Dynamic capital structure with heterogeneous beliefs and market timing

Baozhong Yang *

Department of Finance, Robinson College of Business, Georgia State University, 35 Broad St., Atlanta, GA 30303, United States

Abstract

This paper builds a dynamic trade-off model of corporate financing with differences in belief between the insider manager and outside investors. The optimal leverage depends on differences of opinion and can differ significantly from that in standard trade-off models. The manager’s market timing behavior leads to several stylized facts, such as the low average debt ratios of firms in the cross section, the substantial presence of zero-debt firms that pay larger dividends and keep higher cash balances than other firms, and negative long-run abnormal returns following stock issuance. Market timing behavior leads to substantial losses of firm value through excessive financing activities. Market timing and debt conservatism depend negatively on shareholder control of the firm.

© 2013 Elsevier B.V. All rights reserved.

JEL classification: G31 G32 G34 G35

Keywords: Capital structure Heterogeneous beliefs Market timing Zero-debt firms Debt conservatism

1. Introduction

The idea that managers attempt to “time the market”, i.e., take advantage of the (possibly perceived) mispricing of their firms’ securities through corporate financing activities, has been proposed to explain many empirical facts that are difficult to reconcile with the rational expectations framework. In their survey, Graham and Harvey (2001) find that two-thirds of CFOs agree that the amount by which their stock is undervalued or overvalued is an important or very important consideration for equity issuance. Jenter (2005) and Jenter et al. (2011) provide empirical evidence that managers attempt to time the market in their corporate financing activities. Baker and Wurgler (2002) propose that market timing is the dominant factor driving capital structure changes.1

---

* Tel.: +1 404 413 7350; fax: +1 404 413 7312.
E-mail address: bzyang@gsu.edu.

1 The ensuing empirical literature, while confirming the short-term effects of market timing on capital structure, shows that firms adjust leverage in the opposite direction afterward, consistent with a trade-off model with transaction costs (e.g., Leary and Roberts (2005), Altı (2006), Flannery and Rangan (2006), Kayhan and Titman (2007), Huang and Ritter (2009)).
This paper first observes that market timing behavior does not require superior information on the manager's side or the irrationality of either the manager or investors. Any differences in belief between the manager and outside investors can lead to market timing behavior. People can be rational and still "agree to disagree" (Morris, 1994). In other words, they can be aware of each other's beliefs but insist on their own. Even sophisticated agents, such as CEOs, stock analysts, and economists, frequently disagree. This paper seeks to understand through a quantitative model the following questions: How does marketing timing arising from differences in belief affect a firm's capital structure? What impact does market timing have on a firm's value?

We develop a dynamic trade-off model of corporate financing with differences in belief between the manager insider and outside investors. In the discrete-time infinite-horizon model, a manager insider and outside investors have different beliefs about the firm's earnings process. They agree to disagree about the interpretation of the earnings information revealed in each period and update their beliefs following Bayes' rule. The manager insider makes all the firm's financing decisions and maximizes the value of its long-term shareholders based on his or her own belief. In each period, the manager makes decisions on long-term debt and equity financing/repurchases, dividend payout, and cash balance changes. The outside investors provide liquidity in the financial markets and determine the prices of securities. The manager's actions are rationally expected by outside investors based on their belief and thus reflected in the market price of the firm's securities.

In capital structure decisions, the tax advantage and default costs of debt are traded off with the benefit of issuing/repurchasing equity "mispred" by outsiders relative to the manager. The optimal leverage is the debt ratio at which the marginal benefit of debt financing equals that of equity financing, and depends on the difference of opinions between the manager and investors. The more optimistic investors are relative to the manager, the higher the marginal benefit of equity issuance and the lower the optimal leverage. The capital structure adjustment toward optimal leverage by the manager appears as market timing behavior to investors, since it is closely related to the firm's market valuation. The quantitative impact of heterogeneous beliefs on capital structure is great: A 60% increase of the outsiders' belief relative to the manager's leads to a 70 percentage point decrease in the optimal leverage ratio in the baseline model, resulting in a negative optimal leverage.

The model produces several interesting empirical predictions on the cross-sectional properties of capital structure. First, the model predicts the low average debt ratios of firms. Structural trade-off models usually produce higher leverage ratios than those in the empirical data. A standard trade-off model in Leland (1994) produces optimal leverage ratios of 70% to 90%, compared to the empirically observed average market leverage ratio of 20–30% for US public firms over the past several decades. In the model, changes in sign in the disagreement between the manager and investors lead to asymmetric changes in optimal leverage. In other words, the decrease in optimal leverage caused by overvaluation is greater than the increase in optimal leverage caused by undervaluation of the same magnitude. Therefore, the firm has, on average, lower leverage ratios than in a standard trade-off model. The nonlinear nature of expected bankruptcy costs, despite a linear recovery value upon bankruptcy in the model, leads to the above-mentioned asymmetry. On the one hand, the probability density function of default and thus the expected bankruptcy cost increases sharply as the firm approaches the default boundary. On the other hand, the marginal tax cost of reducing debt is constant. Together, these imply a greater cost to increase leverage than to decrease leverage.

Second, the model predicts the extreme debt conservatism of a substantial fraction of firms, empirically documented in Graham (2000) and Strebulaev and Yang (2013). Due to debt tax advantages, the traditional trade-off models do not predict the existence of zero-debt firms. In the simulated economy of the baseline model, 4% of firm-year observations have zero debt under plausible parameters. Furthermore, the model predicts that zero-leverage firms pay larger dividends and keep larger cash balances than other firms, consistent with the empirical evidence. The intuition of the substantial presence of firms with zero debt is that sufficiently large positive differences between the beliefs of outside investors and of the manager lead to adjustments toward an optimal leverage of zero. Zero-leverage firms pay higher dividends because stock repurchases are more costly than dividend payouts for them, based on the manager's belief. Since this study models cash as negative debt, zero-debt firms keep large cash balances when the optimal leverage is negative.

Third, the model predicts that equity issuance (repurchases) is followed by negative (positive) long-run abnormal returns. The main intuition behind this result is that the difference between outsiders' and the manager's beliefs is positive when equity is issued and its cross-sectional average converges to zero in the long run. The predictions on long-run returns for equity issuances/repurchases are consistent with the empirical evidence (Ikenberry et al., 1995; Loughran and Ritter, 1995; Ritter, 1991; Spiess and Affleck-Graves, 1995). Finally, this study shows that the above main results are qualitatively robust to changes in parameters and different specifications of the true underlying process of earnings.

Market timing behavior leads to not only large distortions of the optimal leverage, but also the substantial loss of firm value due to excessive equity financing. This paper estimates the deadweight loss due to excess financing to be 15.6% of the firm value for the baseline model. This suggests a potential role for the regulator in the market: While market timing and excessive financing may serve Wall Street firms well, they can hurt social welfare. This deadweight cost arises from heterogeneous beliefs and cannot be eliminated as long as the insider and outsiders disagree about the firm's value. An extension of the model allows the manager and outside shareholders to have varying bargaining powers in making the financial decisions of the firm. This extension generates the following testable predictions: market timing, associated excess costs of financing, and debt conservatism all decline when the power of outside shareholders increases. In particular, this suggests that financial regulation aimed at improving firm shareholder control can significantly improve welfare through the channel of reduced market timing.

2 Shleifer and Vishny (2003) propose a model of market timing in merger and acquisition activities.
In the model, the beliefs of agents can have different degrees of correlation with the observed earnings of the firm. In one case, outside investors assign a greater precision to the information revealed in the firm’s realized earnings and thus their belief is more correlated with these public signals than the manager’s. In this case, the model generates further predictions. First, dividend-paying zero-debt firms are more profitable than other firms. Intuitively, when profitability experiences a positive shock, outsiders increase their beliefs more than insiders, which leads to lower optimal leverage ratios and a higher probability of a zero-debt policy. Second, firms with higher prior stock returns or market-to-book ratios are more likely to issue equity, consistent with empirical evidence (Hovakimian et al., 2001, 2004; Marsh, 1982). Intuitively, higher firm profits lead to greater investor belief relative to the manager’s belief, which implies higher prior stock returns and market-to-book ratios, as well as a greater likelihood of equity issuance. The fact that these additional predictions are consistent with empirical findings supports the hypothesis that outsiders rely more than the insider on a firm’s observable characteristics.

This study’s model is related to several branches of theoretical research. First, the model is related to a vast literature studying dynamic trade-off models and various extensions (DeAngelo et al., 2010; Fischer et al., 1989; Goldstein et al., 2001; Hennessy and Whited, 2005; Hennessy et al., 2010; Ju et al., 2005; Leland, 1994, 1998; Leland and Toft, 1996; Morellec, 2003; Strebulavet, 2007; Titman and Tsplakov, 2007; Tserlukevich, 2008). The model complements this literature by being the first to consider heterogeneous beliefs in a dynamic structural trade-off model of capital structure and to study the quantitative impact of heterogeneous beliefs on capital structure and firm value.

Second, the model is related to a sparse but increasingly rich theoretical literature on heterogeneous beliefs in corporate finance (Adrian and Westerfield, 2009; Giat et al., 2010; Jung and Subramanian, 2010). These studies investigate optimal contracts under heterogeneous beliefs between a principal and an agent. While Jung and Subramanian (2010) also study capital structure, they do not model repeated external financing and therefore do not examine dynamic market timing or its implications. The current paper contributes to the literature by joining the above two strands of research and showing that dynamic capital structure decisions under heterogeneous beliefs provide a simple framework for understanding market timing, debt conservatism, and related issues.

Third, this paper is also related to the now vast literature on heterogeneous beliefs in asset pricing (e.g., Banerjee and Kremer, 2010; Barberis et al., 1998, Basak, 2000; Cao and Ou-Yang, 2009; David, 2008; Scheinkman and Xiong, 2003). A number of papers in this literature also consider leverage, albeit in different settings than in this paper. Buraschi et al. (2011) consider the impact of heterogeneous beliefs of agents on credit spreads and stock returns in a general equilibrium setting, treating corporate leverage as exogenous. Geanakoplos (2010) considers agents with heterogeneous beliefs in general equilibrium with collateral (developed in Geanakoplos and Zame, 2009) and predicts the cyclic behavior of margin or leverage in securities transactions. The notion of leverage used there is mainly concerned about collateral and enforceability of contracts and thus is different from the corporate leverage considered in this paper, which reflects the trade-off of benefits and costs of different securities (debt and equity).

Finally, another related theoretical literature explains the relation between stock returns and equity financing through different mechanisms (Carlson et al., 2006; Dittmar and Thakor, 2007; Li et al., 2009; Lucas and McDonald, 1990; Pastor and Veronesi, 2003; Schultz, 2003). The current model offers a new framework under which stylized facts about stock performance around equity financing can be explained, along with the cross-sectional properties of leverage ratios.

The remainder of this paper is organized as follows. Section 2 sets up the model. Section 3 describes its numerical solution. Section 4 presents the model’s empirical implications through simulations. Section 5 provides robustness checks for different assumptions. Section 6 concludes.

2. The model

2.1. The firm and agents

This paper considers an economy with discrete time periods, denoted by \( t = 1, 2, \ldots \). There is a risk-free asset with a constant risk-free rate of \( r \). All agents in the economy are risk neutral and have a discount rate \( \beta = 1/(1 + r) \). There is a single firm in the economy that generates a stochastic cash flow \( N_t \) in period \( t \). The (log) earnings \( n_t = \log(N_t) \) of the firm are given by

\[
n_t = n_t + \epsilon_t, \tag{1}
\]

\[
s_t = \rho s_{t-1} + (1 - \rho)\bar{\sigma} + \eta_t, \tag{2}
\]

where \( s_t \) is the process of expected earnings, \( \bar{\sigma} \) is the long-run mean of earnings, \( \rho \) is the autocorrelation of earnings, and \( \epsilon_t \) is the temporary shock to earnings. The shocks \( \eta_t \) to the expected earnings are normally distributed, with \( \eta_t \sim N(0, \sigma^2) \). The shocks \( \epsilon_t \) and \( \eta_t \) are mutually independent for \( t = 1, 2, \ldots \). The distribution of the shock \( \epsilon_t \) is discussed below. The realized cash flow \( n_t \) is observable to the public in period \( t \), while the expected earnings \( s_t \) and the shocks \( \epsilon_t \) and \( \eta_t \) are not.

---

3. Note that this assumption only refers to the relative difference between the manager and outsiders’ beliefs and does not imply that either party is more correct than the other.

4. Hong and Stein (2007) provide a nice survey on part of this literature.

5. The assumption of risk neutrality is not essential for the results but makes the model tractable.
There are two types of agents in the model. First, there is a manager, or insider (type $M$), of the firm who determines the firm’s financial policies, to be described later. Second, there is a continuum of outside investors (type $I$) who invest in the firm’s securities and provide liquidity when the firm issues or repurchases securities. Financial markets are assumed to be competitive. In other words, securities are issued at the price that equals the valuation by outside investors, that is, the market price.

The two types of agents have different beliefs about the distribution of the temporary shock $\epsilon_t$. The outside investors (manager) believe(\textit{s}) that the shocks $\epsilon_t$ are normally distributed with mean zero and variance $\sigma^2_{\epsilon_t}$: $\epsilon_t \sim N(0, \sigma^2_{\epsilon_t})$. under agent $i$'s belief, for $i = I, M$. (3)

Intuitively, different agents can have different interpretations about the precision of the public signal of earnings announcement $n_t$, as represented by their different beliefs about the variance of $\epsilon_t$. Investors in this model agree to disagree, meaning that they know each other’s belief but insist on their own. The agents update their beliefs according to the Bayes’ rule. For simplicity, it is assumed that agents do not learn the value of $\sigma_{\epsilon_t}$ over time. The autocorrelation coefficient $\rho$ and the long-run mean $\bar{s}$ and variance $\sigma^2_\epsilon$ of the shocks $\epsilon_t$ are all known to all agents.

At this point, we do not take a stand as to whether outside investors or the manager has the correct belief (or, more generally, whose belief is closer to the truth), because, as shown later, the objectives and strategies of the manager and outsiders depend only on their own beliefs and their beliefs about their counterparty’s beliefs, and not on the true earning process.

### 2.2. Capital structure and financial policy

In each period of the dynamic model, the firm can issue or repurchase any amount of equity or long-term debt. The manager makes all the firm’s financial decisions. In particular, the manager decides the timing and amount of debt and equity issuance and repurchases, the adjustment of cash balances, and the amount of dividends to pay out to shareholders.

Following Leland (1994), this paper assumes that long-term debt pays out a fixed coupon perpetually. The coupon rate (based on par value) is $r$, equal to the risk-free rate. Debt is issued at the fair market price, and thus below par value to reflect the risk of default. The debt obligation can be called at the par value at any time. When the firm needs to raise more debt, it first recalls all their own beliefs and their beliefs about their counterparty’s beliefs, and not on the true earning process.

The debt default when it cannot meet the debt service requirement by issuing more equity. In other words, the default time is determined endogenously by the first time market value of equity drops to zero. Upon default, the firm is liquidated and debt holders recover a fraction $\lambda < 1$ of the unlevered firm value (the continuation value of the firm if it keeps operating with zero debt). The bankruptcy costs of debt reflect costs from several sources that are not modeled explicitly here: for example, legal and negotiation costs, loss of human capital, missed investment opportunities, and damage to relationships with customers and suppliers.

The effective corporate tax rate is assumed to be a constant $\tau$. Coupon payments to debt holders are tax exempt, while dividends are taxable. Therefore, debt has tax benefits over equity financing. For simplicity, I assume that the effective corporate tax rate incorporates the effects of differential personal tax rates on the tax benefits of debt, without explicitly modeling personal tax rates.

Equity issuance or repurchase is costly. Let $I_E$ denote the issuance repurchase amount of equity ($>0$ for issuance and $<0$ for repurchases). In the model, the equity issuance repurchase cost is in a linear-quadratic form, $\text{Cost}(I_E) = \xi_1 I_E + \xi_2 I_E^2 / V_E$.

where $V_t$ is the total expected equity value of the firm after financing. The linear term reflects the substantial deadweight transaction costs of equity issuance (Altinkilic and Hansen, 2000). The quadratic term reflects the price impact of equity issuance/
repurchases on the stock, which is proportional to the fraction of the firm offered/repurhased. The negative/positive price responses to stock issuance/repurhases have been well documented empirically (Mikkelsen and Partch, 1986). For simplicity, these price impacts are summarized by a quadratic cost without the source of the cost being explicitly modeled.

In the model, cash holdings incur a linear tax cost rather than benefits. More precisely, a single variable $b_{t}$ is used to represent debt and cash balance in period $t$; the firm keeps a long-term debt of par value $b_{t}$ if $b_{t} > 0$ and a positive cash balance of $|b_{t}|$ if $b_{t} < 0$. The variable $c_{t} = r_{t}b_{t}$ represents the required coupon payment of debt (if $c_{t} > 0$) or the interest income from cash holdings of the firm (if $c_{t} < 0$). The firm’s net income is given by the formula $(1 - \tau)(N_{t} - c_{t})$, where $c_{t}$ can be either positive or negative.

Finally, it is assumed that the firm does not pay more dividends than its net income, that is,

$$Div_{t} \leq (1 - \tau)(N_{t} - c_{t}).$$

This assumption is realistic because, except for one-time extraordinary dividends, dividends are usually paid from after-tax earnings.

2.3. Agents’ beliefs

Since the expected earnings $s_{t}$ are not observable, the agents update their beliefs about $s_{t}$ using the history of realized earnings $n_{t}$. It is assumed that each type of agent initially has a normal prior about the expected earnings of the firm at date 0,

$$s_{0} = N(q_{0}, \alpha_{0}^{2}).$$

The agents then update their beliefs following the Bayes’ rule. The normality of priors implies normal posterior distributions. The following proposition characterizes the updating process of agents’ beliefs.

**Proposition 1.** The belief of agent $i$ about the state of the firm after earnings become public in period $t$ is normally distributed,

$$s_{t} = N(q_{t}, \alpha_{t}^{2}),$$

where the mean $q_{i}$ and variance $(\alpha_{i}^{2})$ of the belief are given by the following iteration formulas:

$$q_{t} = n_{t} \left( \rho q_{t-1} + (1 - \rho)s_{t} \right) + (1 - n_{t})n_{t},$$

$$\alpha_{t}^{2} = \frac{\rho^{2}(\alpha_{t-1}^{2} + \alpha_{\eta}^{2}) + \alpha_{e}^{2}}{\rho^{2}(\alpha_{t-1}^{2} + \alpha_{\eta}^{2}) + \alpha_{R}^{2} + \alpha_{\rho}^{2}}.$$

The coefficient $n_{t}$ is given by

$$n_{t} = \frac{(\alpha_{e}^{2})^{2}}{\rho^{2}(\alpha_{t-1}^{2})^{2} + \alpha_{\eta}^{2} + (\alpha_{R}^{2})^{2}}.$$

This proposition implies that the variance $(\alpha_{i}^{2})$ of agents’ beliefs is deterministic, and thus the mean belief $q_{i}$ is a sufficient statistic for the belief of agent $i$ in period $t$. Below, to simplify terminology, $q_{i}$ is often referred to as the agent’s belief.

The belief-updating formula (4) implies that to obtain the belief $q_{i}$ of expected earnings in period $t$, agent $i$ puts weight $n_{t}$ on information from the past belief $q_{t-1}$ and the long-run mean of earnings $\bar{s}$, and puts weight $(1 - n_{t})$ on the new public

---

11 The assumption of cash as negative debt can be found in various theoretical studies, such as those of Hennessy and Whited (2005), DeAngelo et al. (2010), and Hennessy et al. (2010). Gamba and Triantis (2008), however, model cash holdings and long-term debt separately. Since this study focuses on the effects of asymmetric beliefs, for tractability, we do not model cash holdings and debt as separate variables.

12 While in this model the agents have heterogeneous beliefs and update their beliefs in a Bayesian way, in reality agents may exhibit behavioral biases and deviate from the Bayes’ rule. There is a large literature on psychology, behavioral economics, and behavioral finance that study these issues. For example, Tversky and Kahneman (1974), De Bondt (1993), and Bloomfield and Hales (2002) document evidence for the representative bias, or people’s tendency to extrapolate from recent trends. Michael Sinkey (2012) documents evidence that experts can exhibit both signal reassessment in that they use past signals despite that these signals have been incorporated into their priors and confirmatory bias in that they view ambiguous signals as supporting their own hypotheses (see also Mahoney, 1977 and Darley and Gross, 1983). In general, such behavioral biases can cause differences in belief to persist, because agents tend to hold on to their biased beliefs. Since the main intuition behind the results is that the manager and investors have different beliefs, the results should hold when agents deviate from the Bayes’ rule. The author has also considered a model in which the agents exhibit the representativeness bias and the results are qualitatively similar. The results are available upon request.
information from realized earnings $n_t$. From Eq. (6), the updating coefficient $\pi_t$ increases with the variance of short-term shocks $(\sigma_x^2)^2$. Therefore, the agent who assigns a higher precision to the public signal $n_t$ also updates his or her belief faster by assigning greater weight to $n_t$.

For simplicity, this paper considers a stationary version of the model in which the precision of the manager’s and investors’ beliefs has reached the stationary level. The following corollary gives the updating rules of beliefs in the stationary case.

**Corollary 2.** In the stationary case, the belief of agent $i$ is distributed as

$$s_i \sim N\left(q_t^i, \left(\sigma^2_t\right)^2\right), \quad i = 1, M,$$

where the mean belief $q_t^i$ follows the updating formula

$$q_t^i = \pi_t \left(p q_{t-1}^i + (1-p) s_t\right) + (1-\pi_t) n_t.$$  \hspace{1cm} (7)

the variance $\left(\sigma^2_t\right)^2$ of the belief is given by

$$\left(\sigma^2_t\right)^2 = \frac{\left(\alpha_t^2 + (1-\rho^2) \left(\sigma_x^2\right)^2\right)^2 + 4\rho^2 \alpha_t^2 \sigma_x^2 - (1-\rho^2) \left(2\sigma_x^2\right)^2}{2\rho^2},$$  \hspace{1cm} (8)

and the coefficient $\pi_t$ is given by

$$\pi_t = \frac{\left(\sigma_x^2\right)^2}{\rho^2 \left(\sigma^2_t\right)^2 + \alpha_t^2 + \left(\sigma_x^2\right)^2}.$$  \hspace{1cm} (9)

2.4. The model’s timeline

This section describes the detailed timeline of events of the model, shown in Fig. 1.

- When period $t$ starts, the manager has mean belief $q_{t-1}^M$ about expected earnings, and outside investors have mean belief $q_{t-1}^I$.
- The firm has either long-term debt outstanding or a cash balance, summarized by the variable $c_t$. The variable $c_t$ represents required debt coupon payments if $c_t \geq 0$, and interest income from cash holdings if $c_t < 0$.
- A profit $N_t = \exp(n_t)$ is realized and becomes public information. Outside investors and the manager use the information from $n_t$ to update their beliefs to $q_t^I$ and $q_t^M$. The firm’s market prices of equity and debt are adjusted according to the change in the outside investors’ belief.
- If the stock price drops to zero, the firm defaults on the coupon payment and is liquidated with the proceeds returned to debt holders. The bankruptcy process leads to a proportional loss of the (unlevered) firm value.
- If the firm does not default, the manager chooses the amount of external equity issuance/repurchase $I_{E,t}$ (issuance if $>0$ and repurchase if $<0$), the amount of debt issuance/reduction (or cash balance adjustment) $I_{D,t}$, and the amount of dividends $Div_t$. 

![Fig. 1. The model’s timeline.](image-url)
After all external financing transactions are completed, the firm makes coupon and tax payments and then pays dividends to shareholders. The financing decisions satisfy the following budget balancing condition:

\[(1 - \tau)(N_t - c_t) + I_{E,t} + I_{D,t} - \text{Cost}(I_{E,t}) - \text{Div}_t = 0.\]  

(10)

- Period \( t \) ends.

2.5. Equity and debt values

This section examines the representation of the firm’s equity and debt values. In period \( t \) (after the cash flow is realized), the state of the economy can be completely characterized by the following five state variables: \( n_t \), the (log) earnings of the firm; \( \theta_t \), the total number of shares of the firm (at the beginning of period \( t \)); \( q_t^M \) and \( q_t^I \), the manager’s and outside investors’ beliefs about the state of the firm’s earnings process; and \( c_t \), the required debt interest payment.

The market values of the firm’s equity and debt are denoted by \( S_t \) and \( D_t \), respectively. Then \( S_t^t = S_t(n_t, \theta_t, q_t^M, q_t^I, c_t) \) and \( D_t^t = D_t(n_t, \theta_t, q_t^M, q_t^I, c_t) \) are functions of the five state variables. Note that these functions refer to security values after earnings are realized and the agents update their beliefs, but before the firm makes financing decisions in period \( t \). Let \( \ell_t \) \( i = M, I \) denote the information set of agent \( i \) after earnings are realized in period \( t \), and let \( \mathcal{F}_t^i \) denote the filtration generated by \( \ell_t \).

The market value of equity in period \( t \) is given by the discounted dividends to current shareholders,

\[ S_t^t = S_t^t(n_t, \theta_t, q_t^M, q_t^I, c_t) = \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t^t \left[ \frac{\theta_t}{\theta_{s+1}} \text{Div}_s 1_{T_s<s} \right]. \]  

(11)

where \( \beta = 1/(1 + r) \) is the discount rate of risk-neutral investors, \( \text{Div}_s \) is the dividend payout in period \( s \), \( \mathbb{E}_t^t[\cdot] = \mathbb{E}_t^t[\cdot | \mathcal{F}_t^t] \) is the expectation conditional on the outside investors’ information set in period \( t \), and \( T_s \) is the time of default. There is no liquidation payoff to shareholders, since the price of equity at default is zero. In Eq. (11), the ratio \( \frac{\theta_t}{\theta_{s+1}} \) reflects the necessary ownership adjustment to changes in the total number of shares of the firm. Note that \( \theta_{s+1} \) represents the shares outstanding \( after \) financing in period \( s \). There is no liquidation payoff to shareholders, since the price of equity at default is zero.

The market value of debt is given by

\[ D_t^t = D_t^t(n_t, \theta_t, q_t^M, q_t^I, c_t) = \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t^t \left[ \left( \frac{\theta_t}{\theta_{s+1}} \text{Div}_s \right) 1_{T_s<s} \right] \]  

(12)

where \( T_s \) is the time of retirement of existing debt in period \( t \) (recall that when new debt is issued, old debt is retired) and \( T_B \) is the time of default. The first term inside the expectation term in Eq. (12) is the coupon payment received by current debt holders if the firm does not default and debt is not retired in period \( s \). The second term is the full retirement value received by debt holders, that is, the par value \( (c_t/r) \) plus interest \( (c_t) \), when debt is retired in period \( s \) (e.g., when new debt is issued or when the firm issues equity to retire debt). The third term is the recovery value of debt upon default. In liquidation, debt holders recover a fraction \( \lambda \) of the market value of the unlevered firm.

2.6. Agents’ objective and the equilibrium

The manager is initially endowed with a fixed share of the firm’s stock at date 0. Since the focus of this paper is on corporate financing and not on insider trading, it is assumed that, as an insider, the manager is completely restricted from personally selling or buying the firm’s stock.\(^{13}\) The manager’s objective is to maximize the expected present value of his or her dividend stream. This assumption on the manager’s objective function is equivalent to the assumption that the manager maximizes long-term shareholder welfare, as in Myers and Majluf (1984). Their model differs from this study’s in that the manager and outside investors have asymmetric information but not asymmetric beliefs.

The manager’s value of the firm’s equity in period \( t \) is given by

\[ S_t^M = S_t^M(n_t, \theta_t, q_t^M, q_t^I, c_t) = \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t^M \left[ \frac{\theta_t}{\theta_{s+1}} \text{Div}_s 1_{T_s<s} \right]. \]  

(13)

\(^{13}\) In reality, there exist legal restrictions on insider trading to avoid the exploitation of outside investors. Although such rules are not perfect, they provide a bound on the manager’s ability to trade personally based on market conditions.
where $E^M_t[\cdot] = E^M_t[\cdot|\theta_t]$ is the expectation conditional on the manager's information set and belief in period $t$ and $\theta_t$ is the ownership adjustment to changes in shares outstanding.

Outside investors provide liquidity when the firm seeks external financing in the form of equity or debt. Since it is assumed that capital markets are competitive, the outside investors' objective is to break even in the financial transactions they participate in, based on their belief. Outside shareholders do make one decision, the default time $T_B$. Due to limited liability, the firm defaults when the market value of equity first drops to zero.

In period $t$, the manager's strategy consists of an $F^M_t$-adapted triple $(c_t + 1, \theta_{t+1}, DI_v)$, where $c_t + 1$ is the new coupon level, $\theta_{t+1}$ is the new shares outstanding, and $DI_v$ is the dividend payment. Note that these variables determine the firm's capital structure and payout policies. The outside investors' decision is the stopping time $T_B$. Below the notion of a (subgame-perfect) equilibrium in the model is defined.

**Definition 1.** An equilibrium is a sequence of strategies $((c_t + 1, \theta_{t+1}, DI_v))_{t \geq 0}$ and $T_B$ such that

1) $(c_t + 1, \theta_{t+1}, DI_v)$ is $F^M_t$-adapted for $t \geq 0$ and $T_B$ is $F^M_0$-adapted.

2) Given $T_B$ and the future strategies $((c_s + 1, \theta_{s+1}, DI_v))_{s \geq t + 1}$, the manager's strategy $(c_t + 1, \theta_{t+1}, DI_v)$ maximizes his or her expected equity value in period $t$.

3) Given the manager's strategy $((c_t + 1, \theta_{t+1}, DI_v))_{t \geq 0}$, the default time $T_B$ is the first time the market value of equity drops to zero.

### 2.7. Equilibrium analysis

In equilibrium, the manager and outside shareholders solve dynamic optimization problems. It is a standard technique to represent the dynamic problems in a recursive form, or with the Hamilton Jacobi equations. The Hamilton Jacobi equation for the manager's problem is derived below. First, one can rewrite Eq. (13) as

\[
S^M_t(n_t, \theta_t, q_t^M, q_t^I, c_t) = \frac{\theta_t}{\theta_{t+1}} \left( DI_v) + \beta E^M_t \left[ S^M_{t+1} \left( n_{t+1}, \theta_{t+1}, q_{t+1}^M, q_{t+1}^I, c_{t+1} \right) \right] \right) 1_{T_B > t},
\]

In period $t$, the manager's optimal decision $(c_t + 1, \theta_{t+1}, DI_v)$ thus satisfies the Hamilton–Jacobi equation

\[
(c_t + 1, \theta_{t+1}, DI_v) = \arg \max_{(c_t, DI_v)} \frac{\theta_t}{\theta_{t+1}} \left( DI_v + \beta E^M_t \left[ S^M_{t+1} \left( n_{t+1}, \theta_{t+1}, q_{t+1}^M, q_{t+1}^I, c_{t+1} \right) \right] \right) 1_{T_B > t},
\]

s.t. $\theta = \frac{1}{1-a} \theta_t, \quad a < 1$,

\[
I_E = a \left( DI_v + \beta S^M_t(\theta, c) \right),
\]

\[
I_D = \begin{cases} 
\beta D^M_t(\theta, c) - \frac{1}{r} c_t, & \text{if } c_t > c, \text{ and } c > 0, \\
\frac{1}{r} (c - c_t), & \text{if } c_t \leq c, \text{ or } c \leq 0.
\end{cases}
\]

\[
(1-\gamma)(N_t - c_t) + I_E + I_D - (c_E I_E + S^I_t) = 0,
\]

where $S^M_t(\theta, c) = E^M_t \left[ S^M_{t+1} \left( n_{t+1}, \theta_{t+1}, q_{t+1}^M, q_{t+1}^I, c_{t+1} \right) \right]$ and $D^M_t(\theta, c) = E^M_t \left[ D^M_{t+1} \left( n_{t+1}, \theta_{t+1}, q_{t+1}^M, q_{t+1}^I, c_{t+1} \right) \right]$ are the expected continuation market values of equity and debt, given the choice of coupon level $c$ and shares outstanding $\theta$.

The variable $a$ in the optimization problem represents the fraction of the claim to the current and future cash flows of the firm offered to new shareholders (if $a > 0$), or the fraction repurchased (if $a < 0$). The new shares outstanding, $\theta$, are related to the fraction offered, $a$, and $\theta_t$ in Eq. (16). Eq. (17) means that the new shareholders get a fair fraction of the firm: $I_E$ on the left-hand side is the amount of money raised and the expression on the right-hand side is the fair market value of equity sold to outside investors. Eq. (19) represents the break-even condition for new bond holders. In the first case of Eq. (19), when $c > c_t$ and $c > 0$, more new debt is issued and old debt is retired. The term $\beta D^M_t(\theta, c)$ represents the amount the firm can raise from increasing the coupon level to $c$, and the cost of debt retirement is $\frac{1}{r}$. In the second case of Eq. (19), when $c \leq c_t$ or $c \leq 0$, the firm is retiring part or all of its debt pro rata and the retirement cost is $\frac{1}{r} c_t$. Finally, Eq. (20) gives the budget-balancing condition for the firm, that is, a replication of Eq. (10). It relates the decision on dividend payout to the financing actions $I_E$ and $I_D$.

Let $B_t = 1_{T_B > t}$ be the indicator of the default decision in period $t$. Then the default decision problem can be rewritten as

\[
B_t = \arg \max_{B \in [0,1]} B \left( DI_v + \beta E^M_t \left[ S^M_{t+1} \left( n_{t+1}, \theta_{t+1}, q_{t+1}^M, q_{t+1}^I, c_{t+1} \right) \right] \right).
\]
The decision rule is simple: The firm defaults in period \( t \) (i.e., \( B_t = 1 \)) if and only if the market value of equity in the hypothetical case of no default is negative. When the firm defaults, the firm is liquidated and the equity value is zero. The collection of decisions \( \{B_t\}_{t \geq 0} \) determines the stopping time \( T_B \) as the first time when \( B_t = 1 \).

### 3. Numerical solution of the model

The model contains five continuous state variables: the log earnings level \( n_t \), the shares outstanding \( n_t \), the manager’s belief \( q^M_t \), the investor’s belief \( q^I_t \), and the coupon level \( c_t \). In general, it is difficult to derive analytical solutions for a dynamic model with multiple state variables, and no closed-form solution exists in this case. Therefore, this study resorts to numerical methods to find the optimal financing policies and the value functions of debt and equity. Details of the numerical procedure are provided in Appendix B.

#### 3.1. Parameter choices

This section describes how parameters in the model are determined. When feasible, the parameters are chosen based on the empirical literature. The baseline parameters of the model are reported in Table 1.

The risk-free rate, or the discount rate for the firm’s cash flows, is set to be \( \tau = 10\% \). The effective corporate tax rate is set at \( \tau = 20\% \). This choice reflects the reduction in tax benefits of a 35\% corporate tax rate by the different personal tax rates on interest income, capital gains, and dividends. The bankruptcy recovery rate is set to be \( \lambda = 0.8 \), which falls in the range of estimates in Andrade and Kaplan (1998) and is close to the recovery rate 0.75 in Leland (1998).

Altinkilic and Hansen (2000) estimate an average issuance cost of 5.38\% from common stock offers from 1990 to 1997. Gomes (2001) estimates a fixed cost of 8\% and a proportional cost of 2.5\% from a data set from 1971 to 1975. There is no fixed cost in this model, and the linear coefficient of equity issuance costs is chosen to be \( c_t = 0.2 \). This implies that an equity issuance of 15\% of the firm’s existing shares (the average issuance/repurchase is set at 0.2) does not affect the main results qualitatively.

The standard deviation \( \sigma_{st} \) of the shock to the hidden process of expected earnings \( s_t \) in Eq. (2) is set to be 0.1. The standard deviations of the temporary shock \( \epsilon_t \) to earnings in Eq. (1) based on outside investors’ and the manager’s beliefs are set to be \( \sigma_{st} = 0.1 \) and \( \sigma_{ct} = 0.3 \), respectively. This assumption implies that, in updating their beliefs, outside investors put more weight on observable earnings than the manager (see the belief-updating Eqs. (7) and (8)). This is a reasonable assumption, since the manager has more access to detailed inside information about the company and may base his or her belief more on “soft” information that is unobservable to outsiders. On the other hand, outside investors may have to rely more on the hard information observable to the public. Note that this assumption does not imply that either the manager or the outside investors are mistaken in their beliefs. Section 5.1 examines the results for the reverse case, where \( \sigma_{ct} = 0 \) and \( \sigma_{st} = 0.3 \), that is, when the manager relies more on the firm’s observable information, and compares these two cases.

---

\[^{14}\] The assignment of a value of zero to the parameter \( c_t \) reduces the number of state variables by one (the state variables are now \( (n_t, \theta_t, q^M_t, c_t) \)) because \( q^I_t = n_t \) and thus helps with the “curse of dimensionality” and speeds up the solution of the model by an order of magnitude. Therefore, this assumption is made for convenience. In untabulated results, assuming a nonzero value of \( c_t \) does not affect the main results qualitatively.

\[^{15}\] In a recent paper, Alam et al. (2010) show that board directors located far from corporate headquarters tend to rely more on hard information than proximate board directors in making decisions such as CEO compensation and turnover.
The variance parameter $\sigma_e$ for the true underlying process is set to be equal to the manager’s belief, $\sigma_e = 0.3$, in the baseline case. Section 5.1 allows this parameter of the true model to vary to check the robustness of the predictions.

Finally, since the earnings process is known to be highly persistent, the autocorrelation of the hidden process $s_t$ underlying earnings is set to be $\rho = 0.8$. The long-run mean of earnings $\bar{s}$ affects all value functions by a constant factor and does not influence the optimal financing policies, and thus is normalized to zero.

Fig. 2. Financial policies under different beliefs. This figure shows the optimal financial policies of the firm in the model for different values of the belief state variables. Parameters of the model are given in Table 1. In Panel A, the circled line is the after-financing market leverage as a function of the market leverage before financing, for the case where the manager’s belief $q^M = 0.0$ and the investors’ belief $q^I = -0.3$, the solid line indicates the case where $q^M = 0.0$ and $q^I = 0.0$, and the line with pluses indicates the case where $q^M = 0.0$ and $q^I = 0.3$. The 45-degree dotted line is where the after-financing leverage equals the before-financing leverage. Panel B plots the cases where $q^I = 0$ and $q^M = -0.3, 0.0, 0.3$. Policy functions are calculated on discrete grid points and then smoothed using interpolation by cubic splines.
Table 2
Optimal leverage under different beliefs. This table reports the optimal leverage ratios of the firm under different values of the mean beliefs of the outside investors and manager in the model. The parameters of the model are given in Table 1. This table displays the case where the manager’s share θ = 0.5. The optimal market leverage ratios for different combinations of the manager’s belief \( q^M \) and outside investors’ belief \( q^I \) are shown.

<table>
<thead>
<tr>
<th>( q^I )</th>
<th>( q^M = -0.6 )</th>
<th>( q^M = -0.3 )</th>
<th>( q^M = 0.0 )</th>
<th>( q^M = 0.3 )</th>
<th>( q^M = 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q^I = 0.0 )</td>
<td>0.52</td>
<td>0.93</td>
<td>0.92</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>( q^I = 0.3 )</td>
<td>0.24</td>
<td>0.61</td>
<td>0.89</td>
<td>0.95</td>
<td>0.94</td>
</tr>
<tr>
<td>( q^I = 0.6 )</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.14</td>
<td>0.63</td>
<td>0.88</td>
</tr>
</tbody>
</table>

3.2. Optimal financing policies

In the traditional trade-off model, the financial policies of the firm are shaped by the tax advantage and bankruptcy costs of debt. The optimal leverage ratio is the debt ratio at which the marginal benefit of debt financing is equal to zero. In the current model, the difference between the manager’s and the market valuation of the firm is another driving force behind capital structure choices. The optimal leverage ratio of the firm is that at which the marginal benefit of debt financing equals the marginal benefit of equity financing. On the one hand, the marginal benefit of debt financing decreases with leverage because marginal bankruptcy costs increase as the firm approaches the default boundary. On the other hand, the marginal benefit of equity financing increases with leverage. This can be viewed as a form of "leverage effect" — the market misvaluation of equity relative to the manager’s belief is more extreme when the firm is more levered. The optimal leverage is thus achieved at the point where the marginal effects of equity and debt cancel out.

The following simplified static example illustrates the intuition on the above-mentioned leverage effect on equity financing. Suppose the manager believes the total value of the firm’s future cash flows is \( V^M \), while outside investors believe it to be \( V^{FM} \), and the firm has outstanding debt \( D < V^M \). Then the equity value of the firm is \( V^M - D \) and \( V^M - D \), according to the manager and outside investors, respectively. The marginal benefit of issuing one dollar of equity on the market is thus equal to \( \frac{V^M - D}{V^M - D} - 1 = \frac{V^M - V^F}{V^M - V^F} \). This ratio is clearly decreasing in \( D \) and thus the marginal benefit of equity financing decreases with the debt level. Note that when the firm has cash holdings, \( D \) is negative, and the marginal benefit of equity financing approaches zero as \( D \) goes to minus infinity. This implies that even overvalued firm will not choose to keep too large a cash balance because the overvaluation effect diminishes when cash increases, and therefore the optimal leverage is always above \(-1\).

The optimal capital structure policies under the different beliefs of the manager and investors are shown in Fig. 2, which plots the market leverage after financing as a function of the leverage before financing. Negative leverage in the figure means that the firm has positive cash holdings and no debt outstanding. The plotted leverage policy function determines how the firm’s leverage would evolve dynamically. In particular, the point where the leverage policy function crosses the 45-degree line determines the optimal leverage ratio. When the current leverage is below the optimal leverage, the firm increases debt and repurchases equity to move toward the optimum, and the leverage policy function is thus above the 45-degree line in that region. Similarly, when leverage is higher than the optimum, the leverage policy function is below the 45-degree line.

Several features of the optimal financial policies stand out in Fig. 2. First, given the same before-financing leverage ratios, the after-financing leverage ratio decreases with the investors’ optimism relative to that of the manager. This is consistent with the intuition that more “overvalued” firms are more likely to choose equity over debt as the financing instrument. Second, the optimal leverage ratio increases with the manager’s belief and decreases with the investors’ belief. This follows from the facts that the marginal benefit of equity financing increases with investor optimism and the marginal benefit of debt financing decreases with leverage — when investors are more optimistic, the optimal leverage has to be lower for the manager to be ambivalent between debt and equity. Third, when investors’ beliefs deviate from the manager’s belief, the resulting changes in optimal leverage are asymmetric in positive and negative changes of beliefs. Higher investor belief leads to greater decreases in optimal leverage than lower investor belief leads to increases.

The above-mentioned asymmetric effect of heterogeneous beliefs on optimal leverage is more apparent in Table 2, which shows the optimal leverage for different values of the belief variables. For example, when the manager’s belief is equal to the long-run mean \( (q^M = 0) \), the average of the optimal leverage for the cases in which the investors’ belief \( q^I \) is \( 0.3 \) and \( -0.3 \) is \( (0.89 + 0.14) / 2 = 0.52 \), less than the optimal leverage 0.6 when \( q^I = 0 \), and the average optimal leverage for the cases \( q^I = 0.6 \) and \( -0.6 \) is \( (0.92 - 0.1) / 2 = 0.41 \), even smaller.

The main cause of this asymmetry is the nonlinear nature of bankruptcy costs and the existence of the default boundary. On the one hand, the marginal cost of bankruptcy increases sharply as the firm approaches the default boundary. On the other hand, the marginal tax cost of debt reduction is a constant (equal to the tax rate). Therefore, the marginal cost to increase the debt level and approach the default boundary is greater than that to decrease it toward zero or negative debt levels. The plot of the firm value function in Fig. 3 further clarifies this point. The total firm value (equity plus debt, excluding cash) is maximized at the optimal

---

16 In this simplified example, it is assumed that the firm value does not depend on the choice of debt level. In the general case, the firm value depends on the capital structure, but the intuition of the argument is similar.
leverage ratio. The firm value function, however, is asymmetrically bell shaped around the optimal leverage, decreasing faster when leverage exceeds the optimum than when leverage is less than the optimum.

Table 2 also shows that moderate changes in the difference in beliefs can lead to large changes in the optimal leverage. For example, when $q^M = 0$ and $q^I$ changes from 0 to 0.6, the optimal leverage changes by 70 percentage points, from 0.60 to −0.10. The optimal leverage becomes negative when the difference $q^I - q^M$ is sufficiently large, a phenomenon that cannot be observed in traditional trade-off models. Therefore, the quantitative impact of heterogeneous beliefs on firm capital structure is economically significant.

4. Empirical implications of the model

The analysis of the optimal financing policies in the previous section provides intuition about the optimal leverage and financial policies of the firm. However, there is still a gap from these results to empirical predictions. First, the investors' and manager's beliefs are endogenously determined in the model and not directly observable in the real world. Second, in the dynamic model, the firm's history is important in shaping the leverage decisions and other characteristics of the firm. Therefore, to examine the empirical implications of the model, this study conducts simulations based on the optimal financial policies and empirically analyzes the simulated data. Simulation-based analysis is particularly suitable for examining nonlinear and dynamic models. Many previous studies (Berk et al., 1999; Hennessy and Whited, 2005; Leary and Roberts, 2005; Strebulavaev, 2007; Tserlukievich, 2008) have used simulation techniques to study problems in asset returns and capital structure.

We generate a panel data consisting of 1000 firms' histories by simulating the manager's optimal financial policies obtained in Section 3. The initial earnings and beliefs are randomized to reflect the unconditional variance of these variables specified in the baseline model. Then a simulation is run for 1000 firms over 100 periods (years). The first 20 years of data are excluded from the final sample to focus on the steady state of firms' capital structure decisions. When firms default, they are thereafter excluded from the sample (but kept in the sample before the default to avoid survivorship bias). The final simulated sample consists of 75,091 firm-year observations of 1000 unique firms over 80 years. To distinguish the effects of heterogeneous beliefs from those of other factors, a simulated panel data is also obtained for the benchmark model with symmetric beliefs in which outside investors have the same belief as that of the manager, with all other aspects identical to those in the main model. The benchmark sample consists of 77,120 firm-year observations of 1000 firms over 80 years. Further details of the simulation are provided in Appendix B.

For comparison, we consider an empirical panel data set that runs for 30 years. In 1973, the 1000 firms in Compustat with the longest history prior to that time are chosen. This choice is made so that relatively mature firms are studied (the average age of firms in this panel is about 10 years), comparable to the simulated sample from the steady-state solution of the model. The final empirical sample consists of 17,268 annual observations of these 1000 firms between 1973 and 2002 (inclusive).

---

17 Randomly choosing 1000 firms in 1973 or choosing all firms in 1973 does not change the results in any material manner.
variable” minus cash holdings) to market assets. Book assets are set at a constant so that the median MA/BA is equal to the median ratio from the empirical data. The models, respectively. The first 20 years of simulated data are excluded. In the simulated data, belief as that of the manager. Simulations based on optimal financial policies generate random samples of 1000 firms for 100 years for the benchmark and main models. The benchmark model is the same as the main model except that the outside investors share the same belief as that of the manager. Simulations based on optimal financial policies generate random samples of 1000 firms for 100 years for the benchmark and main models, respectively. The first 20 years of simulated data are excluded. In the simulated data, “Market leverage” is the ratio of the net market debt to book assets, “Book leverage” is the ratio of the book debt to book assets, “MA/BA” is the ratio of market assets to book assets, and “Profitability” is the ratio of earnings before interests and taxes to book assets. Panels B and C report summary statistics for the simulated data from the benchmark and main models. The main model is that in which investors and manager have differences in belief. The benchmark model is the same as the main model except that the outside investors share the same belief as that of the manager. Simulations based on optimal financial policies generate random samples of 1000 firms for 100 years for the benchmark and main models, respectively. The first 20 years of simulated data are excluded. In the simulated data, “Market leverage” is the ratio of the net market debt (market debt minus cash holdings) to market assets. Book assets are set at a constant so that the median MA/BA is equal to the median ratio from the empirical data. The variable “Book leverage” is the ratio of the market debt to book assets and “Profitability” is the ratio of earnings before interests and taxes to book assets.

4.1. Distribution of leverage ratios

This section compares the distribution of leverage ratios and other firm characteristics among the empirical sample and the simulated samples for the main and benchmark models. First, the main variables in the empirical analysis are described below. Market assets is the sum of the market value of debt and equity. Market leverage is defined as the ratio of market debt to market assets. To facilitate comparison, Book assets is normalized to a constant so that the median market-to-book ratio in the main simulated data is equal to that in the empirical panel data. Book leverage is the ratio of the market value of debt to book assets. Market-to-Book is the ratio of market assets to book assets, or Tobin’s Q. Profitability is the ratio of operating earnings (before interest and taxes) to book assets.

Table 3 reports the summary statistics for the simulated and empirical data. On the one hand, the comparison of the empirical data (panel A) and simulated benchmark data (panel B) confirms the stylized facts about the distribution of leverage ratios. On the other hand, there is a significant mismatch between the benchmark model and the empirical data: The empirical mean market leverage (23%) is significantly lower than that of the benchmark-simulated data (74%), while the cross-sectional standard deviation of the empirical market leverage (29%) is much higher than that of the benchmark data (12%).

Structural trade-off models usually produce higher leverage ratios than those of the empirical data. A benchmark financing model in Leland (1994) produces optimal leverage ratios as high as 70–90%, compared to the empirical average market leverage ratio of 20–30% for US public firms in Compustat over the last several decades.18 This study proposes differences in belief as an independent factor contributing to this stylized fact.

The mean and standard deviation of market leverage (18% and 16%, respectively) in the main model (panel C of Table 3) match more closely those in the empirical data than those in the benchmark model do. The model with heterogeneous beliefs does a better job than the plain trade-off model for two reasons: First, the optimal leverage ratio now changes with the differences in belief, creating a wide variation of leverage ratios in dynamics. Second, the low average leverage is driven by the asymmetry of the effects of high and low investor beliefs on optimal leverage ratios, discussed in Section 3.2. The fact that bankruptcy costs are nonlinear with a higher curvature at higher leverage ratios implies that it is more costly to increase than to decrease leverage. Therefore, despite the fact that the mean difference between the beliefs of investors and the manager is zero, firms respond more to higher investor beliefs than lower investor beliefs and have lower leverage ratios, on average, than in the case of symmetric beliefs.

18 A number of studies have been trying to identify theoretically the reasons why firms are, on average, low levered (Goldstein et al., 2001; Ju et al., 2005; Morellec, 2003; Strebulaev, 2007). Almeida and Philippon (2007), Chen (2010), and Bhamra et al. (2010) argue that the high marginal utility of money in an economic recession leads to higher expected costs of default under the risk-neutral probability, and thus lower firm leverage ratios.
In a visual comparison, Fig. 4 shows the distributions of market leverage ratios in the empirical data, the benchmark model, and the main model. The benchmark model (panel B) generates a leverage distribution very different from the empirical distribution (panel A). In the benchmark model, market leverage ratios are bounded within the interval [32%, 94%] with a mean leverage ratio at 74%, while the empirical market leverage spans a much wider range, [−80%, 100%], with a mean leverage of 23%. The main model (panel C) generates a distribution widely distributed between [−23%, 95%], with a mean leverage of 18%, qualitatively similar to the empirical distribution and dramatically different from that of the benchmark model without heterogeneous beliefs.

Untabulated analysis shows that the average market value of the firm drops 17%, from 10.75 in the benchmark model to 8.92 in the main model. Therefore, the cost of market timing is significant. The deadweight costs of external equity financing are 0.20 in the benchmark model and 1.88 in the main model. Therefore, the excess cost of financing due to market timing is 1.68, accounting for 92% of the total cost of market timing. To sum up, market timing is very costly socially, with the majority of the cost coming from the excess deadweight cost of external financing. The model thus illustrates a contrast of interests between Wall Street investment banks and the government or regulators: While investment banks make lots of profits from excessive financing activities by firms that time markets, regulators should discourage such behavior in view of social welfare.

In the model, the deadweight cost of excessive financing arises from the difference of opinions between the insider manager and outsiders. As long as there are heterogeneous beliefs between the manager and outsiders, typical corporate governance mechanisms such as taking over the firm and replacing the management cannot completely eliminate the cost of excessive financing. On the other hand, mechanisms that limit the manager’s control of the firm can mitigate the above problem. Section 4.4 provides an analysis of the relationship between capital structure and financing costs with the relative control of the manager and outside shareholder on the firm.

4.2. The zero-leverage phenomenon

A substantial fraction of firms exhibits extreme debt conservatism. Graham (2000) discovers that “paradoxically, large, liquid, profitable firms with low expected distress costs use debt conservatively” and leave significant amounts of potential tax benefits unclaimed.

---

**Fig. 4. Distribution of Market Leverage Ratios.** Panel A plots the distribution of market leverage ratios in the empirical panel data. The empirical data cover the period from 1973 to 2002. In 1973, 1000 firms in Compustat with the longest history prior to 1973 are chosen. The final sample consists of all observations of these 1000 firms between 1973 and 2002. In the empirical data, the net book debt is the book debt minus cash holdings; the market leverage is the ratio of the net book debt to market assets. Panels B and C plot the distribution of the market leverage ratios in the simulated panel data for the benchmark model and main model, respectively. In the main model, investors and manager have differences in belief. The benchmark model is the same as the main model except that outside investors share the same belief as that of the manager. Simulations based on optimal financial policies generate random paths of 1000 firms for 100 years for the benchmark and main models. The first 20 years of simulated data are excluded. In the simulated economy, the market leverage is the ratio of the net market debt (market debt minus cash holdings) to market assets.
unusual. Strebulaev and Yang (2013) find that about 10% of large US public firms have zero debt outstanding over the period 1963–2002. Moreover, they find that zero-leverage firms are also more profitable, pay more dividends and taxes, and keep larger cash balances than other firms. Since cash has a tax disadvantage, zero-debt firms have particularly large unrealized tax benefits. Because bankruptcy costs are nonexistent for zero-debt firms, the presence of these firms is not consistent with traditional trade-off models.

The model with baseline parameters produces 4% firms with zero debt, with the minimum net debt ratio being $-23\%$ (for the distribution of leverage ratios, see panel C of Fig. 4). The intuition behind the substantial presence of zero-leverage firms is that the firm may have zero debt (and negative net debt) as the optimal capital structure in face of high investor beliefs relative to the manager. In the simulation, zero leverage is reached when investor beliefs are higher than the manager’s belief for several periods, and the firm makes successive debt reductions until the debt level reaches zero.

Table 4 compares the profitability, dividend payout, and cash holdings between zero-leverage dividend payers and average dividend payers in the empirical data and in the simulated data. The model (panel B of Table 4) predicts that zero-debt firms are much more profitable, pay much higher dividends, and keep a much larger cash balance than average firms, consistent with the empirical evidence in panel A (see also Strebulaev and Yang, 2013). Intuitively, zero-leverage firms are more profitable because investors are more likely to have high beliefs following a positive profitability shock.\footnote{This result on profitability follows from the assumption that outside investors put more weight on observable earnings than the manager. In the reverse case, the opposite result is found (see Section 5.1).} Zero-leverage firms choose to pay higher dividends than average firms because repurchasing “overvalued” stocks is not optimal. In the model, cash is equivalent to negative debt, therefore, zero-debt firms may have a negative optimal (net) leverage ratio and thus keep positive cash balances while other firms keep a zero cash balance.

Another piece of empirical evidence on zero-leverage firms comes from Strebulaev and Yang (2013)’s study of “jump-down” (“jump-up”) zero-leverage firms, that is, firms that decrease (increase) capital structure substantially to become zero-debt firms (to stop the zero-debt policy). They document that jump-down (jump-up) firms experience abnormally high (low) stock returns and large increases (decreases) in cash flow the year prior to the adjustment. This evidence is consistent with the current model’s explanation of zero-leverage behavior.

### 4.3. Comparative statics

This section considers the impact of varying parameters on debt conservatism and the deadweight costs of financing. In calculating the deadweight costs of financing, the costs of financing in the benchmark model are subtracted from that of the main model to reflect the excess costs of financing due to market timing. Fig. 5 shows the results of the comparative statics. Panel A shows the impact of varying the discount rate $r$. Increased interest rates initially lead to higher market leverage and lower fractions of zero-debt firms, and the relation is then reversed for higher values of $r$. The intuition is that higher $r$ reduces the discounted value of future cash flows, and thus reduces the difference in manager’s and investors’ valuations (fixing their beliefs about expected cash flows). This, in turn, generates less market timing behavior and less debt conservatism. The reversion of this relation at high values of $r$ may be due to the smaller tax shields associated with a higher discount rate. The deadweight costs of equity financing decrease monotonically with $r$ due to the above effect on market timing and the discount rate effect.

Panel B of Fig. 5 considers the comparative statics for the tax rate $\tau$. As expected, the degree of debt conservatism depends negatively on $\tau$ because the higher the tax benefits, the greater the optimal leverage. The deadweight costs of equity financing decrease with $\tau$ because equity issuance activities decrease when optimal leverage is higher.
Panel C presents the effects of the default recovery rate $\lambda$. Since lower recovery rates $\lambda$ imply higher costs of debt, the debt conservatism behavior depends negatively on the debt recovery rates $\lambda$, though the difference does not seem to be significant. The lack of sensitivity of debt policy on bankruptcy costs is due to the fact that, because of market timing, the average firm is far from the bankruptcy boundary.

Greater equity financing costs increase the costs of market timing behavior and thus should decrease the degree of debt conservatism. This intuition is confirmed for the different choices of the linear cost coefficient $c_1^E$ and the quadratic cost coefficient $c_2^E$ in panels D and E of Fig. 5. Interestingly, the deadweight costs of financing depends positively on $c_1^E$, but slightly negatively on $c_2^E$. This is because the linear cost captures the deadweight cost of financing, while the quadratic term captures the cost of price pressure. Greater price pressure ($c_2^E$) will cause the manager to issue or repurchase equity less aggressively, thus reducing the deadweight costs.

The greater volatility of the expected earnings process ($\sigma_{\eta}$) leads to smaller differences in the investors’ and manager’s beliefs because they agree on a greater component of the total shocks to earnings. On the contrary, the greater volatility $\sigma_{\xi}^M$ of the manager’s belief of the periodic shock to earnings increases the difference between the manager’s and outsider investors’ beliefs. Therefore, debt conservatism and excess costs of financing should decrease with $\sigma_{\eta}$ and increase with $\sigma_{\xi}^M$. Panels F and G of Fig. 5 confirm this intuition. It is noted here that a small change in $\sigma_{\xi}^M$ (from 0.3 to 0.25) can cause the average cross-sectional leverage ratio to undergo a relatively large change (from 0.17 to 0.25). This again illustrates the large quantitative impact of heterogeneous beliefs on capital structure decisions.

Finally, the autocorrelation $\rho$ of the expected earnings process affects the persistence of beliefs. The more persistent the underlying process, the greater and more persistent the differences in the beliefs of the manager and outside investors.
persistent differences then lead to more aggressive market timing behavior, which can last for many periods, and heightened debt conservatism. This intuition is confirmed in Panel H of Fig. 5.

4.4. Market timing and shareholder control

The previous sections make the assumption that the manager makes all financing decisions based on his or her belief. This section attempts to model the different control rights of the manager and outsiders. For tractability, we do not model the possibility of an explicit contract between the manager and outside investors. Instead, we assume in this section that the manager makes decisions based on a weighted average of the manager’s and investors’ utilities,

$$U^w = w^M U^M + (1 - w^M) U^I,$$

where the weight $w^M$ indicates how much bargaining power the manager possesses relative to outside investors. When $w^M = 1$, the model is such that the manager has complete control over the firm, the same as in previous sections. By adopting this assumption, we model the conflicts of interest of the manager and investors in a reduced form, following Dittmar and Thakor (2007).
The results in Fig. 6 show that when $w^M = 0$, that is, the investors control the firm completely, the model degenerates into a trade-off model where everyone shares the outsiders' belief. As the manager's bargaining power ($w^M$) increases, the manager engages in more market timing behavior; therefore, debt conservatism and the cost of financing increase. Increasing shareholder control of the firm has a great economic impact on market timing behavior: increasing shareholder control from 0% to 50% reduces the costs of excess financing by 45%.

In the wake of the 2007–2009 financial crisis, various financial reforms have tried to improve shareholder control and corporate governance, for example, the granting of proxy access to shareholders in the Dodd–Frank Act. The results in this section provide another rationale of such regulation: to reduce the costs of excessive market timing behavior.

4.5. Long-run performance of financing events

It is empirically well documented that firms have negative long-run abnormal returns up to five years after an initial public offering (Brav and Gompers, 1997; Ritter, 1991; Stigler, 1964) and seasoned equity offers (Loughran and Ritter, 1995; Pontiff and Woodgate, 2008; Spiess and Affleck-Graves, 1995), compared to non-issuing firms with similar characteristics. Furthermore, Ikenberry et al. (1995) show that firms that repurchase their stock experience positive long-run abnormal returns for up to four years after repurchases. These facts are hard to explain within the standard rational expectations framework, since investors would adjust the market prices sooner if long-run abnormal returns are expected.

We calculate the year-by-year abnormal returns following equity issuance/repurchase of the model-generated data and compare these model predictions with the corresponding prior empirical results. Table 5 reports the results. Consistent with the empirical results of Loughran and Ritter (1995), in the simulated data, firms that issue equity have significant negative abnormal returns in each of the five years following the event. Stock repurchases in the model are followed by significantly higher one-year stock returns, similar to Ikenberry et al. (1995). The longer-term returns after stock repurchases in the model, however, are less significant and exhibit mixed patterns. This is similar to the less significant results in Ikenberry et al. (1995), which also shows weaker results after the first year.

Intuitively, equity issuance (repurchases) occurs when the firm is overvalued (undervalued) by investors relative to the manager’s belief. The main intuition behind the long-run return results is that although investors’ valuation of a firm may differ from that of the manager temporarily, the average difference in beliefs in the long-run vanishes due to the arrival of independent shocks to earnings. This change in belief is reflected in the long-run return of the firm. Sections 5.1 and 5.2 confirm that the results on long-run performance do not depend on the true model of earnings or whether the manager’s or outsiders’ beliefs correlate more with earnings.

---

20 It is noted that this convergence holds only on average. While the time series and cross-sectional mean differences in beliefs converge to zero in the long run, the differences in beliefs for any particular firm do not converge to zero due to the arrival of new shocks in each period.
This section examines the relation between stock returns and equity issuance. A number of empirical studies have identified high stock returns as an important factor in managers’ decisions to issue equity (Hovakimian et al., 2001; Hovakimian et al., 2004; Marsh, 1982). In their survey of corporate executives, Graham and Harvey (2001) document that two-thirds of CFOs agree that “the amount by which our stock is undervalued or overvalued was an important or very important consideration for equity issuance, and nearly as many agree that ‘if our stock price has recently risen, the price at which we can sell is ‘high.’”

We conduct logit regressions of the equity issuance dummy variable on the firm’s characteristic variables for the simulated economy of the baseline model. Table 6 reports the results and compares them with those of prior empirical studies. In panel A, the sample consists of all firm-years that issue either equity or debt that year, as in most empirical studies comparing the choice between debt and equity issuance. The first two columns of panel A show that, in the model, high one-year prior stock returns makes equity issuance more likely. Furthermore, another indicator of overvaluation, high market-to-book ratio, also increases the probability that the firm chooses equity over debt. The market leverage ratio is included as a control variable. This evidence is qualitatively consistent with the results of Marsh (1982), Hovakimian et al. (2001), and Hovakimian et al. (2004).

Panel B of Table 6 conducts a related test by pitting equity-issuing firms against firms that do not issue either equity or debt. The results are similar to those in panel A: Higher stock returns or market-to-book ratios significantly increase the probability that a firm issues equity, consistent with the results of Hovakimian et al. (2004).

Intuitively, the market price of a stock is determined by outside investors’ valuations, which in turn depend on the firm’s earnings realization. Therefore, when the firm experiences a positive shock to earnings, the difference between the investors’ and manager’s beliefs increases, stock returns and market-to-book ratios rise, and the manager has a greater incentive to issue equity and reduce leverage.21

The model thus explains the “hot equity issuance” phenomenon documented in empirical research. It is important to note that this result depends only on the fact that investors and manager have different opinions, and not on which party has more accurate information. Section 5.1 verifies that this result persists for different true underlying models of the economy.

5. Alternative assumptions

This section examines the robustness of the model predictions under different assumptions.

---

21 This intuition relies on the fact that outside investors update beliefs more readily on earnings realizations than the manager does. However, the opposite result is true when the manager relies more on earnings than outsiders (see Section 5.2).
Table 6
Equity issuance decisions. This table reports the results of multinomial logit regressions for the simulated panel data from the model, together with empirical results from previous studies. The dependent variable is the dummy for equity issuance. In panel A, the sample consists of the firm-years that issue an amount of debt or equity that is at least 5% of the market value of equity. In panel B, the sample consists of firms that issue equity (5% or more) and those that issue neither equity nor debt. The definitions of the variables for the simulated data are the same as in Table 3. The t-statistics are reported in parentheses. Prior stock returns are the one-year prior stock returns, except in Hovakimian et al. (2001), in which case they are the two-year prior stock returns. In the first columns (Model FM), logit regressions are carried out year by year and then the averaged coefficients and corresponding t-statistics are reported, as in Fama and MacBeth (1973). In the second columns of panels A and B, logit models with fixed year effects are used for the model-generated panel data. The other columns report results from Marsh (1982, Table 3), Hovakimian et al. (2001), Table 5 or HOT, and Hovakimian et al. (2004), Table 6 for panel A and Table 8 for panel B, or HHT.

### Panel A. Equity versus debt

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior stock return</td>
<td>48.3</td>
<td>55.5</td>
<td>1.026</td>
<td>0.542</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>(4.45)</td>
<td>(29.34)</td>
<td>(4.0)</td>
<td>(19.0)</td>
<td>(3.8)</td>
</tr>
<tr>
<td>MA/BA</td>
<td>3.32</td>
<td>7.26</td>
<td>0.41</td>
<td>0.179</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td>(5.41)</td>
<td>(13.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market leverage</td>
<td>−2.71</td>
<td>5.43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.76)</td>
<td>(6.88)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other control variables</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Fixed year effects</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.8451</td>
<td>0.9500</td>
<td>0.37</td>
<td>0.195</td>
<td>0.228</td>
</tr>
<tr>
<td></td>
<td>(59.31)</td>
<td>(98.19)</td>
<td>(10.0)</td>
<td>(18.2)</td>
<td></td>
</tr>
<tr>
<td>No. of equity issuances</td>
<td>9430</td>
<td>9430</td>
<td>349</td>
<td>2231</td>
<td>2082</td>
</tr>
<tr>
<td>No. of observations</td>
<td>17,318</td>
<td>17,318</td>
<td>748</td>
<td>10,123</td>
<td>12,300</td>
</tr>
</tbody>
</table>

### Panel B. Equity versus no issuance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model FM Mean</th>
<th>Model FE Mean</th>
<th>Marsh (2004) Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior stock return</td>
<td>15.4</td>
<td>15</td>
<td>0.370</td>
</tr>
<tr>
<td></td>
<td>(59.31)</td>
<td>(98.19)</td>
<td>(10.0)</td>
</tr>
<tr>
<td>MA/BA</td>
<td>7.04</td>
<td>6.85</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>(43.32)</td>
<td>(45.03)</td>
<td>(18.2)</td>
</tr>
<tr>
<td>Market leverage</td>
<td>1.94</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(17.14)</td>
<td>(21.03)</td>
<td></td>
</tr>
<tr>
<td>Other control variables</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Fixed year effects</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.4487</td>
<td>0.4422</td>
<td></td>
</tr>
<tr>
<td>No. of equity issuances</td>
<td>9430</td>
<td>9430</td>
<td>2082</td>
</tr>
<tr>
<td>No. of observations</td>
<td>66,263</td>
<td>66,263</td>
<td>23,946</td>
</tr>
</tbody>
</table>

5.1. True model of earnings

For simplicity, the previous sections have focused on the case when the manager’s belief is consistent with the true model of earnings. The intuitions behind the results in earlier sections, however, suggest that most of the predictions depend only on the assumption of differences in opinion between the manager and investors, and not on which agent has the correct belief.

Table 7
Model predictions under different true models of earnings. This table reports the main predictions of the model for cases with different parameters for the true model of earnings. The parameters in the different cases are the same as in the baseline case given in Table 1, except for the true standard deviation of the earnings shock $\sigma_e$. For each case, a simulated economy of 1000 firms for 100 years is generated, and the first 20 years of data are dropped. Summary statistics of the market leverage in the simulated economy are reported. The variable “ZL Frac” is the fraction of zero or negative net-debt firms, and “PredictEq” provides the coefficient of the Fama–MacBeth mean regression coefficient of the equity issuance dummy on prior stock returns in logit regressions as in Table 6. The t-statistics are provided in parentheses.

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>Mean</th>
<th>Median</th>
<th>Std</th>
<th>ZL Frac</th>
<th>PredictEq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark case</td>
<td>0.74</td>
<td>0.78</td>
<td>0.11</td>
<td>0.00</td>
<td>66.79</td>
</tr>
<tr>
<td>$\sigma_e = 0.0$ (investors’ belief)</td>
<td>0.35</td>
<td>0.30</td>
<td>0.14</td>
<td>0.00</td>
<td>(29.33)</td>
</tr>
<tr>
<td>$\sigma_e = 0.1$</td>
<td>0.30</td>
<td>0.28</td>
<td>0.15</td>
<td>0.00</td>
<td>95.91</td>
</tr>
<tr>
<td>$\sigma_e = 0.2$</td>
<td>0.23</td>
<td>0.19</td>
<td>0.16</td>
<td>0.00</td>
<td>86.91</td>
</tr>
<tr>
<td>$\sigma_e = 0.3$ (manager’s belief)</td>
<td>0.18</td>
<td>0.13</td>
<td>0.16</td>
<td>0.041</td>
<td>48.35</td>
</tr>
<tr>
<td>$\sigma_e = 0.4$</td>
<td>0.11</td>
<td>0.07</td>
<td>0.16</td>
<td>0.22</td>
<td>23.99</td>
</tr>
</tbody>
</table>
The underlying true model is specified by the volatility of earnings shocks $\sigma_e$ in each period. By setting $\sigma_e$ to different values, the potential true models span the investors' belief ($\sigma_e = \sigma^I_e$) and the manager's belief ($\sigma_e = \sigma^M_e$) and, for further robustness checks, offer other cases in which neither agent's belief is correct. We examine the main predictions of the model for different cases and the results are reported in Table 7.

The results in Table 7 show that in all the cases of the true model of earnings, the dynamic model of heterogeneous beliefs predicts a low average debt ratio and a higher standard deviation in debt ratios compared to the benchmark case with homogeneous beliefs. When $\sigma_e$ is sufficiently large, there is a nontrivial fraction of zero-debt firms. A comparison across different cases shows that the degree of debt conservatism of firms increases with the parameter $\sigma_e$.

The intuition is that more volatile earnings lead to greater differences in belief due to the differential interpretations of the earnings signal by outside investors and the manager, which in turn generate greater deviations of the optimal leverage ratios from the benchmark values. Greater debt conservatism then follows from the asymmetric deviations of optimal leverage: Deviations toward zero or negative leverage ratios tend to be larger than those toward the default boundary.

The results also show that high stock returns can predict equity issuance, regardless of the true model. In unreported results, the long-run abnormal returns of the firm following equity issuances (repurchases) are significantly negative (positive) for all specifications of the true model. These findings confirm the intuition that equity issuance and long-run performance depend only on the relative valuation between the manager and investors, and not on which party has superior information.

### Table 8

Alternative case: the manager's belief is more correlated with earnings. This table reports the main results of the model with the same parameters shown in Table 1, except that $\sigma^I_e = 0.3$ and $\sigma^M_e = 0.0$, that is, the investors and the manager switched their beliefs compared to the baseline model.

| Panel A. Summary statistics of leverage ratios and equity issuance prediction |
|-----------------------------|------------------|------------------|------------------|------------------|------------------|
| Benchmark case              | Mean  | Median | Std   | 10 pct | 25 pct | ZL Frac | PredictEq |
| $\sigma_e = 0.3$ (investors' belief) | 0.47  | 0.45   | 0.15  | 0.30   | 0.35   | 0.00   | −160.36 |
| $\sigma_e = 0.0$ (manager's belief) | 0.41  | 0.36   | 0.17  | 0.23   | 0.31   | 0.00   | −192.67 |

| Panel B. Behavior of zero-debt firms (investors' belief) |
|------------------|------------------|------------------|
| Profitability    | Div/BA | Cash/BA | No. obs. |
| Zero-leverage sample | 0.08   | 0.07   | 0.06   | 25    |
| Total sample     | 0.11   | 0.06   | 0.00   | 75,859|
| Difference t-Stat| −7.34  | 2.10   | 4.83   |

| Panel C. Long-run abnormal returns |
|------------------|------------------|------------------|
| Issuance         | Repurchase       | Issuance         | Repurchase       |
| Investor's belief|                   | Manager's belief|                   |
|                   |                   |                   |                   |
| Year 1           | 0.62              | 10.86            | −0.08            | −1.39            | −0.36            | −16.57           | 0.14             | 7.87             |
| Year 2           | −0.29             | −4.93            | 0.25             | 4.64             | −0.46            | −21.54           | 0.13             | 6.86             |
| Year 3           | −0.18             | −2.98            | 0.11             | 2.00             | −0.17            | −7.80            | 0.05             | 2.50             |
| Year 4           | −0.15             | −2.51            | 0.08             | 1.42             | −0.22            | −10.24           | 0.04             | 2.43             |

5.2. Manager’s belief more correlated with earnings

This section considers the case where the manager’s belief is more correlated with earnings than the investors’ belief. In this case, all parameters are the same as in the baseline case (Table 1), except that $\sigma^I_e = 0.3$ and $\sigma^M_e = 0.0$.

The results for this alternative case are presented in Table 8. First, panel A shows that the leverage ratios are much lower and variable than the benchmark case where beliefs are symmetric. High past returns now predict a lower probability of equity issuance because investors react more slowly than the manager to earnings shocks, and prior returns are positively related to profitability and undervaluation.

There are very few zero-debt firms in this setting. The comparison of the small zero-leverage sample with other firms in panel B of Table 8 shows that zero-debt firms still pay higher dividends and keep higher cash holdings but are less profitable. The intuition is that zero-leverage firms are overvalued by outside investors. The relative optimism of outsiders increases when earnings decline because the manager updates more quickly based on the reported earnings in this case.
Finally, panel C of Table 8 examines the long-run abnormal returns of equity issuance decisions. It shows that the long-run abnormal return results hold for both the case when the outside investors’ belief is the same as in the true model (columns 1 to 4 in panel C) and for the case when the manager’s belief is the same as that in the true model (columns 5 to 8 in panel C).

In sum, some of the predictions – such as low leverage, the existence of zero-debt firms, and long-run negative returns following equity issuances – persist even in the case when the manager updates beliefs more readily on new earnings shocks than outsiders. This provides another robustness check of the intuitions of heterogeneous beliefs’ impact on capital structure decisions. However, some other predictions – such as the profitability of zero-debt firms relative to other firms and stock returns preceding equity issuances – differ from the main case and the empirical stylized facts. These results support the idea that the manager relies less on observable information (earnings) in updating belief than outside investors do.

6. Conclusion

This paper develops a dynamic structural model of corporate financing that deviates from a standard trade-off model only in that the manager and outside investors have differences in belief. In capital structure decisions, the manager trades off the tax advantages and default costs of debt and the benefit of issuing/repurchasing equity (perceived to be) mispriced by investors. As a result, the optimal leverage ratio is the point at which the marginal benefit of debt financing equals that of equity financing. The optimal leverage ratio now depends on the differences of opinion and can differ significantly from that of a standard trade-off model; for example, it may be optimal to eschew debt financing completely. Furthermore, the dependence of optimal leverage on changes in differences of opinion is asymmetric: Strong investor beliefs lead to greater decreases in optimal leverage than weak investor beliefs lead to increases, a feature that dynamically produces low leverage ratios. The model shows that heterogeneous beliefs can have a large impact on a firm’s optimal financing policy.

While parsimonious, the model is able to generate a number of well-documented empirical predictions that are difficult to explain with standard theories. The model predicts the low leverage in cross section, the substantial presence of firms with no debt and their high dividend payouts and cash holdings, and the long-run stock underperformance following equity issuances. When combined with the plausible assumption that outside investors rely more on observable information, the model also predicts the empirically observed high profitability of dividend-paying zero-debt firms and the phenomenon that equity tends to be issued after high stock returns. The results are robust to different model parameters, different specifications of the true model of earnings, and different degrees of the manager’s bargaining power relative to that of investors. The model also indicates that the costs of market timing, most of which are in the form of excessive financing costs, are substantial. An extended version of the model generates the testable predictions that market timing behavior, debt conservatism, and deadweight financing costs decrease with shareholder control relative to the manager. In particular, financial regulation that improves shareholder control can reduce market timing behavior and improve firm welfare.

This model bridges the expanding literature on differences in belief with the literature on structural models of corporate finance. It provides a simple framework for understanding market timing, debt conservatism, and other related phenomena in corporate financing. For simplicity, however, this model does not consider investment policies. A topic for future research is to consider jointly endogenous investment and financing decisions under heterogeneous beliefs. Such a model has the potential to explain the influence of heterogeneous beliefs and market timing on the efficiency of investment policies and the distortion of capital structure decisions.

Appendix A. Proofs

Proof. Proof of Proposition 1 and Corollary 2

The proposition is proved via induction. For simplicity of notation, the super-index $i$ is omitted in this proof and it is assumed that all beliefs refer to those of agent $i$. Assume that the belief of agent $i$ conditional on information in period $t − 1$ is

$$s_{t−1} \sim N(\eta_{t−1}, \sigma_{t−1}^2).$$

From Eq. (2) and the fact that the shock $\eta_t$ is independent of history, the belief conditional on information in period $t − 1$ about $s_t$ is

$$s_t \sim N(\rho q_{t−1} + (1−\rho)\bar{s}, \rho^2 \sigma_{t−1}^2 + \sigma_n^2).$$

(A1)

In period $t$, from Eqs. (1) and (3), the agent observes the signal

$$n_t = s_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2).$$

(A2)
Therefore, the agent updates the normal prior in (A1) based on a normal likelihood function in (A2). This yields a normal posterior belief

$$s_t \sim N(q_t, \sigma_t^2),$$

where the mean $q_t$ is the average of the prior mean and the signal $n_t$ weighted by precision,

$$q_t = \pi_t (\rho q_{t-1} + (1-\rho)\bar{s}) + (1-\pi_t)n_t,$$

$$\sigma_t^2 = \frac{\rho^2 \sigma_{t-1}^2 + \sigma_n^2}{\rho^2 \sigma_{t-1}^2 + \sigma_n^2 + \sigma_e^2},$$

and

$$\pi_t = \sigma_e^2 \frac{\sigma_t^2}{\rho^2 (\sigma_{t-1})^2 + \sigma_n^2 + \sigma_e^2}.$$

Now the induction is complete.

In the stationary case, the variance $\sigma_t^2$ of the belief does not change with time, that is, $\sigma_t^2 = \sigma^2$ is a constant. Substituting this in Eq. (A4), one obtains a quadratic equation in $\sigma^2$ and the solution is given by

$$\sigma^2 = \sqrt{\left(\sigma_0^2 + (1-\rho^2)\sigma_e^2\right)^2 + 4\rho^2\sigma_0^2\sigma_n^2 - \sigma_0^2 - \left(1-\rho^2\right)\sigma_e^2}.\quad (A6)$$

Appendix B. Procedure of numerical solution

The numerical solution of the model is obtained through a dynamic programming iteration. Since the state variables are continuous, discretization is used. In particular, nine points, geometrically distributed between $[-1.2, 1.2]$, are used for $n_t = q_t I$ and $q_t M$; 11 grid points, equally spaced between $[0, 1]$, are used for $\theta_t$; and 46 points are used for $c_t$, geometrically distributed away from zero in the interval $[-1.1, 3.5]$. The value functions are represented as functions on the grid points, and piecewise linear interpolation is used when function values on non-grid points are needed.

The expectation is computed using the Gauss–Hermite quadrature method with $n = 11$ sample points (Abramowitz and Stegun, 1972). In the quadrature method, the Gaussian integral is approximated by

$$\int_{-\infty}^{\infty} e^{-x^2} f(x) \, dx \approx \sum_{i=1}^{n} w_i f(x_i),\quad (B1)$$

where $x_i$ are the roots of the Hermite polynomial $H_n(x)$ and the associated weights are given by

$$w_i = \frac{2^n n! \sqrt{\pi}}{n^2 [H_{n-1}(x_i)]^2}.\quad (B2)$$

In the numerical solution of the model, the optimal financing policies, default time, and value functions are computed through backward reduction using the recursive formulas in Section 2.5. Stationary solutions are found by iterating the recursive procedure until the value and policy functions between adjacent iterations are less than $10^{-4}$. Normally, the procedure converges within 300 iterations.

References


23 The use of finer grids does not change the results qualitatively.