7-1  a. A portfolio is made up of a group of individual assets held in combination. An asset that would be relatively risky if held in isolation may have little, or even no risk if held in a well-diversified portfolio.

b. The feasible, or attainable, set represents all portfolios that can be constructed from a given set of stocks. This set is only efficient for part of its combinations.

c. An efficient portfolio is that portfolio which provides the highest expected return for any degree of risk. Alternatively, the efficient portfolio is that which provides the lowest degree of risk for any expected return.

d. The efficient frontier is the set of efficient portfolios out of the full set of potential portfolios. On a graph, the efficient frontier constitutes the boundary line of the set of potential portfolios.

e. An indifference curve is the risk/return trade-off function for a particular investor and reflects that investor's attitude toward risk. The indifference curve specifies an investor's required rate of return for a given level of risk. The greater the slope of the indifference curve, the greater is the investor's risk aversion.

f. The optimal portfolio for an investor is the point at which the efficient set of portfolios--the efficient frontier--is just tangent to the investor's indifference curve. This point marks the highest level of satisfaction an investor can attain given the set of potential portfolios.

g. The Capital Asset Pricing Model (CAPM) is a general equilibrium market model developed to analyze the relationship between risk and required rates of return on assets when they are held in well-diversified portfolios. The SML is part of the CAPM.

h. The Capital Market Line (CML) specifies the efficient set of portfolios an investor can attain by combining a risk-free asset and the risky market portfolio M. The CML states that the expected return on any efficient portfolio is equal to the riskless rate plus
a risk premium, and thus describes a linear relationship between expected return and risk.

i. The characteristic line for a particular stock is obtained by regressing the historical returns on that stock against the historical returns on the general stock market. The slope of the characteristic line is the stock's beta, which measures the amount by which the stock's expected return increases for a given increase in the expected return on the market.

j. The beta coefficient (\( \beta \)) is a measure of a stock's market risk. It measures the stock's volatility relative to an average stock, which has a beta of 1.0.

k. Arbitrage Pricing Theory (APT) is an approach to measuring the equilibrium risk/return relationship for a given stock as a function of multiple factors, rather than the single factor (the market return) used by the CAPM. The APT is based on complex mathematical and statistical theory, but can account for several factors (such as GNP and the level of inflation) in determining the required return for a particular stock.

l. The Fama-French 3-factor model has one factor for the excess market return (the market return minus the risk free rate), a second factor for size (defined as the return on a portfolio of small firms minus the return on a portfolio of big firms), and a third factor for the book-to-market effect (defined as the return on a portfolio of firms with a high book-to-market ratio minus the return on a portfolio of firms with a low book-to-market ratio).

m. Most people don’t behave rationally in all aspects of their personal lives, and behavioral finance assume that investors have the same types of psychological behaviors in their financial lives as in their personal lives.

7-2 Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because \( \sigma_A > \sigma_B \) and \( CV_A > CV_B \).
7-1  a. A plot of the approximate regression line is shown in the following figure:

![Graph showing a regression line with labeled axes and points](image)

The equation of the regression line is

\[ k_x = a + b k_m. \]

The stock's approximate beta coefficient is given by the slope of the regression line:

\[ b = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta Y}{\Delta X} = \frac{23 - (-14)}{37.2 - (-26.5)} = \frac{37}{63.7} = 0.6. \]

The intercept, \( a \), seems to be about 3.5. Using a calculator with a least squares regression routine, we find the exact equation to be \( k_x = 3.7 + 0.56 k_m \), with \( r = 0.96 \).

b. The arithmetic average return for Stock X is calculated as follows:

\[ k_{\text{avg}} = \frac{-14.0 + 23.0 + \ldots + 18.2}{7} = 10.6\%. \]

The arithmetic average rate of return on the market portfolio, determined similarly, is 12.1\%.
For Stock X, the estimated standard deviation is 13.1 percent:

\[ \sigma_X = \sqrt{\frac{(-14.0 - 10.6)^2 + (23.0 - 10.6)^2 + \ldots + (18.2 - 10.6)^2}{7 - 1}} = 13.1\% . \]

The standard deviation of returns for the market portfolio is similarly determined to be 22.6 percent. The results are summarized below:

<table>
<thead>
<tr>
<th></th>
<th>Stock X</th>
<th>Market Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return, ( \bar{k}_{avg} )</td>
<td>10.6%</td>
<td>12.1%</td>
</tr>
<tr>
<td>Standard deviation, ( \sigma )</td>
<td>13.1</td>
<td>22.6</td>
</tr>
</tbody>
</table>

Several points should be noted: (1) \( \sigma_M \) over this particular period is higher than the historic average \( \sigma_M \) of about 15 percent, indicating that the stock market was relatively volatile during this period; (2) Stock X, with \( \sigma_X = 13.1\% \), has much less total risk than an average stock, with \( \sigma_{avg} = 22.6\% \); and (3) this example demonstrates that it is possible for a very low-risk single stock to have less risk than a portfolio of average stocks, since \( \sigma_X < \sigma_M \).

c. Since Stock X is in equilibrium and plots on the Security Market Line (SML), and given the further assumption that \( \bar{k}_X = \bar{k}_m \) and \( \hat{k}_X = \hat{k}_M \)--and this assumption often does not hold--then this equation must hold:

\[ \bar{k}_X = k_{sr} + (\bar{k} - k_{sr})b_X. \]

This equation can be solved for the risk-free rate, \( k_{rf} \), which is the only unknown:

\[
egin{align*}
10.6 &= k_{sr} + (12.1 - k_{sr})0.56 \\
10.6 &= k_{sr} + 6.8 - 0.56k_{rf} \\
0.44k_{rf} &= 10.6 - 6.8 \\
k_{rf} &= 3.8 / 0.44 = 8.6\% .
\end{align*}
\]

d. The SML is plotted below. Data on the risk-free security (\( b_{rf} = 0 \), \( k_{rf} = 8.6\% \)) and Security X (\( b_X = 0.56 \), \( \bar{k}_X = 10.6\% \)) provide the two points through which the SML can be drawn. \( k_M \) provides a third point.
e. In theory, you would be indifferent between the two stocks. Since they have the same beta, their relevant risks are identical, and in equilibrium they should provide the same returns. The two stocks would be represented by a single point on the SML. Stock Y, with the higher standard deviation, has more diversifiable risk, but this risk will be eliminated in a well-diversified portfolio, so the market will compensate the investor only for bearing market or relevant risk. In practice, it is possible that Stock Y would have a slightly higher required return, but this premium for diversifiable risk would be small.
a. The regression graph is shown above. b will depend on students' freehand line. Using a calculator, we find b = 0.62.

b. Because b = 0.62, Stock Y is about 62 percent as volatile as the market; thus, its relative risk is about 62 percent of that of an average firm.

c. 1. Total risk (\( \sigma^2_r \)) would be greater because the second term of the firm's risk equation, \( \sigma^2_r = b^2 \sigma^2_m + \sigma^2_{e^r} \), would be greater.

2. CAPM assumes that company-specific risk will be eliminated in a portfolio, so the risk premium under the CAPM would not be affected.

d. 1. The stock's variance would not change, but the risk of the stock to an investor holding a diversified portfolio would be greatly reduced.

2. It would now have a negative correlation with \( k_m \).

3. Because of a relative scarcity of such stocks and the beneficial net effect on portfolios that include it, its "risk premium" is likely to be very low or even negative. Theoretically, it should be negative.

e. The following figure shows a possible set of probability distributions. We can be reasonably sure that the 100-stock portfolio comprised of b = 0.62 stocks as described in Condition 2 will be less risky than the "market." Hence, the distribution for Condition 2 will be more
peaked than that of Condition 3. This statement can also be made on the basis of an analytical approach as shown by the material following the graph.

\[ \hat{k}_Y = \hat{k}_{100} = \hat{k}_M = 9.8\%. \quad \sigma_i^2 = b_i^2 \sigma_n^2. \]

For Condition 2, with 100 stocks in the portfolio, \( \sigma_i^2 = 0 \), so

\[ \sigma_p^2 = (0.62)^2 \sigma_n^2, \quad \sigma_p = \sqrt{(0.62)^2 \sigma_n^2} = 0.62 \sigma_n. \]

Since \( \sigma_p \) is only 62 percent of \( \sigma_n \), the probability distribution for Condition 2 is clearly more peaked than that for Condition 3; thus, we can be reasonably confident of the relevant locations of the distributions for Conditions 2 and 3.

With regard to Condition 1, the single-asset portfolio, we can be sure that its probability distribution is less peaked than that for the 100-stock portfolio. Analytically, since \( b = 0.62 \) both for the single stock portfolio and for the 100-stock portfolio,

\[ \sigma_Y^2 = (0.62 \sigma_n)^2 + \sigma_i^2 > (0.62 \sigma_n)^2 + 0 = \sigma_p^2. \]

We can also say on the basis of the available information that \( \sigma_Y \) is smaller than \( \sigma_n \); Stock Y's market risk is only 62 percent of the "market," but it does have company-specific risk, while the market portfolio does not. However, we know from the given data that \( \sigma_Y = 13.8\% \), while \( \sigma_n = 19.6\% \). Thus, we have drawn the distribution for the single stock portfolio more peaked than that of the market. The relative rates of return are not reasonable. The return for any stock should be

\[ k_i = k_{RF} + (k_n - k_{RF}) b_i. \]

Stock Y has \( b = 0.62 \), while the average stock (M) has \( b = 1.0 \); therefore,

\[ k_Y = k_{RF} + (k_n - k_{RF}) 0.62 < k_n = k_{RF} + (k_n - k_{RF}) 1.0. \]

A disequilibrium exists--Stock Y should be bid up to drive its yield down. More likely, however, the data simply reflect the fact that past returns are not an exact basis for expectations of future returns.

7-3  a. \( k_i = k_{RF} + (k_n - k_{RF}) b_i = k_{RF} + (k_n - k_{RF}) \frac{r_m \sigma_i}{\sigma_n}. \)

b. CML: \( \hat{k}_p = k_{RF} + \left( \frac{k_n - k_{RF}}{\sigma_n} \right) \sigma_p. \) SML: \( k_i = k_{RF} + \left( \frac{k_n - k_{RF}}{\sigma_n} \right) r_m \sigma_i. \)
With some arranging, the similarities between the CML and SML are obvious. When in this form, both have the same market price of risk, or slope, \( (k_M - k_{RF})/\sigma_M \).

The measure of risk in the CML is \( \sigma_p \). Since the CML applies only to efficient portfolios, \( \sigma_p \) not only represents the portfolio's total risk, but also its market risk. However, the SML applies to all portfolios and individual securities. Thus, the appropriate risk measure is not \( \sigma_i \), the total risk, but the market risk, which in this form of the SML is \( r_{IM}\sigma_i \), and is less than for all assets except those which are perfectly positively correlated with the market, and hence have \( r_{IM} = +1.0 \).

7-4 a. Using the CAPM:
\[ k_i = k_{RF} + (k_M - k_{RF})b_i = 7\% + (1.1)(6.5\%) = 14.15\% . \]

b. Using the 3-factor model:
\[ k_i = k_{RF} + (k_M - k_{RF})b_i + (k_{SMB})c_i + (k_{HML})d_i \\
= 7\% + (1.1)(6.5\%) + (5\%)(0.7) + (4\%)(-0.3) = 16.45\% . \]