Chapter 8
Time Value of Money
ANSWERS TO SELECTED END-OF-CHAPTER QUESTIONS

8-1  a. PV (present value) is the value today of a future payment, or stream of payments, discounted at the appropriate rate of interest. PV is also the beginning amount that will grow to some future value. The parameter i is the periodic interest rate that an account pays. The parameter INT is the dollars of interest earned each period. FVₙ (future value) is the ending amount in an account, where n is the number of periods the money is left in the account. FVAₙ is the value today of a future stream of equal payments (an annuity) and FVAₙ is the ending value of a stream of equal payments, where n is the number of payments of the annuity. PMT is equal to the dollar amount of an equal, or constant cash flow (an annuity). In the EAR equation, m is used to denote the number of compounding periods per year, while i Nom is the nominal, or quoted, interest rate.

b. FVIFᵢᵣ,ₙ is the future value interest factor for a lump sum left in an account for n periods paying i percent interest per period. PVIFᵢᵣ,ₙ is the present value interest factor for a lump sum received n periods in the future discounted at i percent per period. FVIFAᵢᵣ,ₙ is the future value interest factor for an ordinary annuity of n periodic payments paying i percent interest per period. PVIFAᵢᵣ,ₙ is the present value interest factor for an ordinary annuity of n periodic payments discounted at i percent interest per period. All the above factors represent the appropriate PV or FV when the lump sum or ordinary annuity payment is $1. Note that the above factors can also be defined using formulas.

c. The opportunity cost rate (i) of an investment is the rate of return available on the best alternative investment of similar risk.

d. An annuity is a series of payments of a fixed amount for a specified number of periods. A single sum, or lump sum payment, as opposed to an annuity, consists of one payment occurring now or at some future time. A cash flow can be an inflow (a receipt) or an outflow (a deposit, a cost, or an amount paid). We distinguish between the terms cash flow and PMT. We use the term cash flow for uneven streams, while we use the term PMT for annuities, or constant payment amounts. An uneven cash flow stream is a series of cash flows in which the amount varies from one period to the next. The PV (or FVₙ) of an uneven payment stream is merely the sum of the present values (or future values) of each individual payment.
e. An ordinary annuity has payments occurring at the end of each period. A deferred annuity is just another name for an ordinary annuity. An annuity due has payments occurring at the beginning of each period. Most financial calculators will accommodate either type of annuity. The payment period must be equal to the compounding period.

f. A perpetuity is a series of payments of a fixed amount that last indefinitely. In other words, a perpetuity is an annuity where \( n \) equals infinity. Consol is another term for perpetuity. Consols were originally bonds issued by England in 1815 to consolidate past debt.

g. An outflow is a deposit, a cost, or an amount paid, while an inflow is a receipt. A time line is an important tool used in time value of money analysis; it is a graphical representation which is used to show the timing of cash flows.

h. Compounding is the process of finding the future value of a single payment or series of payments. Discounting is the process of finding the present value of a single payment or series of payments; it is the reverse of compounding.

i. Annual compounding means that interest is paid once a year. In semiannual, quarterly, monthly, and daily compounding, interest is paid 2, 4, 12, and 365 times per year respectively. When compounding occurs more frequently than once a year, you earn interest on interest more often, thus increasing the future value. The more frequent the compounding, the higher the future value.

j. The effective annual rate is the rate that, under annual compounding, would have produced the same future value at the end of 1 year as was produced by more frequent compounding, say quarterly. The nominal (quoted) interest rate, \( i_{\text{nom}} \), is the rate of interest stated in a contract. If the compounding occurs annually, the effective annual rate and the nominal rate are the same. If compounding occurs more frequently, the effective annual rate is greater than the nominal rate. The nominal annual interest rate is also called the annual percentage rate, or APR. The periodic rate, \( i_{\text{PER}} \), is the rate charged by a lender or paid by a borrower each period. It can be a rate per year, per 6-month period, per quarter, per month, per day, or per any other time interval (usually one year or less).

k. An amortization schedule is a table that breaks down the periodic fixed payment of an installment loan into its principal and interest components. The principal component of each payment reduces the remaining principal balance. The interest component is the interest payment on the beginning-of-period principal balance. An amortized loan is one that is repaid in equal periodic amounts (or "killed off" over time).
1. The terminal value is the future value of an uneven cash flow stream.

8-2 The opportunity cost rate is the rate of interest one could earn on an alternative investment with a risk equal to the risk of the investment in question. This is the value of i in the TVM equations, and it is shown on the top of a time line, between the first and second tick marks. It is not a single rate—the opportunity cost rate varies depending on the riskiness and maturity of an investment, and it also varies from year to year depending on inflationary expectations (see Chapter 5).

8-3 True. The second series is an uneven payment stream, but it contains an annuity of $400 for 8 years. The series could also be thought of as a $100 annuity for 10 years plus an additional payment of $100 in Year 2, plus additional payments of $300 in Years 3 through 10.

8-4 True, because of compounding effects—growth on growth. The following example demonstrates the point. The annual growth rate is i in the following equation:

\[ \$1(1 + i)^{10} = \$2. \]

The term \((1 + i)^{10}\) is the FVIF for i percent, 10 years. We can find i in one of two ways:

1. Using a financial calculator input \(N = 10, PV = -1, PMT = 0, FV = 2\), and \(I = ?\). Solving for I you obtain 7.18%.

2. Using a financial calculator, input \(N = 10, I = 10, PV = -1, PMT = 0\), and \(FV = ?\). Solving for FV you obtain \$2.59. This formulation recognizes the "interest on interest" phenomenon.

8-5 For the same stated rate, daily compounding is best. You would earn more "interest on interest."

8-6 False. One can find the present value of an embedded annuity and add this PV to the PVs of the other individual cash flows to determine the present value of the stream of cash flows.

8-7 The concept of a perpetuity implies that payments will be received forever. \(FV \ (Perpetuity) = PV \ (Perpetuity)(1 + i)^\infty = \infty.\)
SOLUTIONS TO END-OF-CHAPTER PROBLEMS

8-1

a. 0 1
   6% 1
   $500(1.06) = $530.00.
   -500  FV = ?

b. 0 1 2
   6% 1 2
   $500(1.06)^2 = $561.80.
   -500  FV = ?

c. 0 1
   6% 1
   $500(1/1.06) = $471.70.
   PV = ? 500

d. 0 1 2
   6% 1 2
   $500(1/1.06)^2 = $445.00.
   PV = ? 500

8-2

a. 0 1 2 3 4 5 6 7 8 9 10
   6% 1 2 3 4 5 6 7 8 9 10
   $500(FVIF_{6%,10}) =
   -500  FV = ?
   $500(1.7908) = $895.40.

b. 0 1 2 3 4 5 6 7 8 9 10
   12% 1 2 3 4 5 6 7 8 9 10
   $500(FVIF_{12%,10}) =
   -500  FV = ?
   $500(1.1058) = $1,552.90.

c. 0 1 2 3 4 5 6 7 8 9 10
   6% 1 2 3 4 5 6 7 8 9 10
   $500(FVIF_{6%,10}) =
   PV = ? 500
   $500(0.5584) = $279.20.

d. 0 1 2 3 4 5 6 7 8 9 10
   12% 1 2 3 4 5 6 7 8 9 10
   PV = ? 1,552.90

$1,552.90(FVIF_{12%,10}) = $1,552.90(FVIF_{6%,10}) =
$1,552.90(0.3220) = $500.03; i = 6%: $1,552.90(0.5584) = $867.14.

The present value is the value today of a sum of money to be received in the future. For example, the value today of $1,552.90 to be received 10 years in the future is about $500 at an interest rate of 12 percent, but it is approximately $867 if the interest rate is 6 percent. Therefore, if you had $500 today and invested it at 12 percent, you would end up with $1,552.90 in 10 years. The present value depends on the interest rate because the interest rate determines the amount of interest you forgo by not having the money today.
8-3  a. $400 = $200(FVIF_{7\%},n)
-200  400
2 = FVIF_{7\%},n
n \approx 10\ years.

With a financial calculator, enter I = 7, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 10.24. Override I with the other values to find N = 7.27, 4.19, and 1.00.

b. 2 = FVIF_{10\%},n
-200  400
n \approx 7\ years.

c. 2 = FVIF_{18\%},n
-200  400
n \approx 4\ years.

d. 2 = FVIF_{100\%},n
-200  400
n = 1\ year.

8-4 The general formula is \( FVA_n = PMT(FVIFA_{i,n}) \).

a. 400  400  400  400  400  400  400  400  400  400
FV = ?

\( FVA_{10} = ($400)15.9374 = $6,374.96. \)

With a financial calculator, enter N = 10, I = 10, PV = 0, and PMT = -400. Then press the FV key to find FV = $6,374.97.

b. ($200)5.5256 = $1,105.12.

With a financial calculator, enter N = 5, I = 5, PV = 0, and PMT = -200. Then press the FV key to find FV = $1,105.13.

c. ($400)5 = $2,000.00.

With a financial calculator, enter N = 5, I = 0, PV = 0, and PMT = -400. Then press the FV key to find FV = $2,000.
d. To solve Part d using a financial calculator, repeat the procedures discussed in Parts a, b, and c, but first switch the calculator to "BEG" mode. Make sure you switch the calculator back to "END" mode after working the problem.

\[
\begin{array}{ccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
400 & 400 & 400 & 400 & 400 & 400 & 400 & 400 & 400 \\
\end{array}
\]

FVA \(_n\) (Annuity due) = PMT(FVIFA\(_{i,n}\)) (1 + i). Therefore,

\[
\text{FVA}_{10} = \$400(15.9374)(1.10) = \$7,012.46.
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
200 & 200 & 200 & 200 & 200 \\
\end{array}
\]

FVA \(_n\) = PMT(FVIFA\(_{i,n}\)) (1 + i). Therefore,

\[
\text{FVA}_5 = \$200(5.5256)(1.05) = \$1,160.38.
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
400 & 400 & 400 & 400 & 400 \\
\end{array}
\]

FVA \(_n\) = PMT(FVIFA\(_{i,n}\)) (1 + i). Therefore,

\[
\text{FVA}_5 = \$400(5)(1.00) = \$2,000.00.
\]

8-5 The general formula is PVA \(_n\) = PMT(PVIFA\(_{i,n}\)).

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\text{PV} = ? & 400 & 400 & 400 & 400 & 400 & 400 & 400 & 400 \\
\end{array}
\]

With a financial calculator, simply enter the known values and then press the key for the unknowns. Except for rounding errors, the answers are as given below.

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{PV} = ? & 200 & 200 & 200 & 200 \\
\end{array}
\]

\[
\$200(4.3295) = \$865.90.
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{PV} = ? & 400 & 400 & 400 & 400 \\
\end{array}
\]

\[
\$400(5) = \$2,000.00.
\]

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
400 & 400 & 400 & 400 & 400 & 400 & 400 & 400 \\
\end{array}
\]

PVA \(_n\) (Annuity due) = PMT(PVIFA\(_{i,n}\))(1 + i). Therefore,

\[
\$400(6.1446)(1.10) = \$2,703.62.
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
200 & 200 & 200 & 200 \\
\end{array}
\]

PVA \(_n\) (Annuity due) = $200(4.3295)(1.05) = $909.20.
8-6  a.  

```
                 0  1  2  3  4  5
     PV = ?
     8% 100 400 400 400 300
```

With a financial calculator, simply enter the cash flows (be sure to enter \( CF_0 = 0 \)), enter \( I = 8 \), and press the NPV key to find \( NPV = PV = 1,125.25 \) for the first problem. Override \( I = 8 \) with \( I = 0 \) to find the next PV for Cash Stream A. Repeat for Cash Stream B to get \( NPV = PV = 1,300.32 \).

b. \( PV_A = 100 + 400 + 400 + 400 + 300 = 1,600 \).
\( PV_B = 300 + 400 + 400 + 400 + 100 = 1,600 \)

8-7  These problems can all be solved using a financial calculator by entering the known values shown on the time lines and then pressing the I button.

a.  

```
            0  1
     +700  i = ?  -749
     7 percent: \( 700 = 749(PVIF_{i,1}) \); \( PVIF_{i,1} = 0.9346 \).
```

b.  

```
            0  1
     -700  i = ?  +749
     7 percent.
```

c.  

```
            0  10
     +85,000  i = ?  -201,229

$201,229/$85,000 = 2.3674 = PVIF_{i,10}; \( i = 9\% \).
```

d.  

```
            0  1  2  3  4  5
     +9,000  -2,684.80  -2,684.80  -2,684.80  -2,684.80  -2,684.80

$9,000/$2,684.80 = 3.3522 = PVIFA_{i,5}; \( i = 15\% \).
```

8-8  a.  

```
            0  1  2  3  4  5
     -500  i = 12\%  FV = ?
```

\( PV = 800(5)(1.00) = 2,200.00 \).
With a financial calculator, enter N = 5, I = 12, PV = -500, and PMT = 0, and then press FV to obtain FV = $881.17. With a regular calculator, proceed as follows:

\[ FV_n = PV (1 + i)^n = 500 (1.12)^5 = 500 (1.7623) = 881.15. \]

b.  
\[
\begin{array}{cccccccccccc}
0 & 6\% & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
-500 & & & & & & & & & & & \\
\end{array}
\]

Enter the time line values into a financial calculator to obtain FV = $895.42, or

\[
PV_n = PV \left( 1 + \frac{i}{m} \right)^{mn} \\
= 500 \left( 1 + \frac{0.12}{2} \right)^{10} \\
= 500 (FVIF_{6\%, 10}) = 500 (1.7908) = 895.40.
\]

c.  
\[
\begin{array}{cccccccc}
0 & 3\% & 4 & 8 & 12 & 16 & 20 \\
-500 & & & & & & & \\
\end{array}
\]

Enter the time line values into a financial calculator to obtain FV = $903.06, or

\[
FV_n = 500 \left( 1 + \frac{0.12}{4} \right)^{40} = 500 (1.03)^{20} = 500 (1.8061) = 903.05.
\]

d.  
\[
\begin{array}{cccccccc}
0 & 1\% & 12 & 24 & 36 & 48 & 60 \\
-500 & & & & & & & \\
\end{array}
\]

Enter the time line values into a financial calculator to obtain FV = $908.35, or

\[
FV_n = 500 \left( 1 + \frac{0.12}{12} \right)^{120} = 500 (1.01)^{60} = 500 (1.8167) = 908.35.
\]

a.  
\[
\begin{array}{cccccccc}
0 & 6\% & 2 & 4 & 6 & 8 & 10 \\
500 & & & & & & & \\
\end{array}
\]

Enter the time line values into a financial calculator to obtain PV = $279.20, or
PV = \left( \frac{1}{1 + \frac{1}{m}} \right)^n = 500 \left( \frac{1}{1 + \frac{0.12}{2}} \right)^{10}
= 500 \left( \frac{1}{1.06} \right)^{10} = 500(PVIF_{6\%}, 10) = 500(0.5584) = 279.20.

b. 0 3% 4 8 12 16 20
PV = ?

Enter the time line values into a financial calculator to obtain PV = $276.84, or

PV = 500 \left( \frac{1}{1 + \frac{0.12}{4}} \right)^{40} = 500 \left( \frac{1}{1.03} \right)^{20} = 500(0.5537) = 276.85.

c. 0 1% 1 2 12
PV = ?

Enter the time line values into a financial calculator to obtain PV = $443.72, or

PV = 500 \left( \frac{1}{1 + \frac{0.12}{12}} \right)^{12} = 500 \left( \frac{1}{1.01} \right)^{-12} = 500(0.8874) = 443.70.

8-10 a. 0 6% 1 2 3 9 10
400 400 400 400 400

Enter N = 5 x 2 = 10, I = 12/2 = 6, PV = 0, PMT = -400, and then press FV to get FV = $5,272.32.

b. Now the number of periods is calculated as N = 5 x 4 = 20, I = 12/4 = 3, PV = 0, and PMT = -200. The calculator solution is $5,374.07.

Note that the solution assumes that the nominal interest rate is compounded at the annuity period.
c. The annuity in Part b earns more because some of the money is on deposit for a longer period of time and thus earns more interest. Also, because compounding is more frequent, more interest is earned on interest.

8-11 a. Universal Bank: Effective rate = 7%.

Regional Bank:

Effective rate = \left( 1 + \frac{0.06}{4} \right)^4 - 1.0 = (1.015)^4 - 1.0
= 1.0614 - 1.0 = 0.0614 = 6.14%.

With a financial calculator, you can use the interest rate conversion feature to obtain the same answer. You would choose the Universal Bank.

b. If funds must be left on deposit until the end of the compounding period (1 year for Universal and 1 quarter for Regional), and you think there is a high probability that you will make a withdrawal during the year, the Regional account might be preferable. For example, if the withdrawal is made after 10 months, you would earn nothing on the Universal account but \((1.015)^{3} - 1.0 = 4.57\%\) on the Regional account.

Ten or more years ago, most banks and S&Ls were set up as described above, but now virtually all are computerized and pay interest from the day of deposit to the day of withdrawal, provided at least $1 is in the account at the end of the period.

8-12 a. With a financial calculator, enter \(N = 5\), \(I = 10\), \(PV = -25000\), and \(FV = 0\), and then press the PMT key to get \(PMT = 6,594.94\). Then go through the amortization procedure as described in your calculator manual to get the entries for the amortization table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment</th>
<th>Interest</th>
<th>Repayment of Principal</th>
<th>Remaining Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$6,594.94</td>
<td>$2,500.00</td>
<td>$4,094.94</td>
<td>$20,905.06</td>
</tr>
<tr>
<td>2</td>
<td>6,594.94</td>
<td>2,090.51</td>
<td>4,504.43</td>
<td>16,400.63</td>
</tr>
<tr>
<td>3</td>
<td>6,594.94</td>
<td>1,640.06</td>
<td>4,954.88</td>
<td>11,445.75</td>
</tr>
<tr>
<td>4</td>
<td>6,594.94</td>
<td>1,144.58</td>
<td>5,450.36</td>
<td>5,995.39</td>
</tr>
<tr>
<td>5</td>
<td>6,594.93*</td>
<td>599.54</td>
<td>5,995.39</td>
<td>0</td>
</tr>
</tbody>
</table>

\*$32,974.69 \quad 7,974.69 \quad 25,000.00$

*The last payment must be smaller to force the ending balance to zero.

b. Here the loan size is doubled, so the payments also double in size to $13,189.87.
c. The annual payment on a $50,000, 10-year loan at 10 percent interest would be $8,137.27. Because the payments are spread out over a longer time period, more interest must be paid on the loan, which raises the amount of each payment. The total interest paid on the 10-year loan is $31,372.70 versus interest of $15,949.37 on the 5-year loan.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th></th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z:</td>
<td>-422.41</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B:</td>
<td>-1,000.00</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

a. With a financial calculator, for Z, enter N = 10, PV = -422.41, PMT = 0, FV = 1000, and press I to get I = 9.00%. For B, enter N = 10, PV = -1000, PMT = 80, FV = 1000, and press I to get I = 8%. (Alternatively, enter the values exactly as shown on the time line in the CF register, and use the IRR key to obtain the same answer.)

b. With a calculator, for the "zero coupon bond," enter N = 10, I = 6, PMT = 0, FV = 1000, and press PV to get the value of the security today, $558.39. The profit would be $558.39 - $422.41 = $135.98, and the percentage profit would be $135.98/$422.41 = 32.2%.

For the "coupon bond," enter N = 10, I = 6, PMT = 80, FV = 1000, and then press PV to get PV = $1,147.20. The profit is $147.20, and the percentage profit is 14.72%.

c. Here we compound cash flows to obtain a "terminal value" at Year 10, and then find the interest rate which equates the TV to the cost of the security.

There are no intermediate cash flows with Security Z, so its TV is $1,000, and, as we saw in Part a, 9% causes the PV of $1,000 to equal the cost, $422.41. For Security B, we must compound the cash flows over 10 years at 6%. Enter N = 10, I = 6, PV = 0, PMT = 80, and then press PV to get the PV of the 10 year annuity of $80 per year: FV = $1,054.46. Then add the $1,000 to be received at Year 10 to get TV = $2,054.46. Then enter N = 10, PV = -1000, PMT = 0, FV = 2054.46, and press I to get I = 7.47%.

So, if the firm buys Security Z, its actual return will be 9% regardless of what happens to interest rates--this security is a zero coupon bond which has zero reinvestment rate risk. However, if the firm buys the 8% coupon bond, and rates then fall, its "true" return over the 10 years will be only 7.47%, which is an average of the old 8% and the new 6%.

d. The value of Security Z would fall from $422.41 to $321.97, so a loss of $100.44, or 23.8%, would be incurred. The value of Security B would fall to $773.99, so the loss here would be $226.01, or 22.6% of the $1,000 original investment. The percentage losses for the two bonds is close, but only because the zero's original return was 9% versus 8% for the coupon bond.
The "actual" or "true" return on the zero would remain at 9%, but the "actual" return on the coupon bond would rise from 8% to 9.17% due to reinvestment of the $80 coupons at 12%.

8-14 a. First, determine the annual cost of college. The current cost is $12,500 per year, but that is escalating at a 5 percent inflation rate:

<table>
<thead>
<tr>
<th>Year</th>
<th>Current Cost</th>
<th>Years from Now</th>
<th>Inflation Adjustment</th>
<th>Cash Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12,500</td>
<td>5</td>
<td>(1.05)^5</td>
<td>$15,954</td>
</tr>
<tr>
<td>2</td>
<td>12,500</td>
<td>6</td>
<td>(1.05)^6</td>
<td>16,751</td>
</tr>
<tr>
<td>3</td>
<td>12,500</td>
<td>7</td>
<td>(1.05)^7</td>
<td>17,589</td>
</tr>
<tr>
<td>4</td>
<td>12,500</td>
<td>8</td>
<td>(1.05)^8</td>
<td>18,468</td>
</tr>
</tbody>
</table>

Now put these costs on a time line:

13 14 15 16 17 18 19 20 21
---------------------------
-15,954 -16,751 -17,589 -18,468

How much must be accumulated by age 18 to provide these payments at ages 18 through 21 if the funds are invested in an account paying 8 percent, compounded annually?

\[
\begin{align*}
$15,954(\text{PVIF}_{8\%,0}) & = $15,954 \\
$16,751(\text{PVIF}_{8\%,1}) & = 15,510 \\
$17,589(\text{PVIF}_{8\%,2}) & = 15,079 \\
$18,468(\text{PVIF}_{8\%,3}) & = 14,660 \\
\text{Total} & = $61,203
\end{align*}
\]

Thus, the father must accumulate $61,203 by the time his daughter reaches age 18.

b. She has $7,500 now (age 13) to help achieve that goal. Five years hence that $7,500, when invested at 8 percent, will be worth $11,020:

\[
$7,500(1.08)^5 = $11,020.
\]

c. The father needs to accumulate only $61,203 - $11,020 = $50,183. The key to completing the problem at this point is to realize the series of deposits represent an ordinary annuity rather than an annuity due, despite the fact the first payment is made at the beginning of the first year. The reason it is not an annuity due is there is no interest paid on the last payment which occurs when the daughter is 18. Thus,

\[
$50,183 = \text{PMT}(\text{FVIFA}_{8\%,6}).
\]

\[
\text{PMT} = $6,841.
\]
Another way to approach the problem is to treat the series of payments as a five-year annuity due with a lump sum deposit at the end of Year 5: 

\[ 50,183 = FVA_{5, 8\%} + PMT = PMT \left( (PVIFA_{8\%, 5}) \times (1.08) \right) + PMT = PMT \left[ 5.8666 \times (1.08) + 1 \right]. \]

Therefore, \( PMT = \frac{50,183}{7.3353} = 6,841. \)

8-15

<table>
<thead>
<tr>
<th>14%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>?</td>
<td>2,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With a calculator, enter \( N = 6, I = 14, PMT = 0, \) and \( FV = 2000, \) and press \( PV \) to get \( PV = 911.17. \)

\[ 2,000 \times (1/1.14)^6 = 2,000 \times 0.4556 = 911.20. \]

$1,000 today is worth more. The present value of $2,000 at 14 percent over six years is $911.20, which is less than $1,000.00. Alternatively, the future value of $1,000 is: 

\[ FV = 1,000 \times (1.14)^6 = 1,000 \times 2.1950 = 2,195. \]

8-16

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>?</td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>

With a calculator, enter \( N = 5, PV = -6, PMT = 0, FV = 12, \) and then solve for \( I = 14.87\%. \)

b. The calculation described in the quotation fails to take account of the compounding effect. It can be demonstrated to be incorrect as follows:

\[ 6,000,000 \times (1.20)^5 = 6,000,000 \times (2.4883) = 14,929,800, \]

which is greater than $12 million. Thus, the annual growth rate is less than 20 percent; in fact, it is about 15 percent, as shown in Part a.

8-17

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

\[ 4,000,000/8,000,000 = 0.50, \]

which is slightly less than the \( PVIF_{1, 7\%} \) for 7 percent in 10 years. Thus, the expected rate of return is just over 7 percent. With a calculator, enter \( N = 10, PV = -4, PMT = 0, FV = 8, \) and then solve for \( I = 7.18\%. \)
8-18  

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13,250</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
<td>2,345.05</td>
</tr>
</tbody>
</table>

\[ PVA_{10} = PMT(PVIFA_{i, 10}) \]
\[ $13,250 = $2,345.05(PVIFA_{i, 10}) \]
\[ PVIFA_{i, 10} = 5.6502 \]
\[ i = 12\% . \]

With a calculator, enter \( N = 10, \ PV = -13250, \ PMT = 2345.05, \ FV = 0, \) and then solve for \( I = 12\% . \)

8-19  

\[ 85,000 \]
\[ -8,273.59 \]
\[ -8,273.59 \]
\[ -8,273.59 \]
\[ -8,273.59 \]
\[ -8,273.59 \]

\[ $85,000 / $8,273.59 = 10.2737 = PVIFA_{i,n} \] for a 30-year annuity.

With a calculator, enter \( N = 30, \ PV = 85000, \ PMT = -8273.59, \ FV = 0, \) and then solve for \( I = 9\% . \)

8-20 a.  

\[ PV = ? \]
\[ -10,000 \]
\[ -10,000 \]
\[ -10,000 \]
\[ -10,000 \]

With a calculator, enter \( N = 4, \ I = 7, \ PMT = -10000, \) and \( FV = 0. \) Then press \( PV \) to get \( PV = 33,872.11. \)

b. (1) At this point, we have a 3-year, 7\% annuity whose value is $26,243.16. You can also think of the problem as follows:

\[ $33,872(1.07) \] \[ $10,000 = $26,243.04. \]

(2) Zero after the last withdrawal.

8-21  

\[ 12,000 \]
\[ -1,500 \]
\[ -1,500 \]
\[ -1,500 \]
\[ -1,500 \]

\[ PVA_n = PMT(PVIFA_{i,n}) . \]
\[ $12,000 = $1,500(PVIFA_{9,15}) \]
\[ PVIFA_{9,15} = 8.000 . \]

With a calculator, enter \( I = 9, \ PV = 12000, \ PMT = -1500, \) and \( FV = 0. \) Press \( N \) to get \( N = 14.77 = 15 \) years. Therefore, it will take approximately 15 years to pay back the loan.
With a financial calculator, get a "ballpark" estimate of the years by entering $I = 12$, $PV = 0$, $PMT = -1250$, and $FV = 10000$, and then pressing the N key to find $N = 5.94$ years. This answer assumes that a payment of $1,250 will be made 94/100th of the way through Year 5.

Now find the FV of $1,250 for 5 years at 12%; it is $7,941.06$. Compound this value for 1 year at 12% to obtain the value in the account after 6 years and before the last payment is made; it is $7,941.06(1.12) = 8,893.99$. Thus, you will have to make a payment of $10,000 - 8,893.99 = 1,106.01 at Year 6, so the answer is: it will take 6 years, and $1,106.01 is the amount of the last payment.

8-23 $PV = 100/0.07 = 1,428.57$. $PV = 100/0.14 = 714.29$.

When the interest rate is doubled, the PV of the perpetuity is halved.

8-24

<table>
<thead>
<tr>
<th>0</th>
<th>8.24%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV = ?</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>1,050</td>
<td></td>
</tr>
</tbody>
</table>

Discount rate: Effective rate on bank deposit:

$$\text{EAR} = (1 + 0.08/4)^4 - 1 = 8.24\%.$$

Find PV of above stream at 8.24%:

$$PV = 893.26 \text{ using the cash flow register.}$$

Also get $PV = 893.26$ using the TVM register, inputting $N = 4$, $I = 8.24$, $PMT = 50$, and $FV = 1000$.

8-25 This can be done with a calculator by specifying an interest rate of 5% per period for 20 periods with 1 payment per period, or 10% interest, 20 periods, 2 payments per year. Either way, we get the payment each 6 months:

$$\begin{align*}
N &= 10 \times 2 = 20. \\
I &= 10\%/2 = 5. \\
PV &= -10000. \\
FV &= 0.
\end{align*}$$

Solve for $PMT = 802.43$. Set up amortization table:
### 8-26
First, find PMT by using a financial calculator: N = 5, I/YR = 15, PV = -1000000, and FV = 0. Solve for PMT = $298,315.55. Then set up the amortization table:

<table>
<thead>
<tr>
<th>Period</th>
<th>Beg Bal</th>
<th>Payment</th>
<th>Interest</th>
<th>Principal</th>
<th>End Bal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1,000,000.00</td>
<td>$298,315.55</td>
<td>$150,000.00</td>
<td>$148,315.55</td>
<td>$851,684.45</td>
</tr>
<tr>
<td>2</td>
<td>851,684.45</td>
<td>298,315.55</td>
<td>127,752.67</td>
<td>170,562.88</td>
<td>681,121.57</td>
</tr>
</tbody>
</table>

Fraction that is principal = $170,562.88/$298,315.55 = 0.5718 = 57.18%.

### 8-27

#### a. Begin with a time line:

Since the first payment is made today, we have a 5-period annuity due. The applicable interest rate is I = 12/2 = 6 per period, N = 5, PV = 0, and PMT = -100. Setting the calculator on "BEG," we find FVA (Annuity due) = $597.53. That will be the value at July 1, 2001, or t = 2.5. Now we must compound out to t = 10, or for 7.5 years at an EAR of 12.36%, or 15 semiannual periods at 6%.

$$
\frac{\$597.53}{10 - 2.5 = 7.5 \text{ years} \at\ 12.36\%} \to \$1,432.02.
$$

or

$$
\frac{\$597.53}{20 - 5 = 15 \text{ periods} \at\ 6\%} \to \$1,432.02.
$$

#### b.

The time line depicting the problem is shown above. Because the payments only occur for 5 periods throughout the 40 quarters, this
problem cannot be immediately solved as an annuity problem. The problem can be solved in two steps:

(1) Discount the $1,432.02 back to the end of Quarter 5 to obtain the PV of that future amount at Quarter 5.

(2) Then solve for PMT using the value solved in Step 1 as the FV of the five-period annuity due.

Step 1: Input the following into your calculator: \( N = 35, I = 3, PMT = 0, FV = 1432.02 \), and solve for PV at Quarter 5. \( PV = 508.92 \).

Step 2: The PV found in Step 1 is now the FV for the calculations in this step. Change your calculator to the BEGIN mode. Input the following into your calculator: \( N = 5, I = 3, PV = 0, FV = 508.92 \), and solve for \( PMT = 93.07 \).

8-28 Here we want to have the same effective annual rate on the credit extended as on the bank loan that will be used to finance the credit extension.

First, we must find the EAR = EFF% on the bank loan. Enter NOM% = 15, \( N = P/YR = 12 \), and press EFF% to get EAR = 16.08%.

Now recognize that giving 3 months of credit is equivalent to quarterly compounding—interest is earned at the end of the quarter, so it is available to earn interest during the next quarter. Therefore, enter \( P/YR = 4, EFF% = EAR = 16.08\% \), and press NOM% to find the nominal rate of 15.19 percent.

Therefore, if a 15.19 percent nominal rate is charged and credit is given for 3 months, the cost of the bank loan will be covered.

Alternative solution: We need to find the effective annual rate (EAR) the bank is charging first. Then, we can use this EAR to calculate the nominal rate that should be quoted to the customers.

Bank EAR: \( EAR = (1 + \frac{i_{Nom}}{m})^m - 1 = (1 + 0.15/12)^{12} - 1 = 16.08\% \).

Nominal rate that should be quoted to customers:

\[
16.08\% = (1 + \frac{i_{Nom}}{4})^4 - 1 \\
1.1608 = (1 + \frac{i_{Nom}}{4})^4 \\
1.0380 = 1 + \frac{i_{Nom}}{4} \\
i_{Nom} = 0.0380(4) = 15.19\%.
\]

8-29 Information given:

1. Will save for 10 years, then receive payments for 25 years.

2. Wants payments of $40,000 per year in today's dollars for first payment only. Real income will decline. Inflation will be 5
percent. Therefore, to find the inflated fixed payments, we have this time line:

```
<table>
<thead>
<tr>
<th>0</th>
<th>5%</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000</td>
<td></td>
<td></td>
<td>FV = ?</td>
</tr>
</tbody>
</table>
```

Enter N = 10, I = 5, PV = -40000, PMT = 0, and press FV to get FV = $65,155.79.

3. He now has $100,000 in an account which pays 8 percent, annual compounding. We need to find the FV of the $100,000 after 10 years. Enter N = 10, I = 8, PV = -100000, PMT = 0, and press FV to get FV = $215,892.50.

4. He wants to withdraw, or have payments of, $65,155.79 per year for 25 years, with the first payment made at the beginning of the first retirement year. So, we have a 25-year annuity due with PMT = 65,155.79, at an interest rate of 8 percent. (The interest rate is 8 percent annually, so no adjustment is required.) Set the calculator to "BEG" mode, then enter N = 25, I = 8, PMT = 65155.79, FV = 0, and press PV to get PV = $751,165.35. This amount must be on hand to make the 25 payments.

5. Since the original $100,000, which grows to $215,892.50, will be available, we must save enough to accumulate $751,165.35 - $215,892.50 = $535,272.85.

6. The $535,272.85 is the FV of a 10-year ordinary annuity. The payments will be deposited in the bank and earn 8 percent interest. Therefore, set the calculator to "END" mode and enter N = 10, I = 8, PV = 0, FV = 535272.85, and press PMT to find PMT = $36,949.61.