Patents, R&D and Lag Effects: Evidence from Flexible Methods for Count Panel Data on Manufacturing Firms

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Appendices A and B
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Appendix A: Details on Unobserved Effects Count Data Models

Fixed Effects Methods

For the Poisson case, if $(y_{it} \mid x_{it}, \nu_i) \sim$ i.i.d. Poisson$(\theta_{it}\nu_i)$, then it can be shown that the conditional joint density for the *i-th* observation is

$$f(y_i \mid x_i, y_{i.}) = \frac{(y_{i}.)!}{\prod_{t=1}^{T} y_{it}!} \prod_{t=1}^{T} \left(\frac{\theta_{it}}{\theta_{i.}} \right)^{y_{it}}. \tag{A1}$$

The individual effects drop out upon conditioning. Hence, the conditional Poisson MLE of β can be obtained by maximizing the conditional likelihood function $\sum_{i=1}^{N} \log\ f(y_i \mid x_i, y_{i.})$, where $f(.)$ is given in (A1).[1] The first two moments of the conditional Poisson and other models presented below are given in Table B1 in Appendix B.[2]

Let ϕ denote the dispersion parameter of the negative binomial distribution such that $\delta_i^{-1} = \frac{\nu_i}{\phi_i}$. If $(y_{it} \mid x_{it}, \delta_i)$ is i.i.d. as negative binomial type 1 with mean θ_{it}/δ_i, the conditional maximum likelihood method can be used to estimate β. The Negbin conditional density for the *i-th* observation in which δ_i drops out can be obtained as

$$f(y_i \mid x_i, y_{i.}) = \frac{\Gamma(\theta_{i.})\,\Gamma(y_{i.}+1)}{\Gamma(\theta_{i.}+y_{i.})} \prod_{t=1}^{T} \frac{\Gamma(\theta_{it}+y_{it})}{\Gamma(\theta_{it})\,\Gamma(y_{it}+1)}. \tag{A2}$$

Given (A2), the ensuing log-likelihood function, which only involves β, is $\sum_{i=1}^{N} \log\ f(y_i \mid x_i, y_{i.})$. Table B1 shows that the mean of the conditional Negbin is the same as that of the conditional Poisson. As compared to Poisson fixed effects model, the Negbin fixed effects model has more general variance-covariance structure.

Mixture Models

Consider the Poisson specification with density

$$f(y_{it} \mid x_{it}, \nu_i) = \exp(-\theta_{it}\nu_i)\,(\theta_{it}\nu_i)^{y_{it}} / \Gamma(y_{it}+1),$$

where $\theta_{it} = \exp(x_{it}'\beta)$ as before. Assume that ν_i is independent of the observed covariates and that y_{it} and y_{is} are independent conditional on x_i and ν_i. Since ν_i is unob-

[1] The first order conditions for the conditional Poisson ML estimator is $\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it} - \frac{y_{i.}}{\theta_{i.}}\theta_{it}\right)x_{it}$. See Cameron and Trivedi (1998) and references there in for a discussion on how these conditions have been used as a basis of estimation using the method of moments.

[2] Except for the conditional Poisson, the moments for the other models have not been provided explicitly in the literature.

servable, we need to integrate it out. This gives the Poisson random effects (Poisson-Gamma) model:

$$f(y_i \mid x_i) = \left[\Pi_{t=1}^{T} \frac{\theta_{it}^{y_{it}}}{\Gamma(y_{it}+1)}\right] \frac{\Gamma(y_{i.}+\alpha)}{\Gamma(\alpha)} \alpha^{-y_{i.}} \left(1+\frac{\theta_{i.}}{\alpha}\right)^{-(\alpha+y_{i.})}, \tag{A3}$$

The unknown parameter vector for Poisson-Gamma mixture model is $(\beta\ \alpha)$. In the Poisson random effect specification with $E(y_{it} \mid x_i, \nu_i) = \exp(x_{it}'\beta)\nu_i$, if x_{it}'s are constant, we have randomness only across individuals. If x_{it}'s are constant, there is no variation across time. This is a potential problem with the Poisson random effects model given in (A3). We consider alternative models that exhibit randomness both across individuals and across time.

The most common distribution for ξ_i in (4) of the paper is the beta density

$$g_b(\xi_i) = \frac{1}{B(a,b)} \xi_i^{a-1}(1-\xi_i)^{b-1}, \tag{A4}$$

where $B(.)$ is the beta function:

$$B(\gamma,\omega) = \frac{\Gamma(\gamma)\Gamma(\omega)}{\Gamma(\gamma+\omega)}.$$

In this case, it can readily be shown that

$$\Xi(\theta_{i.}, y_{i.}) = \frac{B(\theta_{i.}+a, y_{i.}+b)}{B(a,b)},$$

and (**??**) reduces to the Negbin random effects model:

$$f(y_i \mid x_i) = \left[\Pi_{t=1}^{T} \frac{\Gamma(y_{it}+\theta_{it})}{\Gamma(\theta_{it})\Gamma(y_{it}+1)}\right] \frac{B(\theta_{i.}+a, y_{i.}+b)}{B(a,b)} \tag{A5}$$

The unknown parameters of the Negbin-Beta model are (β, a, b). The moments given in Table B1 shows that the Negbin-Beta specification is more flexible than that of the Poisson-gamma specification.

In the context of random effects generalized linear models, Liang and Zeger (1986) and Zeger, Liang and Albert (1988) have proposed population-averaged mixed models in which serial correlations are allowed for but the random effects are averaged out. In the empirical section, we estimate population-averaged panel data model based on the Negbin family using GEE. The estimation approach is based on the first two moments of the Negbin distribution, incorporating unobserved effects and serial correlation. As before, let y_i (θ_i) denote a $T \times 1$ vector with *t-th* element y_{it} (θ_{it}). The first-order conditions take the form

$$\sum_{i=1}^{N} D' V_i \left(y_i - \theta_i\right) = 0, \tag{A6}$$

where $D = [diag(\theta_{it})x_i]$ is a $T \times p$ matrix and V_i is a $T \times T$ weighting matrix involving the mean parameters, correlation parameters , and dispersion or scale parameter ϕ. Depending on the assumed correlation structure, the dispersion parameter as well as the correlation parameters can be estimated iteratively using Pearson residuals.

Moments for SPJ Model

To develop the moments associated with the SPJ model, define

$$\varpi(\delta_1, \delta_2) = \frac{1}{\sum_{j=0}^{K} d_j^2} \sum_{j=0}^{K} \sum_{k=0}^{K} \sum_{l=0}^{j} \sum_{m=0}^{k} d_j d_k \Delta_{jk} \Psi_{lm} \text{ B}(\delta_1 + a + j + k - l - m, \ \delta_2 + b) \quad \text{(A7)}$$

for arbitrary constants $\delta_1$and δ_2. The first and second order moments of the SP2 density are also shown in Table B1.

Appendix B: Tables

Table B1: Moments of Some Panel Count Data Models

Model	Mean$(y_{it} \mid x)$ Or Mean$(y_{it} \mid x, y_{i.})$	Var$(y_{it} \mid x)$ Or Var$(y_{it} \mid x, y_{i.})$	Cov$(y_{it}, y_{is} \mid x)$ Or Cov$(y_{it}, y_{is} \mid x, y_{i.})$
Conditional Poisson	$\theta_{it} \frac{y_{i.}}{\theta_{i.}}$	$y_{i.} \left(\frac{\theta_{it}}{\theta_{i.}}\right) \left(1 - \frac{\theta_{it}}{\theta_{i.}}\right)$	$- y_{i.} \left(\frac{\theta_{it}}{\theta_{i.}}\right) \left(1 - \frac{\theta_{is}}{\theta_{i.}}\right)$
Conditional Negbin	$\theta_{it} \frac{y_{i.}}{\theta_{i.}}$	$y_{i.} \left(\frac{\theta_{it}}{\theta_{i.}}\right) \left(1 - \frac{\theta_{it}}{\theta_{i.}}\right) \left(\frac{y_{i.}+\theta_{i.}}{1+\theta_{i.}}\right)$	$- y_{i.} \left(\frac{\theta_{it}}{\theta_{i.}}\right) \left(1 - \frac{\theta_{is}}{\theta_{i.}}\right) \times \left(\frac{y_{i.}+\theta_{i.}}{1+\theta_{i.}}\right)$
Poisson-Gamma	θ_{it}	$\theta_{it} + \alpha^{-1}\theta_{it}^2$	$\alpha^{-1}\theta_{it}\theta_{is}$
Negbin-Beta	$(a-1)^{-1} b\theta_{it}$	$\frac{(a+b-1)(a+\theta_{it}-1)b\theta_{it}}{(a-1)^2(a-2)}$	$\frac{(a+b-1)b\theta_{it}\theta_{is}}{(a-1)^2(a-2)}$
SPJ	$\theta_{it}\varpi(-1,1)$	$\theta_{it}\left[\varpi(-1,1)\left(1-\theta_{it}\varpi(-1,1)\right) + (1+\theta_{it})\,\varpi(-2,2)\right]$	$\theta_{it}\theta_{is}\left[\varpi(-2,2) - \varpi^2(-1,1)\right]$

Note: The moments for the conditional Poisson and Negbin are obtained by additionally conditioning on $y_{i.} = \sum_{t=1}^{T} y_{it}$.

Table B2: Estimates from GMM I[c] Specifications (Details)

Variable	(1)		(2)		(3)		(4)		(5)	
log R&D$_t$	0.403	(5.24)	0.441	(4.83)	0.373	(2.87)	0.687	(4.00)	0.692	(2.68)
log R&D$_{t-1}$	0.176	(2.23)	-0.096	(0.76)	-0.063	(0.40)	-0.215	(0.98)	-0.313	(1.22)
log R&D$_{t-2}$			0.234	(2.58)	0.386	(2.42)	0.322	(1.76)	0.488	(1.96)
log R&D$_{t-3}$					-0.140	(1.05)	0.194	(1.01)	0.259	(1.27)
log R&D$_{t-4}$							-0.478	(2.54)	-0.236	(0.64)
log R&D$_{t-5}$									-0.398	(1.03)
Sum of R&D Elasticity	0.579		0.580		0.555		0.511		0.493	
GMM J-Statistics and [P-values]	88.2 [0.225]		65.14 [0.335]		53.6 [0.177]		36.1 [0.278]		22.3 [0.272]	

[a] Absolute value of t-statistic.

[b] All models include year dummies.

[c] Two-step quasi-differenced GMM estimator using z_{it}^1 as instruments.

Table B3: Estimates from GMM II[c] Specifications (Details)

Variable	(1)		(2)		(3)		(4)		(5)	
log R&D$_t$	0.599	(19.55)	0.563	(17.39)	0.534	(14.38)	0.542	(13.19)	0.485	(10.43)
Patent$_{t-1}$	0.036	(0.94)	0.039	(0.79)	0.069	(1.25)	0.054	(0.84)	0.192	(2.49)
Patent$_{t-2}$			0.114	(2.67)	0.177	(3.10)	0.143	(2.17)	0.091	(1.03)
Patent$_{t-3}$					0.157	((2.92)	0.051	(0.84)	0.054	(0.77)
Patent$_{t-4}$							0.085	(1.98)	0.153	(2.70)
Patent$_{t-5}$									-0.012	(0.22)
Sum of Patent	0.036		0.152		0.402		0.333		0.477	
GMM J-Statistics	105.3		89.1		76.4		66.9		49.3	
and [P-values]	[0.265]		[0.359]		[0.371]		[0.281]		[0.463]	

[a] Absolute value of t-statistic.

[b] All models include year dummies.

[c] Two-step quasi-differenced GMM (LFM) estimator using z_{it}^2 as instruments.

Table B5: Summary Statistics for Patent Numbers - Comparison with HGH Data

Statistic	1982-1992 sample	1972-1979 sample (HGH Data)
Mean	36.8	26.3
Standard deviation	96.0	67.8
First quartile	1	1
Median	5	4
Third quartile	24	19
Fraction of zeros	0.16	0.23
Fraction of at least 100	0.10	0.07

Description: Patents are utility patents granted to firms as distributed by year of application filing.

Table B6: Autoregressive Estimates for log R&D

Equation	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
log R&D$_{t-1}$	0.994 (433.560)	1.121 (60.451)	1.118 (59.828)	1.116 (60.007)	1.110 (59.572)	1.107 (59.357)	1.099 (58.891)	1.115 (60.000)	1.108 (59.632)
log R&D$_{t-2}$		−0.128 (6.885)	−0.107 (3.917)	−0.116 (4.258)	−0.129 (6.988)	−0.126 (6.760)	−0.114 (6.123)	−0.130 (6.999)	−0.120 (6.441)
log R&D$_{t-3}$			−0.018 (1.013)	0.088 (3.339)					
log R&D$_{t-4}$				−0.095 (5.443)					
log Patent$_t$					0.019 (3.155)	0.023 (3.680)	0.026 (4.099)		
log Patent$_{t-1}$					−0.001 (0.231)	0.005 (0.755)	0.013 (1.882)	0.016 (2.721)	0.025 (3.854)
log Patent$_{t-2}$						−0.013 (2.068)	0.002 (0.316)	−0.005 (0.835)	0.010 (1.399)
log Patent$_{t-3}$							−0.009 (1.254)		−0.006 (0.895)
log Patent$_{t-4}$							−0.022 (3.623)		−0.022 (3.505)
$\bar{R}^2$	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986

Notes:

1. Absolute values of t-statistic are in parenthesis.
2. All equations contain a separate intercept for each year.
3. We added 1/3 to the patent variable before taking the log due to the presence of zeros.