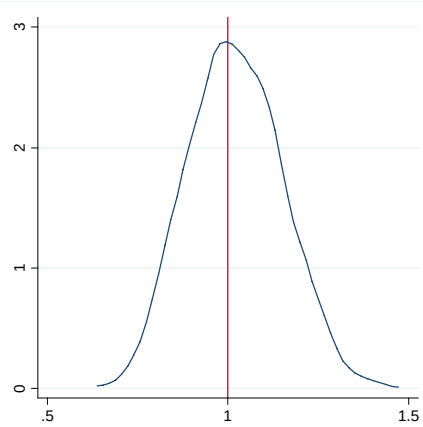
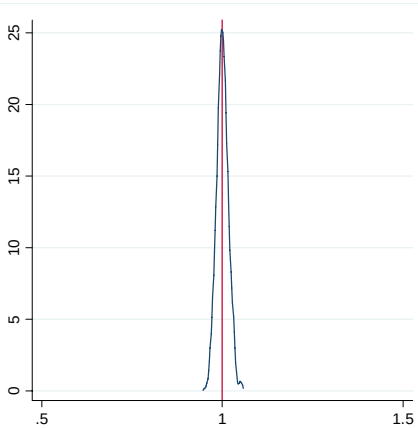
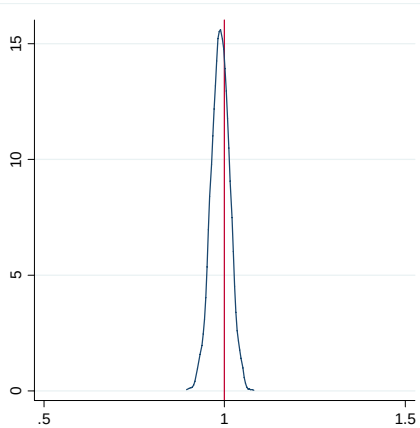
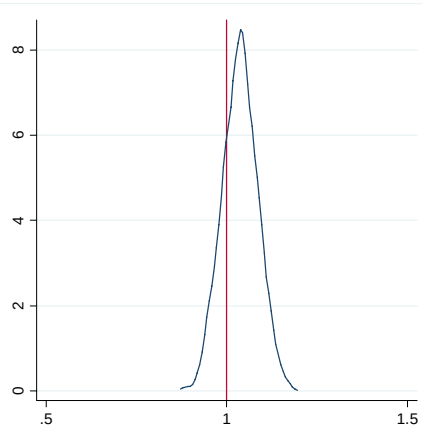
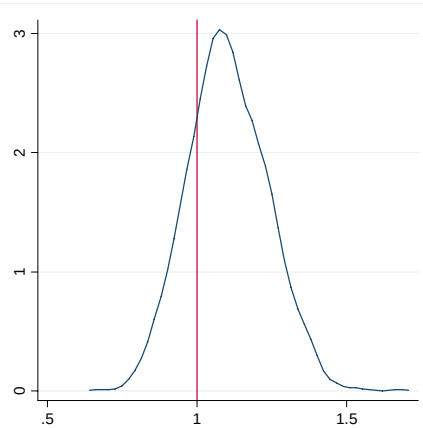
B3.3 Model 3: Heckman-Singer 3 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

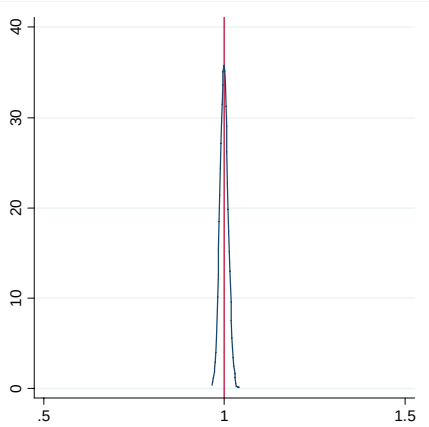
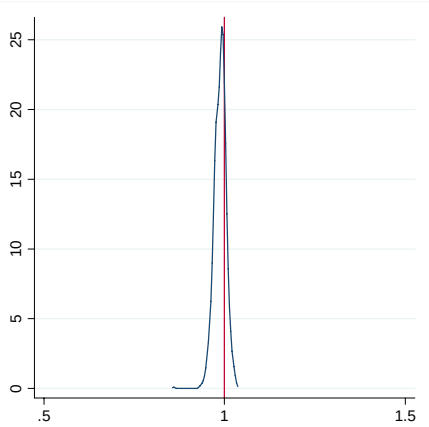
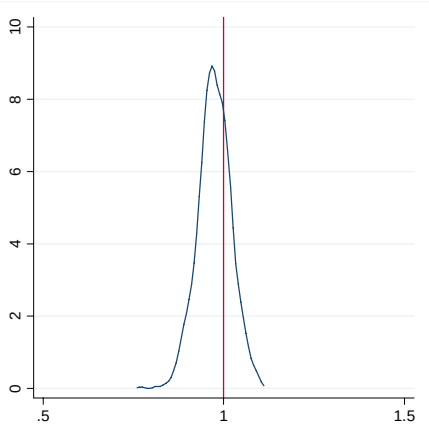
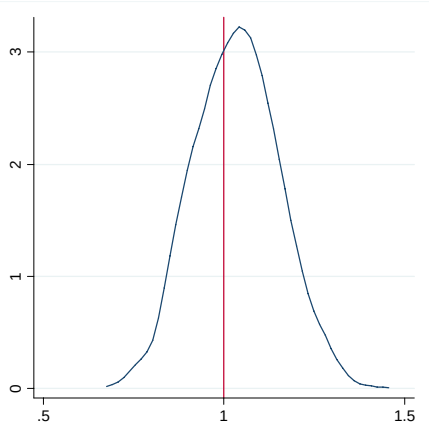
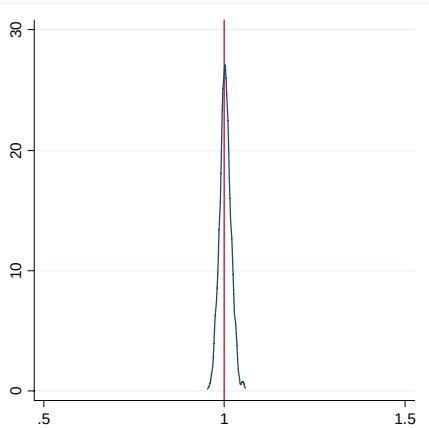
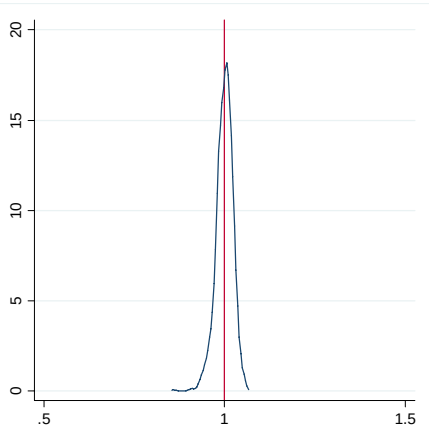
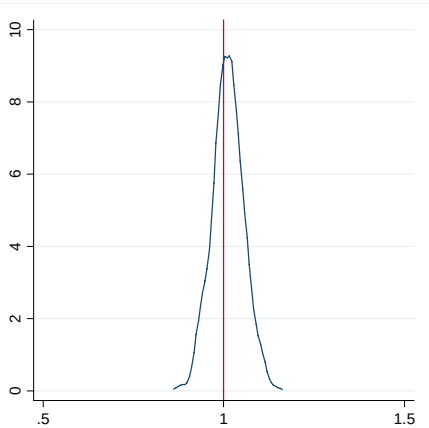
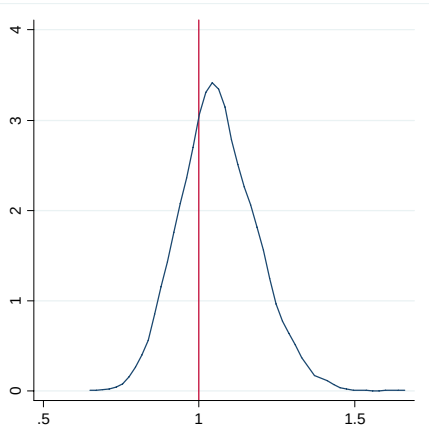
B3.4 Model 4: Heckman Singer 4 Points

	δ_1	δ_2	δ_3	β_2
R1				
R2				

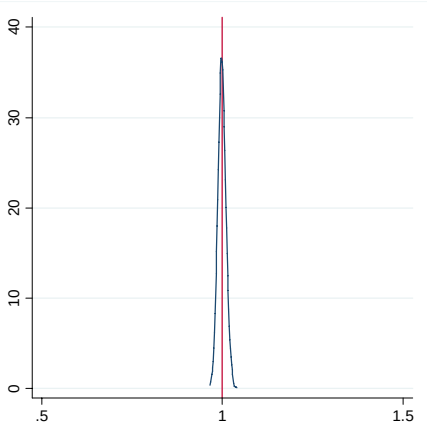
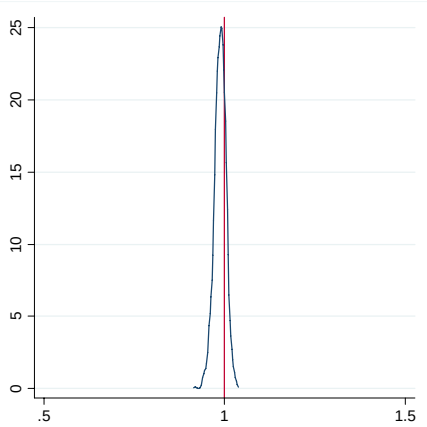
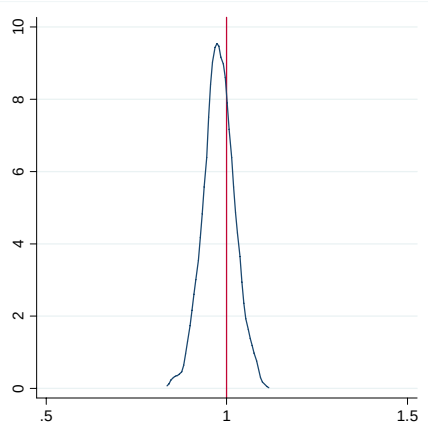
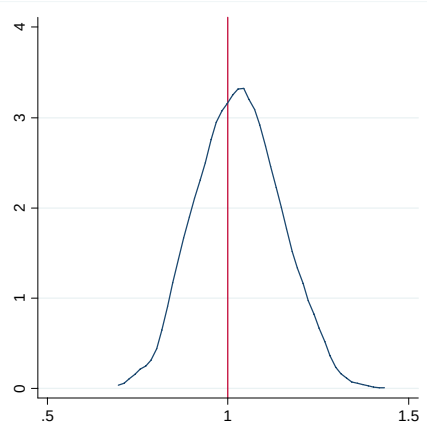
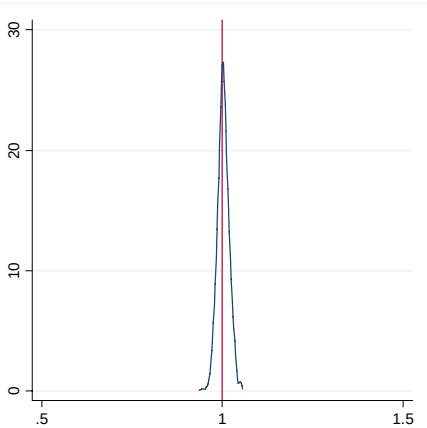
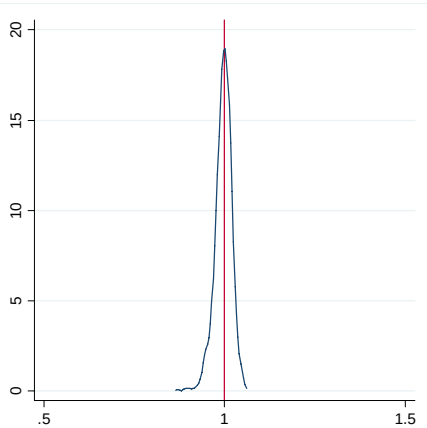
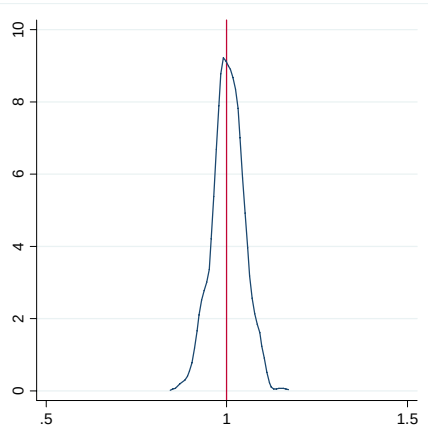
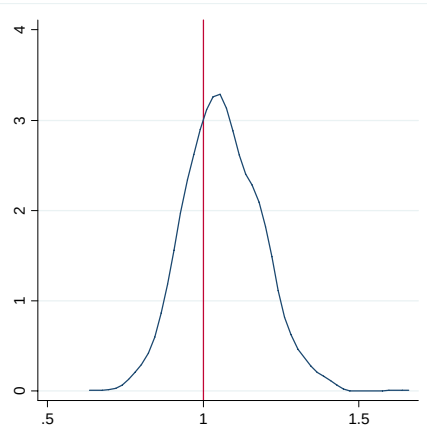
B3.5 Model 5: Polynomial Degree 0

	δ_1	δ_2	δ_3	β_2
R1				
R2				

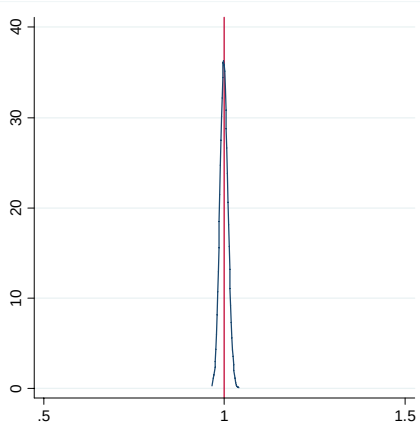
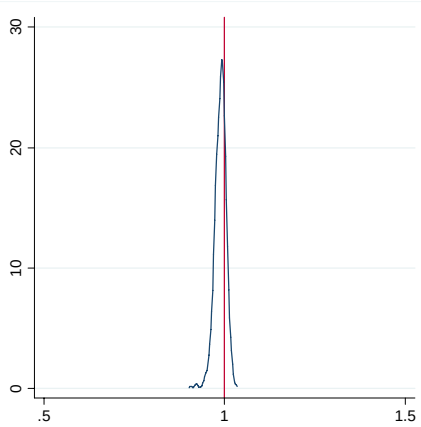
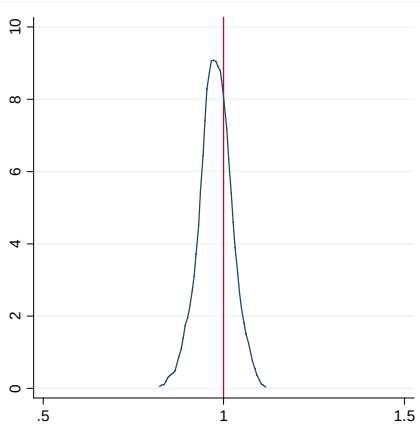
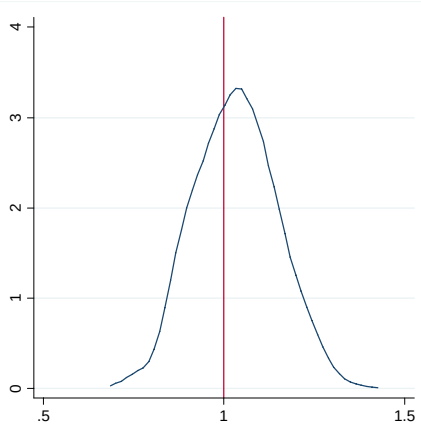
B3.6 Model 6: Polynomial Degree 1

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B3.7 Model 7: Polynomial Degree 2

	δ_1	δ_2	δ_3	β_2
R1				
R2				

B3.8 Model 8: Polynomial Degree 3

	δ_1	δ_2	δ_3	β_2
R1				
R2