Theoretical Explanations of Treatment Effects in Voluntary Contributions Experiments

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Introduction

Public goods experiments are notable in that they produce an array of systematic treatment effects that are inconsistent with the predictions of standard game theory. In response, theorists have proposed alternative models designed to explain these interesting (and often intuitive) patterns in the data. This paper surveys several of these models, and compares their predictions with some of the stylized facts that have emerged from laboratory experiments. In these experiments, a fixed number of individuals decide simultaneously how to divide a resource endowment between private consumption and contribution toward a public good that benefits all individuals equally.

Our focus will be on the most common type of experiment, where the value of the public good is a *linear* function of total contributions. The marginal value of the public good relative to that of the private good is often called the MPCR, which refers to the "marginal per-capita return" from investing in the public good. The MPCR is usually specified to be low enough so that it is individually optimal to contribute nothing, but high enough so that it is socially optimal to contribute fully. Therefore, the Nash equilibrium involves no contributions. The prediction of complete free riding is irrespective of (non-critical) changes in the marginal value of the public good, the number of participants, and the number of rounds, as long as this number is finite and known in advance.

The broad data patterns that have motivated the theoretical work are:[1]

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1) On average, contributions to the public good are a significant fraction of total endowment. In most treatments, more than half of all contribution decisions involve "splitting" the endowment between private consumption and contributions to the public good.
2) Increases in the MPCR are associated with higher contributions, especially for small groups (see the increases from corresponding red to blue bars in Figure 1).
3) An increase in group size is associated with higher contributions, at least for low values of the MPCR and low-to-moderate group sizes (see the left-to-right increases in bar heights on the left side of Figure 1). The group size effect is dominated by the MPCR effect in situations where the product of MPCR and group size is constant. For example, compare the contributions for (MPCR = .75, N=4) shown in the far left blue bar with those for (MPCR = .3, N=10) shown in the second red bar in Figure 1.
4) In most treatments, average contributions begin at about 50 percent of the endowment, and typically decline over time, although not always monotonically. However, contributions do not disappear, even after as many as 60 rounds.
5) As the total number of rounds in a session is varied, contributions toward the end of the session are no lower in long time horizon experiments (40 or 60 rounds) than in short time horizon experiments (10 or 20 rounds).

These observations indicate the Nash equilibrium prediction fails in two respects: the level of contributions deviates from the prediction, and average contributions respond to treatments that have no predicted effect in a Nash equilibrium. A number of explanations for these treatment effects have been proposed; these explanations include altruism, error, reciprocity, fairness, adaptive evolutionary responses, and tacit cooperation (perhaps combined with signals). Before discussing these modeling approaches, it is useful to review the Nash predictions.

In the standard linear voluntary contributions game, there are n individuals, each endowed with E tokens that must be divided between private and public consumption.

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[1] The evidence for many of these patterns is surveyed in Ledyard (1995). In particular, these patterns may be observed in the results reported in Isaac, Walker, and Williams (1994) and Saijo and Nakamura (1995).

The contribution of individual i to the public good is denoted by x_i, and the total contribution from all individuals is denoted by X. The value of private consumption is $v(E - x_i)$, where v is often normalized to be one. The value of the public good to each individual is mX. Thus the MPCR is m/v. The individual's monetary earnings, π_i, are calculated as:

$$\pi_i(x_i) = v(E - x_i) + mX = vE + (m - v)\, x_i + m\Sigma_{j\neq i}\, x_j. \quad (1)$$

The standard specification is for $mn > v$, so that the social optimum is full contribution, and for $v > m$, so that the dominant-strategy Nash equilibrium is zero contribution when this game is only played once. Notice that the equilibrium level of contributions is independent of non-critical changes in the MPCR and group size. When this game is repeated a finite number of times, the usual backward-induction arguments imply that zero contributions should be expected in all rounds.

The theoretical work to be surveyed is motivated by the dramatic difference between the Nash prediction and the empirical regularities described above. We will consider families of models that incorporate:

1) generalized preferences to allow for factors such as altruism, spite, and fairness,
2) noisy decisions caused by unobserved preference shocks or calculation errors,
3) evolutionary adaptation toward decisions with desirable outcomes, and
4) cooperative responses to acceptable earnings or to others' contributions.

For each family of models we will present a representative specification and its empirical implications.

Generalized Preferences

Perhaps the most obvious (and most often explored) explanation for non-zero contributions is that individuals are willing to give up some of their own earnings to raise others' earnings.[2] This is usually modeled as a utility function that depends on one's own earnings and the sum (or average) of other's earnings:

$$u_i = U(\pi_i, \Sigma_{j\neq i}\, x_j), \quad (2)$$

[2] Andreoni and Miller (2002) report data for situations in which a subject could unilaterally give up some money in order to increase the earnings of another randomly selected subject. They report that the amount of such giving depends on the price of helping others.

where U is assumed to be increasing in both arguments. This is an old idea in economics that dates back to Edgeworth's (1881) notion of a "coefficient of sympathy." Following Ledyard (1995), most researchers have used a simple, linear specification for the study of voluntary contributions:[3]

$$u_i \;=\; \pi_i + \alpha\Sigma_{j\neq i}\pi_j \;=\; C + (m - v + \alpha\,(n-1))x_i, \tag{3}$$

where α is an altruism parameter. The constant, C, on the right side of (3) includes all terms that are independent of the person's own contribution, x_i. Notice that the effect of this altruistic concern for the (n-1) others is to introduce the $\alpha(n\text{-}1)$ tern in (3).

With linear altruism, the utility function in (3) is linear in the decisions, x_i, so that the optimal choice is to contribute all or nothing, depending on the relative size of the altruism parameter. In particular, full contribution is optimal if $\alpha > (v - m)/(n - 1)$. Given a distribution of α parameters across individuals, increases in MPCR and group size would tend to increase the probability of full contribution. Heterogeneous linear altruism models explain how the proportion of individuals who contribute responds to these treatment variables, but they cannot explain "splitting."

Levine (1998), for example, uses the results of ultimatum games to estimate a distribution of linear altruism parameters. Unlike public goods games, ultimatum bargaining is a zero-sum game; this may be one reason that Levine concludes that a majority of individuals are characterized by a *negative* altruism parameter, which corresponds to "spite" (Saijo and Nakamura, 1995). Casari and Plott (2002) also find evidence of heterogeneous, linear other-regarding preferences, including altruism and spite as special cases.

Another possible modification of the basic model is to allow individuals to care about the size of their earnings relative to others. Typically these models are framed in terms of reciprocity or fairness. In the altruism model shown in equation (3), contributions do not depend on the amount that others contribute. However, reciprocity (positive or negative) suggests that contributions are made as an "in-kind response to beneficial or harmful acts" (Fehr and Gachter, 2000a). For example, Croson (1998)

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[3] See, for example, Laury (1996), Anderson, Goeree, and Holt (1998), Goeree, Holt, and Laury (2002b). A non-linear Cobb-Douglas specification is tested in Goeree, Holt, and Laury (2002a).

reports a strong correlation between contributions and one's expectation of others' contributions, and Fehr and Gachter (2000b) show that cooperators are willing to punish free riders, even when it is costly to do so.

Related models propose that individuals have a taste for fair (or equitable) outcomes. The easiest way to incorporate these preferences is to subtract an inequality measure from the earnings function. For example, Ledyard (1995) uses the squared deviation of one's own payoff from average earnings as a measure of inequality. This type of fairness is symmetric: it is just as bad to be above the average as below. Bolton and Ockenfels (2000) model utility as depending one's own payoff and one's payoff relative to others. This relative payoff function reaches a maximum when earnings are equally divided among all players. Fehr and Schmidt (1999) present a similar model, but incorporate *asymmetric* inequality aversion. The idea is that an equal division is the preferred outcome, however individuals "suffer more from inequity that is to their material disadvantage than from inequity that is to their material advantage" (p.822).

Noisy Decision Making

In a Nash equilibrium, individuals are assumed to choose the decision with the highest expected utility, even if the utility difference is arbitrarily small. Deviations from this ideal can be caused by transitory (unobserved) random shocks to preferences or by random calculation and recording errors. Note that a random shock to preferences could be something like a change in altruistic attitudes, whereas a calculation or recording error corresponds to a type of bounded rationality. In either case, the effects of these random shocks should be less important when earnings differences are great. Indeed, scaling up payoffs in a variety of laboratory experiments tends to reduce the variance of observed decisions, and to move behavior away from the boundaries (Smith and Walker, 1993, 1997).[4]

The effects of random shocks can be captured with probabilistic choice models in which decisions with higher expected earnings are more likely. The most common

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[4] To our knowledge, there are no public goods experiments that directly consider the effects of independent changes in the scale of payoffs on the variability of decisions.

probabilistic choice model is the logit specification, in which the probability density of choosing decision i is an exponential function of expected earnings, π_i:

$$f(x_i) = K\exp(\pi_i(x_i)/\mu), \tag{4}$$

where K is a constant of integration and μ is an error parameter that determines the degree of rationality. In the limit as μ goes to zero, payoff differences get magnified and probability is concentrated around the decisions with the highest expected payoff.

Anderson, Goeree, and Holt (1998) use this kind of specification to evaluate contributions to the public good.[5] When the earnings are linear in x_i as in (1), then the choice density in (4) is an exponential function of x_i. This combination yields an exponential distribution of contributions (truncated if a maximum contribution is specified). The MPCR determines the effect of one's contribution on one's own payoff, either without altruism, as in (1), or with altruism, as in (3). Hence, the MPCR determines the parameter of the exponential distribution. First consider the intuition for the case of no altruism, where it is an error to contribute anything. The magnitude of the "observed" error is determined by the difference between the marginal values of the private and public goods, $v - m$, which goes down with an increase in the MPCR. As the cost of this "error" decreases, random preference shocks and/or calculation errors are more likely to dominate and result in contribution. A small amount of altruism further reduces the magnitude of the utility cost associated with contributions, and enough altruism may make contributions more likely than not. Goeree, Holt, and Laury (2002a,b) estimate this model, and find that both the altruism and error parameters are significant.

This is an equilibrium model, however, and therefore does not explain the declining time patterns of contributions in early rounds. This brings us to dynamic models of evolution and adjustment.

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[5] This model is an application of McKelvey and Palfrey's (1995) notion of a quantal response equilibrium to the public goods game with a continuum of possible decisions.

Evolution and Adaptation

Like the error models discussed above, models of noisy evolutionary adjustment involve probability distributions over the set of possible contribution decisions. One difference, however, is that the probabilities change over time in evolutionary models, with high-payoff decisions generally becoming more likely. There are many ways to specify such processes. Miller and Andreoni (1991) propose a particularly simple specification, in which the probability, p_i^t, of making contribution level x_i, is assumed to change in a manner that depends on earnings relative to the average for some population. Let X^t denote the average contribution calculated using the probabilities at time t: $X^t = \Sigma_j p_j^t x_j$. Then the expected contribution to the public good is nX^t, and the average earnings level for the population is $v(E\text{-}X^t) + mnX^t$. The idea behind the replicator dynamic is that any specific contribution, x_i, with an expected payoff above the population average should increase in frequency. Similarly, any contribution below this average should decrease in frequency. The ratio form of the replicator dynamic used by Miller and Andreoni (1991) is:

$$p_i^{\,t+1} = p_i^{\,t}\,\frac{v(E - x_i) + mnX^{\,t}}{v(E - X^{\,t}) + mnX^{\,t}} \tag{5}$$

The term on the right is just the previous probability of making contribution x_i times the ratio of the expected payoff for that contribution to the payoff averaged over all contributions.[6]

As long as the process starts out with some dispersion (some positive probability of at least two distinct contribution levels), then the contributions below the average will have higher expected payoffs than those above the average. Hence, the lower contributions will increase in frequency and the higher contributions will decrease in frequency. This process will eventually drive the average contribution down to the Nash level of zero. An interesting feature of this model is that decay can be quite slow, and the rate of decay in contributions is influenced by MPCR and group size. To see this, note that sufficiently high values of each will result in a high value of the term, mnX^t that

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[6] The formula used by Miller and Andreoni ignores the effect of the contribution decision on the average itself, and hence, on the mnX^t in the numerator. This simplifying assumption has little effect on the simulations they carried out.

appears in both the numerator and denominator on the right side of (5). When the numerator and denominator increase in this manner, the ratio gets closer to 1, which translates into a slow decay of the probabilities. In addition, a high value of either MPCR or group size will reduce the impact of changes in the other parameter on decay rates, which is consistent with the average contribution patterns shown in Figure 1.[7] As Miller and Andreoni note, however, the predictions of their model only depend on the product of group size and MPCR, which contradicts the observation that the MPCR effect tends to dominate the group size effect (stylized fact 3, above).

Anderson, Goeree, and Holt (1997) present a gradient-based model of noisy evolutionary adjustment that can explain a declining pattern of contributions when the Nash equilibrium involves full free riding. Unlike the predictions of Miller and Andreoni's model, the decision-making noise keeps the contributions from falling all the way to zero. They prove that their model is stable for a class of games that includes public goods games, and that the steady-state distribution of decisions is a logit equilibrium determined by (4). When the logit equilibrium distribution is concentrated in the range of relatively low contributions, the average contributions will decline over time.

Cooperation and Signaling

There are a number of other models that capture some of the dynamic properties of contribution behavior, positing that subjects recognize the potential gains from cooperation. Those models that involve backward-looking behavior typically specify that expectations or preferences are endogenous and evolve based upon observed behavior. In contrast, other models allow more forward-looking behavior, i.e., that individuals consider the effects of their decisions on others' contributions in future rounds.

Some empirical evidence supports the notion that preferences or expectations may evolve over time. Contributions tend to be lower in late rounds of a session than in early

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[7] The implications of the Miller and Andreoni model are quite specific to the ratio structure that they use. It is straightforward to specify evolutionary models, even with a replicator dynamic, that do not produce such nice predictions. For example, in a "difference form" of the replicator model, the change in the probability associated with a specific contribution would be proportional to the difference between the expected payoff for that contribution and the average contribution. Notice that the mnX^t terms would cancel in taking the difference, so that the rate of decay would not depend on MPCR or group size.

rounds, and contributions by experienced participants are typically lower than by inexperienced participants. One explanation for these patterns is that subjects are learning, but the issue is what is being learned. One possibility is that they are learning to use a dominant strategy. Another possibility is that they are learning what to expect from others, which may affect their attitudes toward others' earnings.[8] In a post-experiment questionnaire performed by Laury, Walker, and Williams (1995) several subjects expressed frustration over the lack of cooperation by other group members. A study by van Dijk, Sonnemans and van Winden (2002) tested attitudes toward others both before and after a 25-round public goods game. They found that attitudes changed, and the direction of this change depended upon the results of the public goods game.

Brandts and Schram (1996) propose a model in which a subset of the subjects are "cooperative gain seekers." These subjects contribute (and forego the benefit of private consumption) if they believe total contributions to the public good (by all "cooperative" subjects, including themselves) will be high enough so that earnings will be higher than if no cooperative players contribute. These cooperative gain seekers will contribute only if they expect enough others to do so as well. Their model is backward-looking in that the cooperative gain seekers have prior beliefs about the number of others of this type. These beliefs evolve over time, based up observed contributions. Declining contributions throughout the experiment would be observed if cooperative gain seekers systematically over-estimate the number of others of this type in their group and reduce contributions in response to observed (lower-than-expected) contributions.

From this type of model in which expectations evolve based on past behavior, it is a small logical step to a forward-looking model. If subjects believe that others' future decisions are based in part upon observed contributions in previous rounds, it may be worthwhile to contribute in early rounds to lessen the decline or to encourage others to contribute. This is the basis of an earlier model put forth by Isaac, Walker, and Williams (1994). The flavor of their model is very similar to that of Brandts and Schram (1996), but it includes a forward-looking component. Subjects are willing to forego current private consumption in order to signal a willingness to contribute to the public good in

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[8] Rabin (1993) argues that preferences in games are affected by others' prior actions.

the upcoming round. They will undertake this signal if future earnings are at least as great as if no one contributes anything to the public good (the Nash equilibrium). However, because subjects consider the impact of their decisions only one round in the future, the time-horizon of the experiment is predicted to have no impact on decisions. Laury (1996) generalizes this model so that subjects consider the impact of their contributions over all remaining rounds of the experiment. In this setting, contributing is more attractive with a longer time horizon, so contributions are expected to be lower late in an experimental session, when there are fewer rounds remaining.

In each of these models, the net cost of contributing in the current round, $(v - m)x_i$, is lower with an increase in MPCR, thus contributions are expected to increase with an increase in MPCR.

Final Observations

The main qualitative predictions of the alternatives to the Nash model are summarized in Table 1. All of these capture the most prominent feature of the data, the MPCR effect. One way to pick up the group size effect is to add altruism. In addition, Miller and Andreoni's ratio-replicator dynamic predicts some interesting non-linearities (a high value of either group size or MPCR will reduce the effect of changing the other). The typically observed pattern of declining contributions can be explained by dynamic models, for example, evolution, evolving preferences and expectations, or signaling in an attempt to alter others' behavior.

The various approaches listed in Table 1 are not necessarily mutually exclusive. It is likely that many of the key elements (for example, altruism, error, learning, endogenous preferences, and signaling) are present in the laboratory. A good model will select the most important of these factors and ignore the others (or include them in a noise terms). The best such models can only be determined from carefully designed experiments coupled with econometric work that estimates parameters representing components of alternative models.

Table 1. Predicted Treatment Effects from Classes of Models

	MPCR	Group Size	Time	Splitting
Nash	0	0	0	no
Heterogeneous Linear Altruism[a,b]	+	+	0	no
Decision Error[c]	+	0	0	yes
Decision Error (with altruism)[c]	+	+	0	yes
Evolutionary (Ratio-Replicator) Dynamic[d]	+	+	-	yes
Forward-Looking Signaling[a]	+	0	-	yes
Cooperative Gain Seeking[e]	+	0	-	yes

[a] From Laury (1996).
[b] From Ledyard (1995).
[c] From Anderson, Goeree, and Holt (1998)
[d] From Miller and Andreoni (1991).
[e] From Brandts and Schram (1996).

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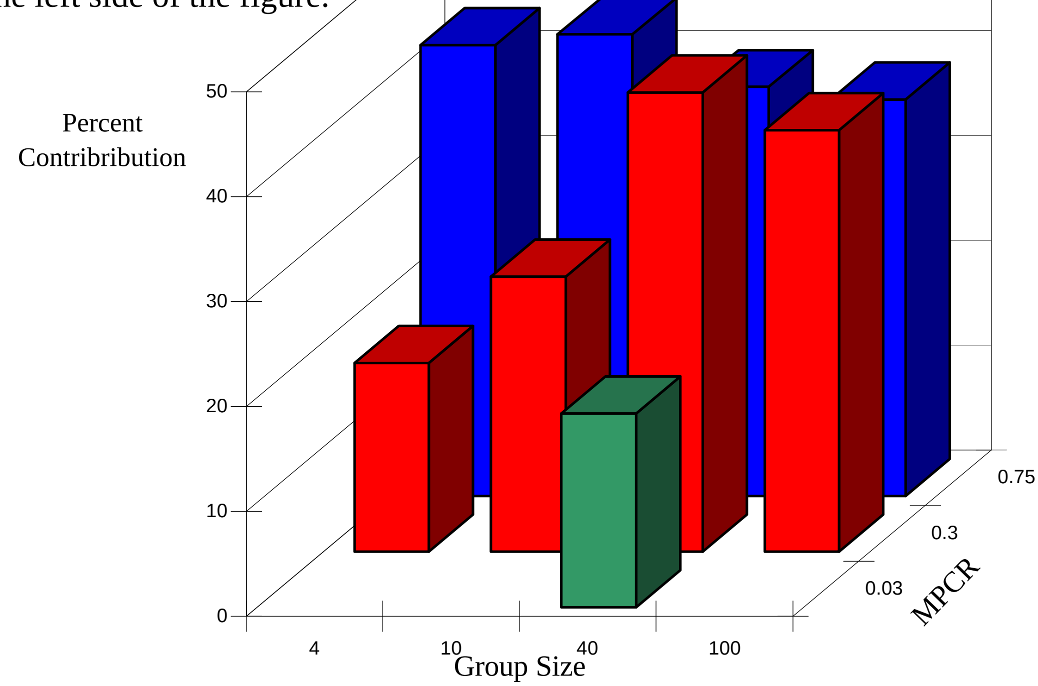
Contribution to Public Good:
Percent of Endowment

Contributions change little with large MPCR and group size, as seen by the red and blue bars in the upper right corner.

Contributions increase as MPCR increases for small group sizes (N=4, 10), as seen by comparing the increase in contributions from red bars to blue bars on the left side of the figure.



Contributions increase as group size increases for low MPCR and moderate group sizes (N=4, 10, 40), as seen by looking from left to right along the row of red bars.