CHAPTER 6
DISCOUNTED CASH FLOW VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. The four pieces are the present value (PV), the periodic cash flow (C), the discount rate (r), and the number of payments, or the life of the annuity, t.

2. Assuming positive cash flows, both the present and the future values will rise.

3. Assuming positive cash flows, the present value will fall and the future value will rise.

4. It’s deceptive, but very common. The basic concept of time value of money is that a dollar today is not worth the same as a dollar tomorrow. The deception is particularly irritating given that such lotteries are usually government sponsored!

5. If the total money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.

6. The better deal is the one with equal installments.

7. Yes, they should. APRs generally don’t provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

8. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue. The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.

9. The problem is that the subsidy makes it easier to repay the loan, not obtain it. However, ability to repay the loan depends on future employment, not current need. For example, consider a student who is currently needy, but is preparing for a career in a high-paying area (such as corporate finance!). Should this student receive the subsidy? How about a student who is currently not needy, but is preparing for a relatively low-paying job (such as becoming a college professor)?
10. In general, viatical settlements are ethical. In the case of a viatical settlement, it is simply an exchange of cash today for payment in the future, although the payment depends on the death of the seller. The purchaser of the life insurance policy is bearing the risk that the insured individual will live longer than expected. Although viatical settlements are ethical, they may not be the best choice for an individual. In a Business Week article (October 31, 2005), options were examined for a 72 year old male with a life expectancy of 8 years and a $1 million dollar life insurance policy with an annual premium of $37,000. The four options were: 1) Cash the policy today for $100,000. 2) Sell the policy in a viatical settlement for $275,000. 3) Reduce the death benefit to $375,000, which would keep the policy in force for 12 years without premium payments. 4) Stop paying premiums and don’t reduce the death benefit. This will run the cash value of the policy to zero in 5 years, but the viatical settlement would be worth $475,000 at that time. If he died within 5 years, the beneficiaries would receive $1 million. Ultimately, the decision rests on the individual on what they perceive as best for themselves. The values that will affect the value of the viatical settlement are the discount rate, the face value of the policy, and the health of the individual selling the policy.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

   \[ PV = \frac{FV}{(1 + r)^t} \]

   \[ PV@10\% = \frac{1,100}{1.10} + \frac{720}{1.10^2} + \frac{940}{1.10^3} + \frac{1,160}{1.10^4} = 3,093.57 \]

   \[ PV@18\% = \frac{1,100}{1.18} + \frac{720}{1.18^2} + \frac{940}{1.18^3} + \frac{1,160}{1.18^4} = 2,619.72 \]

   \[ PV@24\% = \frac{1,100}{1.24} + \frac{720}{1.24^2} + \frac{940}{1.24^3} + \frac{1,160}{1.24^4} = 2,339.03 \]

2. To find the PVA, we use the equation:

   \[ PVA = \frac{C \left(1 - \frac{1}{(1 + r)^t}\right)}{r} \]

   At a 5 percent interest rate:

   \[ X@5\%: \quad PVA = \frac{7,000 \left(1 - \frac{1}{1.05^8}\right)}{.05} = 45,242.49 \]

   \[ Y@5\%: \quad PVA = \frac{9,000 \left(1 - \frac{1}{1.05^5}\right)}{.05} = 38,965.29 \]
And at a 22 percent interest rate:

\[
X@22\%: \text{ PVA} = 7,000 \left\{ \frac{1 - (1/1.22)^8}{.22} \right\} = 25,334.87
\]

\[
Y@22\%: \text{ PVA} = 9,000 \left\{ \frac{1 - (1/1.22)^5}{.22} \right\} = 25,772.76
\]

Notice that the PV of cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At the higher interest rate, these bigger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.

### 3. To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

\[
FV = PV(1 + r)^t
\]

\[
FV@8\% = 700(1.08)^3 + 950(1.08)^2 + 1,200(1.08) + 1,300 = 4,585.88
\]

\[
FV@11\% = 700(1.11)^3 + 950(1.11)^2 + 1,200(1.11) + 1,300 = 4,759.84
\]

\[
FV@24\% = 700(1.24)^3 + 950(1.24)^2 + 1,200(1.24) + 1,300 = 5,583.36
\]

Notice we are finding the value at Year 4, the cash flow at Year 4 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.

### 4. To find the PVA, we use the equation:

\[
PVA = C\left\{ \frac{1 - (1/(1 + r))^t}{r} \right\}
\]

<table>
<thead>
<tr>
<th>Years</th>
<th>PVA@15 yrs:</th>
<th>PVA@40 yrs:</th>
<th>PVA@75 yrs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>39,373.60</td>
<td>54,853.22</td>
<td>57,320.99</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To find the PV of a perpetuity, we use the equation:

\[
PV = \frac{C}{r}
\]

\[
PV = 4,600 / .08 = 57,500.00
\]

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75 year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $179.01.
5. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

\[
PVA = C \{ 1 - [1/(1 + r)]^t \} / r \]

\[
PVA = \$28,000 = C \{ [1 - (1/1.0825)^{15} ] / .0825 \}
\]

We can now solve this equation for the annuity payment. Doing so, we get:

\[
C = \$28,000 / 8.43035 = \$3,321.33
\]

6. To find the PVA, we use the equation:

\[
PVA = C \{ 1 - [1/(1 + r)]^t \} / r \]

\[
PVA = \$65,000 \{ [1 - (1/1.085)^{8} ] / .085 \} = \$366,546.89
\]

7. Here we need to find the FVA. The equation to find the FVA is:

\[
FVA = C \{ [(1 + r)^t - 1] / r \}
\]

FVA for 20 years = \$3,000 \{ (1.105^{20} - 1) / .105 \} = \$181,892.42

FVA for 40 years = \$3,000 \{ (1.105^{40} - 1) / .105 \} = \$1,521,754.74

Notice that because of exponential growth, doubling the number of periods does not merely double the FVA.

8. Here we have the FVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the FVA equation:

\[
FVA = C \{ [(1 + r)^t - 1] / r \}
\]

\[
\$80,000 = C \{ (1.065^{10} - 1) / .065 \}
\]

We can now solve this equation for the annuity payment. Doing so, we get:

\[
C = \$80,000 / 13.49442 = \$5,928.38
\]

9. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

\[
PVA = C \{ 1 - [1/(1 + r)]^t \} / r \]

\[
\$30,000 = C \{ [1 - (1/1.08)^{7} ] / .08 \}
\]

We can now solve this equation for the annuity payment. Doing so, we get:

\[
C = \$30,000 / 5.20637 = \$5,762.17
\]

10. This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

\[
PV = C / r
\]

\[
PV = \$20,000 / .08 = \$250,000.00
\]
11. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

\[ PV = \frac{C}{r} \]

\$280,000 = \frac{\$20,000}{r} \]

We can now solve for the interest rate as follows:

\[ r = \frac{\$20,000}{\$280,000} = .0714 \text{ or } 7.14\% \]

12. For discrete compounding, to find the EAR, we use the equation:

\[ \text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1 \]

\[ \text{EAR} = \left[1 + \left(\frac{.07}{4}\right)\right]^4 - 1 = .0719 \text{ or } 7.19\% \]

\[ \text{EAR} = \left[1 + \left(\frac{.18}{12}\right)\right]^{12} - 1 = .1956 \text{ or } 19.56\% \]

\[ \text{EAR} = \left[1 + \left(\frac{.10}{365}\right)\right]^{365} - 1 = .1052 \text{ or } 10.52\% \]

To find the EAR with continuous compounding, we use the equation:

\[ \text{EAR} = e^a - 1 \]

\[ \text{EAR} = e^{.14} - 1 = .1503 \text{ or } 15.03\% \]

13. Here we are given the EAR and need to find the APR. Using the equation for discrete compounding:

\[ \text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1 \]

We can now solve for the APR. Doing so, we get:

\[ \text{APR} = m\left(\left[1 + \text{EAR}\right]^{1/m} - 1\right) \]

\[ \text{APR} = 2\left(\left[1.1220\right]^{1/2} - 1\right) = .1185 \text{ or } 11.85\% \]

\[ \text{APR} = 12\left(\left[1.0940\right]^{1/12} - 1\right) = .0902 \text{ or } 9.02\% \]

\[ \text{APR} = 52\left(\left[1.0860\right]^{1/52} - 1\right) = .0826 \text{ or } 8.26\% \]

Solving the continuous compounding EAR equation:

\[ \text{EAR} = e^a - 1 \]

We get:

\[ \text{APR} = \ln(1 + \text{EAR}) \]

\[ \text{APR} = \ln(1 + .2380) \]
APR = .2135 or 21.35%
14. For discrete compounding, to find the EAR, we use the equation:

$$\text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1$$

So, for each bank, the EAR is:

- **First National:** $\text{EAR} = \left[1 + \left(\frac{.1310}{12}\right)\right]^{12} - 1 = .1392$ or 13.92%
- **First United:** $\text{EAR} = \left[1 + \left(\frac{.1340}{2}\right)\right]^2 - 1 = .1385$ or 13.85%

Notice that the higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

15. The reported rate is the APR, so we need to convert the EAR to an APR as follows:

$$\text{APR} = m\left[\left(1 + \text{EAR}\right)^{1/m} - 1\right]$$

$$\text{APR} = 365\left[(1.14)^{1/365} - 1\right] = .1311$ or 13.11%

This is deceptive because the borrower is actually paying annualized interest of 14% per year, not the 13.11% reported on the loan contract.

16. For this problem, we simply need to find the FV of a lump sum using the equation:

$$\text{FV} = \text{PV}(1 + r)^t$$

It is important to note that compounding occurs semiannually. To account for this, we will divide the interest rate by two (the number of compounding periods in a year), and multiply the number of periods by two. Doing so, we get:

$$\text{FV} = 1,400\left[1 + \left(\frac{.096}{2}\right)\right]^{40} = 9,132.28$$

17. For this problem, we simply need to find the FV of a lump sum using the equation:

$$\text{FV} = \text{PV}(1 + r)^t$$

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

- **FV in 5 years:** $\text{FV} = 6,000\left[1 + \left(\frac{.084}{365}\right)\right]^{5(365)} = 9,131.33$
- **FV in 10 years:** $\text{FV} = 6,000\left[1 + \left(\frac{.084}{365}\right)\right]^{10(365)} = 13,896.86$
- **FV in 20 years:** $\text{FV} = 6,000\left[1 + \left(\frac{.084}{365}\right)\right]^{20(365)} = 32,187.11$
18. For this problem, we simply need to find the PV of a lump sum using the equation:

\[
PV = \frac{FV}{(1 + r)^t}
\]

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

\[
PV = \frac{$45,000}{(1 + .11/365)^{6(365)}} = $23,260.62
\]

19. The APR is simply the interest rate per period times the number of periods in a year. In this case, the interest rate is 25 percent per month, and there are 12 months in a year, so we get:

\[
APR = 12(25\%) = 300\%
\]

To find the EAR, we use the EAR formula:

\[
EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1
\]

\[
EAR = (1 + .25)^{12} - 1 = 1,355.19\%
\]

Notice that we didn’t need to divide the APR by the number of compounding periods per year. We do this division to get the interest rate per period, but in this problem we are already given the interest rate per period.

20. We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

\[
PVA = C \left\{1 - \left[\frac{1}{1 + r}\right]^t \right\} / r
\]

\[
$61,800 = $C \left\{1 - \left[\frac{1}{1 + (.074/12)}\right]^{60} \right\} / (.074/12)
\]

Solving for the payment, we get:

\[
C = $61,800 / 50.02385 = $1,235.41
\]

To find the EAR, we use the EAR equation:

\[
EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1
\]

\[
EAR = [1 + (.074 / 12)]^{12} - 1 = .0766 \text{ or } 7.66\%
\]

21. Here we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

\[
PVA = C \left\{1 - \left[\frac{1}{1 + r}\right]^t \right\} / r
\]

\[
$17,000 = $300 \left\{1 - (1/1.009)^t \right\} / .009
\]
Now we solve for $t$:

$$
1/1.009^t = 1 - \{[(17,000)/(300)](0.009)\}
$$
$$
1.009^t = 0.49
$$
$$
t = \ln \left(\frac{1}{0.49}\right) = \ln 2.0408 / \ln 1.009 = 79.62 \text{ months}
$$

22. Here we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$$
FV = PV(1 + r)
$$
$$
\$4 = \$3(1 + r)
$$
$$
r = 4/3 - 1 = 33.33\% \text{ per week}
$$

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

$$
APR = (52)33.33\% = 1,733.33\%
$$

And using the equation to find the EAR:

$$
EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1
$$
$$
EAR = \left[1 + .3333\right]^{52} - 1 = 313,916,515.69\%
$$

23. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$$
PV = \frac{C}{r}
$$
$$
\$63,000 = \$1,200 / r
$$

We can now solve for the interest rate as follows:

$$
r = \$1,200 / \$63,000 = 0.0190 \text{ or 1.90\% per month}
$$

The interest rate is 1.90% per month. To find the APR, we multiply this rate by the number of months in a year, so:

$$
APR = (12)1.90\% = 22.86\%
$$

And using the equation to find an EAR:

$$
EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1
$$
$$
EAR = \left[1 + .0190\right]^{12} - 1 = 25.41\%
$$

24. This problem requires us to find the FVA. The equation to find the FVA is:

$$
FVA = C\left\{\left[1 + \left(\frac{r}{m}\right)\right]^m - 1\right\} / \left(\frac{r}{m}\right)
$$
$$
FVA = \$250\left\{\left[1 + (.10/12}\right]^{360} - 1\right\} / (.10/12) = \$565,121.98
$$
25. In the previous problem, the cash flows are monthly and the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate times with the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

\[
\text{EAR} = \left(1 + \frac{\text{APR}}{m}\right)^m - 1 \\
\text{EAR} = \left[1 + \frac{(.10)}{12}\right]^{12} - 1 = .1047 \text{ or } 10.47\% 
\]

Using the FVA equation, we get:

\[
\text{FVA} = C\left\{\frac{\left[(1 + r)^t - 1\right]}{r}\right\} \\
\text{FVA} = \$3,000\left\{\frac{\left[(1.1047)^30 - 1\right]}{.1047}\right\} = \$539,686.21
\]

26. The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

\[
\text{PVA} = C\left\{\frac{\left[1 - \frac{1}{(1 + r)^t}\right]}{r}\right\} \\
\text{PVA} = \$1,500\left\{\frac{\left[1 - (1/1.0075)^{16}\right]}{.0075}\right\} = \$22,536.47
\]

27. The cash flows are annual and the compounding period is quarterly, so we need to calculate the EAR to make the interest rate comparable with the timing of the cash flows. Using the equation for the EAR, we get:

\[
\text{EAR} = \left[1 + \frac{\text{APR}}{m}\right]^m - 1 \\
\text{EAR} = \left[1 + \frac{(.11)}{4}\right]^4 - 1 = .1146 \text{ or } 11.46\% 
\]

And now we use the EAR to find the PV of each cash flow as a lump sum and add them together:

\[
\text{PV} = \frac{\$900}{1.1146} + \frac{\$850}{1.1146^2} + \frac{\$1,140}{1.1146^4} = \$2,230.20 
\]

28. Here the cash flows are annual and the given interest rate is annual, so we can use the interest rate given. We simply find the PV of each cash flow and add them together.

\[
\text{PV} = \frac{\$2,800}{1.0845} + \frac{\$5,600}{1.0845^2} + \frac{\$1,940}{1.0845^3} + \frac{\$1,940}{1.0845^4} = \$8,374.62
\]

Intermediate

29. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

\[.06(10) = .6\]

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

\[(1 + r)^t\]
Setting the two equal, we get:

\[(.06)(10) = (1 + r)^{10} - 1\]

\[r = 1.6^{1/10} - 1 = .0481\] or 4.81%

30. Here we need to convert an EAR into interest rates for different compounding periods. Using the equation for the EAR, we get:

\[\text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1\]

\[\text{EAR} = .18 = (1 + r)^2 - 1; \quad r = (1.18)^{1/2} - 1 = .0863\] or 8.63% per six months

\[\text{EAR} = .18 = (1 + r)^4 - 1; \quad r = (1.18)^{1/4} - 1 = .0422\] or 4.22% per quarter

\[\text{EAR} = .18 = (1 + r)^{12} - 1; \quad r = (1.18)^{1/12} - 1 = .0139\] or 1.39% per month

Notice that the effective six month rate is not twice the effective quarterly rate because of the effect of compounding.

31. Here we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

\[\text{FV} = 5,000 \left[1 + \left(\frac{.025}{12}\right)\right]^6 = 5,062.83\]

This is the balance in six months. The FV in another six months will be:

\[\text{FV} = 5,062.83 \left[1 + \left(\frac{.17}{12}\right)\right]^6 = 5,508.70\]

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

\[\text{Interest} = 5,508.70 - 5,000.00 = 508.70\]

32. We need to find the annuity payment in retirement. Our retirement savings ends and the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: \(\text{FVA} = 600\left[\left[1 + \left(\frac{.12}{12}\right)\right]^{360} - 1\right] / \left(\frac{.12}{12}\right) = 2,096,978.48\)

Bond account: \(\text{FVA} = 300\left[\left[1 + \left(\frac{.07}{12}\right)\right]^{360} - 1\right] / \left(\frac{.07}{12}\right) = 365,991.30\)

So, the total amount saved at retirement is:

\[2,096,978.48 + 365,991.30 = 2,462,969.78\]

Solving for the withdrawal amount in retirement using the PVA equation gives us:

\[\text{PVA} = 2,462,969.78 = \text{C}\left[1 - \left\{\text{I} / \left[1 + \left(\frac{.09}{12}\right)\right]^{300}\right\} / \left(\frac{.09}{12}\right)\right]\]
\[ C = \frac{\$2,462,969.78}{119.1616} = \$20,669.15 \text{ withdrawal per month} \]
33. We need to find the FV of a lump sum in one year and two years. It is important that we use the number of months in compounding since interest is compounded monthly in this case. So:

\[
\text{FV in one year } = 1(1.0108)^{12} = \$1.14
\]

\[
\text{FV in two years } = 1(1.0108)^{24} = \$1.29
\]

There is also another common alternative solution. We could find the EAR, and use the number of years as our compounding periods. So we will find the EAR first:

\[
\text{EAR} = (1 + .0108)^{12} - 1 = .1376 \text{ or } 13.76\%
\]

Using the EAR and the number of years to find the FV, we get:

\[
\text{FV in one year } = 1(1.1376)^{1} = \$1.14
\]

\[
\text{FV in two years } = 1(1.1376)^{2} = \$1.29
\]

Either method is correct and acceptable. We have simply made sure that the interest compounding period is the same as the number of periods we use to calculate the FV.

34. Here we are finding the annuity payment necessary to achieve the same FV. The interest rate given is a 10 percent APR, with monthly deposits. We must make sure to use the number of months in the equation. So, using the FVA equation:

\[
\text{FVA in 40 years } = C \left[ \frac{1 - (1 + \frac{.11}{12})^{-480}}{\frac{.11}{12}} \right] = \frac{\$1,000,000}{8,600.127} = \$116.28
\]

\[
\text{FVA in 30 years } = C \left[ \frac{1 - (1 + \frac{.11}{12})^{-360}}{\frac{.11}{12}} \right] = \frac{\$1,000,000}{2,804.52} = \$356.57
\]

\[
\text{FVA in 20 years } = C \left[ \frac{1 - (1 + \frac{.11}{12})^{-240}}{\frac{.11}{12}} \right] = \frac{\$1,000,000}{865.638} = \$1,155.22
\]

Notice that a deposit for half the length of time, i.e. 20 years versus 40 years, does not mean that the annuity payment is doubled. In this example, by reducing the savings period by one-half, the deposit necessary to achieve the same terminal value is about nine times as large.

35. Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

\[
\text{FV} = \$4 = 1(1 + r)^{12/3}
\]

\[
r = .4142 \text{ or } 41.42\%
\]
36. Since we have an APR compounded monthly and an annual payment, we must first convert the interest rate to an EAR so that the compounding period is the same as the cash flows.

\[
\text{EAR} = \left[1 + \left(\frac{.10}{12}\right)\right]^{12} - 1 = .104713 \text{ or } 10.4713\%
\]

\[
PVA_1 = $90,000 \left\{\frac{1 - (1 / 1.104713)^2}{.104713}\right\} = $155,215.98
\]

\[
PVA_2 = $45,000 + $65,000\left\{\frac{1 - (1/1.104713)^2}{.104713}\right\} = $157,100.43
\]

You would choose the second option since it has a higher PV.

37. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

\[
\text{PV} = C \left\{\frac{1}{(r - g)} - \left[\frac{1}{(r - g)}\right] \times \frac{(1 + g)/(1 + r)}\right\}
\]

\[
\text{PV} = $1,000,000\left\{\frac{1}{(.09 - .05)} - \left[\frac{1}{(.09 - .05)}\right] \times \frac{(1 + .05)/(1 + .09)}\right\}^{25}
\]

\[
\text{PV} = $15,182,293.68
\]

38. Since your salary grows at 4 percent per year, your salary next year will be:

Next year’s salary = $50,000 (1 + .04)
Next year’s salary = $52,000

This means your deposit next year will be:

Next year’s deposit = $52,000(.02)
Next year’s deposit = $1,040

Since your salary grows at 4 percent, you deposit will also grow at 4 percent. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

\[
\text{PV} = C \left\{\frac{1}{(r - g)} - \left[\frac{1}{(r - g)}\right] \times \frac{(1 + g)/(1 + r)}\right\}
\]

\[
\text{PV} = $1,040\left\{\frac{1}{(.10 - .04)} - \left[\frac{1}{(.10 - .04)}\right] \times \frac{(1 + .04)/(1 + .10)}\right\}^{40}
\]

\[
\text{PV} = $15,494.64
\]

Now, we can find the future value of this lump sum in 40 years. We find:

\[
\text{FV} = \text{PV}(1 + r)^t
\]

\[
\text{FV} = $15,494.64(1 + .10)^{40}
\]

\[
\text{FV} = $701,276.07
\]

This is the value of your savings in 40 years.
39. The relationship between the PVA and the interest rate is:

PVA falls as \( r \) increases, and PVA rises as \( r \) decreases
FVA rises as \( r \) increases, and FVA falls as \( r \) decreases

The present values of $7,000 per year for 10 years at the various interest rates given are:

\[
PVA@10\% = \frac{7,000 \times [1 - (1/1.10)^{10}]}{.10} = $43,011.97
\]

\[
PVA@5\% = \frac{7,000 \times [1 - (1/1.05)^{10}]}{.05} = $54,052.14
\]

\[
PVA@15\% = \frac{7,000 \times [1 - (1/1.15)^{10}]}{.15} = $35,131.38
\]

40. Here we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

\[
FVA = $20,000 = 225 \times \left[ \frac{1 - \left(1 + \frac{.09}{12}\right)^t}{\frac{.09}{12}} \right]
\]

Solving for \( t \), we get:

\[
1.0075^t = 1 + \frac{20,000}{225} \times \frac{.09}{12}
\]

\[
t = \frac{\ln 1.66667}{\ln 1.0075} = 68.37 \text{ payments}
\]

41. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

\[
PVA = $55,000 = 1,120 \times \left[ \frac{1 - \left(1 + \frac{1}{1 + r}\right)^{60}}{r} \right]
\]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:

\[
r = 0.682\%
\]

The APR is the periodic interest rate times the number of periods in the year, so:

\[
APR = 12 \times (0.682\%) = 8.18\%
\]
42. The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,100 monthly payments is:

\[
PVA = $1,100 \left[ \frac{1 - \left( \frac{1}{1 + \left( \frac{0.068}{12} \right)} \right)^{360}}{\left( \frac{0.068}{12} \right)} \right] = $168,731.02
\]

The monthly payments of $1,100 will amount to a principal payment of $168,731.02. The amount of principal you will still owe is:

\[
$220,000 - 168,731.02 = $51,268.98
\]

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

\[
\text{Balloon payment} = $51,268.98 \left(1 + \frac{0.068}{12}\right)^{360} = $392,025.82
\]

43. We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

PV of Year 1 CF: $1,500 / 1.10 = $1,363.64
PV of Year 3 CF: $1,800 / 1.10^3 = $1,352.37
PV of Year 4 CF: $2,400 / 1.10^4 = $1,639.23

So, the PV of the missing CF is:

\[
$6,785 - 1,363.64 - 1,352.37 - 1,639.23 = $2,429.76
\]

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

\[
$2,429.76(1.10)^2 = $2,940.02
\]

44. To solve this problem, we simply need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

\[
$1,000,000 + $1,400,000/1.09 + $1,800,000/1.09^2 + $2,200,000/1.09^3 + $2,600,000/1.09^4 + $3,000,000/1.09^5 + $3,400,000/1.09^6 + $3,800,000/1.09^7 + $4,200,000/1.09^8 + $4,600,000/1.09^9 + $5,000,000/1.09^{10} = $19,733,830.26
\]

45. Here we are finding interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the number of payments. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = 0.80($2,400,000) = $1,920,000
Using the PVA equation:

\[ PVA = \frac{1,920,000 - \frac{1 - \left[ 1 / (1 + r) \right]^{360}}{r}}{13,000} \]

Unfortunately this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:

\[ r = 0.598\% \]

The APR is the monthly interest rate times the number of months in the year, so:

\[ APR = 12(0.598\%) = 7.17\% \]

And the EAR is:

\[ EAR = (1 + .00598)^{12} - 1 = .0742 \text{ or } 7.42\% \]

46. The profit the firm earns is just the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

\[ PV = \frac{145,000}{1.13^3} = 100,492.27 \]

And the firm’s profit is:

\[ \text{Profit} = 100,492.27 - 94,000.00 = 6,492.27 \]

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

\[ \frac{94,000}{145,000} = \left(1 + \frac{1}{1 + r}\right)^3 \]

\[ r = \left(\frac{145,000}{94,000}\right)^{1/3} - 1 = .1554 \text{ or } 15.54\% \]

47. We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 17 payments, so the PV of the annuity is:

\[ PVA = \frac{2,000\left[1 - \left(1/1.10\right)^{17}\right]}{.10} = 16,043.11 \]

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at \( t = 8 \). To find the value today, we find the PV of this lump sum. The value today is:

\[ \text{PV} = \frac{16,043.11}{1.10^8} = 7,484.23 \]

48. This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:
\[ PVA_2 = 1,500 \left[ \frac{1 - 1}{1 + (0.10/12)^{96}} \right] / (0.10/12) = 98,852.23 \]
Note that this is the PV of this annuity exactly seven years from today. Now we can discount this lump sum to today. The value of this cash flow today is:

\[
PV = \frac{98,852.23}{\left[1 + \left(\frac{.13}{12}\right)\right]^{84}} = $39,985.62
\]

Now we need to find the PV of the annuity for the first seven years. The value of these cash flows today is:

\[
PVA_1 = \frac{1,500 \times \left\{1 - \frac{1}{\left[1 + \left(\frac{.13}{12}\right)^{84}\right]}\right\}}{\left(\frac{.13}{12}\right)} = $82,453.99
\]

The value of the cash flows today is the sum of these two cash flows, so:

\[
PV = $39,985.62 + 82,453.99 = $122,439.62
\]

49. Here we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate, and payments. First we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

\[
FVA = 1,000 \times \left[\left(1 + \left(\frac{.095}{12}\right)\right)^{180} - 1\right] / \left(\frac{.095}{12}\right) = $395,948.63
\]

Now we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

\[
FV = 395,948.63 = PV \times e^{.09(15)}
\]

\[
PV = 395,948.63 \times e^{-.135} = $102,645.83
\]

50. To find the value of the perpetuity at \( t = 7 \), we first need to use the PV of a perpetuity equation. Using this equation we find:

\[
PV = $5,000 / .057 = $87,719.30
\]

Remember that the PV of a perpetuity (and annuity) equations give the PV one period before the first payment, so, this is the value of the perpetuity at \( t = 14 \). To find the value at \( t = 7 \), we find the PV of this lump sum as:

\[
PV = 87,719.30 / 1.057^7 = $59,507.30
\]

51. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

\[
PVA = $20,000 = 1,916.67 \times \left\{\left(1 - \frac{1}{(1 + r)^{12}}\right) / r \right\}
\]

Again, we cannot solve this equation for \( r \), so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

\[
r = 2.219\% \text{ per month}
\]
So the APR is:

$$\text{APR} = 12(2.219\%) = 26.62\%$$

And the EAR is:

$$\text{EAR} = (1.02219)^{12} - 1 = .3012 \text{ or } 30.12\%$$

**52.** The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

\[
\text{Monthly rate} = \frac{.10}{12} = .00833
\]

To get the semiannual interest rate, we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

\[
\text{Semiannual rate} = (1.00833)^6 - 1 = .0511 \text{ or } 5.11\%
\]

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

\[
PVA @ t = 9: $6,000 \left\{ 1 - \left( \frac{1}{1.0511} \right)^{10} \right\} / .0511 = $46,094.33
\]

Note, this is the value one period (six months) before the first payment, so it is the value at \( t = 9 \). So, the value at the various times the questions asked for uses this value 9 years from now.

\[
PV @ t = 5: \frac{$46,094.33}{1.0511^8} = $30,949.21
\]

Note, you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

\[
\text{EAR} = (1 + .0083)^{12} - 1 = .1047 \text{ or } 10.47\%
\]

So, we can find the PV at \( t = 5 \) using the following method as well:

\[
PV @ t = 5: \frac{$46,094.33}{1.1047^4} = $30,949.21
\]

The value of the annuity at the other times in the problem is:

\[
PV @ t = 3: \frac{$46,094.33}{1.0511^{12}} = $25,360.08
\]
\[
PV @ t = 3: \frac{$46,094.33}{1.1047^6} = $25,360.08
\]
\[
PV @ t = 0: \frac{$46,094.33}{1.0511^{18}} = $18,810.58
\]
\[
PV @ t = 0: \frac{$46,094.33}{1.1047^9} = $18,810.58
\]

**53. a.** Calculating the PV of an ordinary annuity, we get:

\[
PVA = $950 \left\{ 1 - (1/1.095)^8 \right\} / .095 = $5,161.76
\]
b. To calculate the PVA due, we calculate the PV of an ordinary annuity for $t - 1$ payments, and add the payment that occurs today. So, the PV of the annuity due is:

\[
PVA = 950 + 950 \left\{\frac{1 - (1/1.095)^7}{.095}\right\} = 5,652.13
\]

54. We need to use the PVA due equation, that is:

\[
PVA_{\text{due}} = (1 + r) \text{ PVA}
\]

Using this equation:

\[
PVA_{\text{due}} = 61,000 = \left(1 + \frac{.0815}{12}\right) \times C \left\{\frac{1 - \frac{1}{1 + (.0815/12)^{60}}}{(.0815/12)}\right\}
\]

\[
60,588.50 = \frac{C\left\{1 - \frac{1}{(1 + .0815/12)^{60}}\right\}}{(.0815/12)}
\]

\[
C = 1,232.87
\]

Notice, when we find the payment for the PVA due, we simply discount the PV of the annuity due back one period. We then use this value as the PV of an ordinary annuity.

55. The payment for a loan repaid with equal payments is the annuity payment with the loan value as the PV of the annuity. So, the loan payment will be:

\[
PVA = 36,000 = C \left\{\frac{1 - \frac{1}{(1 + .09)^5}}{.09}\right\}
\]

\[
C = 9,255.33
\]

The interest payment is the beginning balance times the interest rate for the period, and the principal payment is the total payment minus the interest payment. The ending balance is the beginning balance minus the principal payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal payment is:

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Payment</th>
<th>Principal Payment</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$36,000.00</td>
<td>$9,255.33</td>
<td>$3,240.00</td>
<td>$6,015.33</td>
<td>$29,984.67</td>
</tr>
<tr>
<td>2</td>
<td>29,984.67</td>
<td>9,255.33</td>
<td>2,698.62</td>
<td>6,556.71</td>
<td>23,427.96</td>
</tr>
<tr>
<td>3</td>
<td>23,427.96</td>
<td>9,255.33</td>
<td>2,108.52</td>
<td>7,146.81</td>
<td>16,281.15</td>
</tr>
<tr>
<td>4</td>
<td>16,281.15</td>
<td>9,255.33</td>
<td>1,465.30</td>
<td>7,790.02</td>
<td>8,491.13</td>
</tr>
<tr>
<td>5</td>
<td>8,491.13</td>
<td>9,255.33</td>
<td>764.20</td>
<td>8,491.13</td>
<td>0.00</td>
</tr>
</tbody>
</table>

In the third year, $2,108.52 of interest is paid.

Total interest over life of the loan = $3,240 + 2,698.62 + 2,108.52 + 1,465.30 + 764.20 = $10,276.64
This amortization table calls for equal principal payments of $7,200 per year. The interest payment is the beginning balance times the interest rate for the period, and the total payment is the principal payment plus the interest payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal principal reduction is:

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Payment</th>
<th>Principal Payment</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$36,000.00</td>
<td>$10,440.00</td>
<td>$3,240.00</td>
<td>$7,200.00</td>
<td>$28,800.00</td>
</tr>
<tr>
<td>2</td>
<td>28,800.00</td>
<td>9,792.00</td>
<td>2,592.00</td>
<td>7,200.00</td>
<td>21,600.00</td>
</tr>
<tr>
<td>3</td>
<td>21,600.00</td>
<td>9,144.00</td>
<td>1,944.00</td>
<td>7,200.00</td>
<td>14,400.00</td>
</tr>
<tr>
<td>4</td>
<td>14,400.00</td>
<td>8,496.00</td>
<td>1,296.00</td>
<td>7,200.00</td>
<td>7,200.00</td>
</tr>
<tr>
<td>5</td>
<td>7,200.00</td>
<td>7,848.00</td>
<td>648.00</td>
<td>7,200.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

In the third year, $1,944 of interest is paid.

Total interest over life of the loan = $3,240 + 2,592 + 1,944 + 1,296 + 648 = $9,720

Notice that the total payments for the equal principal reduction loan are lower. This is because more principal is repaid early in the loan, which reduces the total interest expense over the life of the loan.

Challenge

The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

\[ \text{EAR} = .11 = (1 + \frac{\text{APR}}{12})^{12} - 1; \quad \text{APR} = 12[(1.11)^{1/12} - 1] = 10.48\% \]

And the post-retirement APR is:

\[ \text{EAR} = .08 = (1 + \frac{\text{APR}}{12})^{12} - 1; \quad \text{APR} = 12[(1.08)^{1/12} - 1] = 7.72\% \]

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

\[ \text{PVA} = 20,000\left\{1 - \left[1 / (1 + .0772/12)^{12(20)}\right]\right\} / (.0772/12) = 2,441,554.61 \]

\[ \text{PV} = 750,000 / [1 + (.0772/12)]^{240} = 160,911.16 \]

So, at retirement, he needs:

\[ 2,441,544.61 + 160,911.16 = 2,602,465.76 \]
He will be saving $2,100 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

\[ FVA = 2000\left( \frac{1 + \left( \frac{.1048}{12} \right)^{12(10)} - 1}{\left( \frac{.1048}{12} \right)} \right) = 421,180.66 \]
After he purchases the cabin, the amount he will have left is:

$421,180.66 – 325,000 = $96,180.66

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

\[ \text{FV} = 96,180.66 \left[1 + \left(\frac{.1048}{12}\right)^{12(20)}\right] = 775,438.43 \]

So, when he is ready to retire, based on his current savings, he will be short:

$2,602,465.76 – 775,438.43 = $1,827,027.33

This amount is the FV of the monthly savings he must make between years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

\[ \text{FVA} = 1,827,027.33 = C \left[ \left(1 + \left(\frac{.1048}{12}\right)^{12(20)}\right) - 1 \right] / \left(\frac{.1048}{12}\right) \]

\[ C = 2,259.65 \]

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the $1. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

\[ \text{PV} = 1 + 380 \left\{ 1 - \left[1 / (1 + .08/12)^{12(3)}\right] \right\} / (.08/12) = 12,127.49 \]

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

\[ \text{PV} = 15,000 / \left[1 + (.08/12)^{12(3)}\right] = 11,808.82 \]

The PV of the decision to purchase is:

$28,000 – 11,808.82 = $16,191.18

In this case, it is cheaper to lease the car than buy it since the PV of the leasing cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

\[ 28,000 – \text{PV of resale price} = 12,127.49 \]

\[ \text{PV of resale price} = 15,872.51 \]

The resale price that would make the PV of the lease versus buy decision is the FV of this value, so:

\[ \text{Breakeven resale price} = 15,872.51 \left[1 + (.08/12)^{12(3)}\right] = 20,161.86 \]
59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

\[ \text{EAR} = \left[1 + \left(\frac{.055}{365}\right)\right]^{365} - 1 = 5.65\% \]

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

\[
\begin{align*}
\text{PV} &= 8,000,000 + \frac{4,000,000}{1.0565} + \frac{4,800,000}{1.0565^2} + \frac{5,700,000}{1.0565^3} \\
&\quad + \frac{6,400,000}{1.0565^4} + \frac{7,000,000}{1.0565^5} + \frac{7,500,000}{1.0565^6} \\
\text{PV} &= 36,764,432.45
\end{align*}
\]

The player wants the contract increased in value by $750,000, so the PV of the new contract will be:

\[
\text{PV} = 36,764,432.45 + 750,000 = 37,514,432.45
\]

The player has also requested a signing bonus payable today in the amount of $9 million. We can simply subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

\[
\text{PV} = 37,514,432.45 - 9,000,000 = 28,514,432.45
\]

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365 / 4). The effective quarterly rate is:

\[
\text{Effective quarterly rate} = \left[1 + \left(\frac{.055}{365}\right)\right]^{91.25} - 1 = .01384 \text{ or } 1.384\%
\]

Now we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

\[
\begin{align*}
\text{PVA} &= 28,514,432.45 = C\left\{1 - \left(\frac{1}{1.01384}\right)^{24}\right\} / .01384 \\
C &= 1,404,517.39
\end{align*}
\]

60. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $20,000 you must repay in one year, and the $17,200 you borrow today. The interest rate of the loan is:

\[
\begin{align*}
20,000 &= 17,200(1 + r) \\
r &= (20,000 - 17,200) / 17,200 = .1628 \text{ or } 16.28\%
\end{align*}
\]

Because of the discount, you only get the use of $17,200, and the interest you pay on that amount is 16.28%, not 14%.
61. Here we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$$\text{APR} = 12[(1.09)^{1/12} - 1] = 8.65\%$$

To find the value today of the back pay from two years ago, we will find the FV of the annuity, and then find the FV of the lump sum. Doing so gives us:

$$\text{FVA} = \left(\frac{44,000}{12}\right)\left[\left(1 + \left(\frac{.0865}{12}\right)\right)^{12} - 1\right] / \left(\frac{.0865}{12}\right) = 45,786.76$$

$$\text{FV} = 45,786.76(1.09) = 49,907.57$$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year’s back pay:

$$\text{FVA} = \left(\frac{46,000}{12}\right)\left[\left(1 + \left(\frac{.0865}{12}\right)\right)^{12} - 1\right] / \left(\frac{.0865}{12}\right) = 47,867.98$$

Next, we find the value today of the five year’s future salary:

$$\text{PVA} = \left(\frac{49,000}{12}\right)\left[\left\{1 - \left(\frac{1}{\left[1 + \left(\frac{.0865}{12}\right)\right]^{12(5)}}\right)\right\} / \left(\frac{.0865}{12}\right)\right] = 198,332.55$$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

$$\text{Award} = 49,907.57 + 47,867.98 + 198,332.55 + 100,000 + 20,000 = 416,108.10$$

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

62. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

$$\text{Loan repayment amount} = 10,000(1.09) = 10,900$$

The amount you will receive today is the principal amount of the loan times one minus the points.

$$\text{Amount received} = 10,000(1 - .03) = 9,700$$

Now, we simply find the interest rate for this PV and FV.
\$10,900 = \$9,700(1 + r)\\
\[ r = \left(\frac{10,900}{9,700}\right) - 1 = .1237 \text{ or } 12.37\% \]
63. This is the same question as before, with different values. So:

\[ \text{Loan repayment amount} = \$10,000(1.12) = \$11,200 \]

\[ \text{Amount received} = \$10,000(1 - .02) = \$9,800 \]

\[ \$11,200 = \$9,800(1 + r) \]
\[ r = (\$11,200 / \$9,800) - 1 = .1429 \text{ or } 14.29\% \]

The effective rate is not affected by the loan amount since it drops out when solving for \( r \).

64. First we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the $1,500 application fee, you will need to borrow $221,500 to have $220,000 after deducting the fee. Solving for the payment under these circumstances, we get:

\[ \text{PVA} = \$221,500 = C \left\{ \left[ 1 - 1/(1.006)\right]^{360} \right\}/.006 \]
\[ C = \$1,503.52 \]

We can now use this amount in the PVA equation with the original amount we wished to borrow, $220,000. Solving for \( r \), we find:

\[ \text{PVA} = \$220,000 = \$1,503.52\left\{ \left[ 1 - \left[ 1 / (1 + r)\right]^{360} \right] / r \right\} \]

Solving for \( r \) with a spreadsheet, on a financial calculator, or by trial and error, gives:

\[ r = 0.6057\% \text{ per month} \]
\[ \text{APR} = 12(0.6057\%) = 7.27\% \]
\[ \text{EAR} = (1 + .006057)^{12} - 1 = 7.52\% \]

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

\[ \text{APR} = 7.20\% \]
\[ \text{EAR} = [1 + (.072/12)]^{12} - 1 = 7.44\% \]

65. Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $40.08 per month for three years, so the interest rate on the loan is:

\[ \text{PVA} = \$1,000 = \$40.08\left\{ \left[ 1 - \left[ 1 / (1 + r)\right]^{36} \right] / r \right\} \]

Solving for \( r \) with a spreadsheet, on a financial calculator, or by trial and error, gives:

\[ r = 2.13\% \text{ per month} \]
APR = 12(2.13%) = 25.60%

EAR = (1 + .0213)^12 – 1 = 28.83%

It’s called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

66. Here we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

\[ PVA = \$90,000 \left\{ \frac{1 - (1/1.08)^{20}}{.08} \right\} = \$883,633.27 \]

This amount is the same for all three parts of this question.

a. If your friend makes equal annual deposits into the account, this is an annuity with the FVA equal to the amount needed in retirement. The required savings each year will be:

\[ FVA = \$883,633.27 = C\left(1.08^{30} - 1\right)/.08 \]

\[ C = \$7,800.21 \]

b. Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

\[ FV = \$883,633.27 = PV(1.08)^{30} \]

\[ PV = \$87,813.12 \]

c. In this problem, we have a lump sum savings in addition to an annual deposit. Since we already know the value needed at retirement, we can subtract the value of the lump sum savings at retirement to find out how much your friend is short. Doing so gives us:

\[ FV \text{ of trust fund deposit} = \$25,000(1.08)^{10} = \$53,973.12 \]

So, the amount your friend still needs at retirement is:

\[ FV = \$883,633.27 - 53,973.12 = \$829,660.15 \]

Using the FVA equation, and solving for the payment, we get:

\[ \$829,660.15 = C\left(1.08^{30} - 1\right)/.08 \]

\[ C = \$7,323.77 \]

This is the total annual contribution, but your friend’s employer will contribute $1,500 per year, so your friend must contribute:

\[ \text{Friend's contribution} = \$7,323.77 - 1,500 = \$5,823.77 \]
67. We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 18.20%:

\[
PVA = $10,000 = $200 \left[ \frac{1 - (1/1.0152)^t}{.0152} \right]
\]

where \( .0152 = .182/12 \)

Solving for \( t \), we get:

\[
\frac{1}{1.0152} = 1 - \left( \frac{10,000}{200} \right)(.0152)
\]
\[
\frac{1}{1.0152} = .2417
\]
\[
t = \frac{\ln (1/.2417)}{\ln 1.0152}
\]
\[
t = 94.35 \text{ months}
\]

Without fee and annual rate = 8.20%:

\[
PVA = $10,000 = $200 \left[ \frac{1 - (1/1.006833)^t}{.006833} \right]
\]

where \( .006833 = .082/12 \)

Solving for \( t \), we get:

\[
\frac{1}{1.006833} = 1 - \left( \frac{10,000}{200} \right)(.006833)
\]
\[
\frac{1}{1.006833} = .6583
\]
\[
t = \frac{\ln (1/.6583)}{\ln 1.006833}
\]
\[
t = 61.39 \text{ months}
\]

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

\[
PVA = $10,200 = $200 \left[ \frac{1 - (1/1.006833)^t}{.006833} \right]
\]

where \( .006833 = .082/12 \)

Solving for \( t \), we get:

\[
\frac{1}{1.006833} = 1 - \left( \frac{10,200}{200} \right)(.006833)
\]
\[
\frac{1}{1.006833} = .6515
\]
\[
t = \frac{\ln (1/.6515)}{\ln 1.006833}
\]
\[
t = 62.92 \text{ months}
\]

68. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:

\[
FV_1 = $800(1.11)^5 = $1,348.05
\]
\[
FV_2 = $800(1.11)^4 = $1,214.46
\]
\[
FV_3 = $900(1.11)^3 = $1,230.87
\]
FV₄ = $900(1.11)^2 = $1,108.89

FV₅ = $1,000(1.11)^1 = $1,110.00

Value at year six = $1,348.05 + 1,214.46 + 1,230.87 + 1,108.89 + 1,110.00 + 1,000 = $7,012.26

Finding the FV of this lump sum at the child’s 65th birthday:

FV = $7,012.26(1.07)^{59} = $379,752.76

The policy is not worth buying; the future value of the deposits is $379,752.76, but the policy contract will pay off $350,000. The premiums are worth $29,752.76 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

PV = $800/1.11 + $800/1.11^2 + $900/1.11^3 + $900/1.11^4 + $1,000/1.11^5 + $1,000/1.11^6 = $3,749.04

And the value today of the $350,000 at age 65 is:

PV = $350,000/1.07^{59} = $6,462.87
PV = $6,462.87/1.11^6 = $3,455.31

The premiums still have the higher cash flow. At time zero, the difference is $2,148.25. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $293.73, the difference in the cash flows at time zero, for six years at an 11 percent interest rate, and then for 59 years at a seven percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth $29,752.76, the difference in the cash flows at time 65!

69. The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

\[
PVA = \frac{C \{1 - [1 / (1 + .085/12)]^{360}\}}{(.085/12)}
\]

\[
C = $3,460.11
\]

Now, at time = 8, we need to find the PV of the payments which have not been made. The balloon payment will be:

\[
PVA = \frac{C \{1 - [1 / (1 + .085/12)]^{12(22)}\}}{(.085/12)}
\]

PVA = $412,701.01

70. Here we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs are:
PVA = $15,000\{1 - [1 / (1 + r)^4]} / r$
And the FV of the savings is:

\[ FVA = $5,000 \left\{ \frac{(1 + r)^6 - 1}{r} \right\} \]

Setting these two equations equal to each other, we get:

\[ $15,000 \left\{ \frac{1 - \left\{ \frac{1}{1 + r} \right\}^4}{r} \right\} = $5,000 \left\{ \frac{(1 + r)^6 - 1}{r} \right\} \]

Reducing the equation gives us:

\[ (1 + r)^6 - 4.00(1 + r)^4 + 30.00 = 0 \]

Now we need to find the roots of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

\[ r = 14.52\% \]

**71.** Here we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

\[ PV = \frac{$15,000}{r} \]

And the PV of the annuity is:

\[ PVA = \frac{$20,000 \left\{ 1 - \left\{ \frac{1}{1 + r} \right\}^{10} \right\}}{r} \]

Setting them equal and solving for \( r \), we get:

\[ \frac{$15,000}{r} = \frac{$20,000 \left\{ 1 - \left\{ \frac{1}{1 + r} \right\}^{10} \right\}}{r} \]

\[ \frac{$15,000}{$20,000} = 1 - \left\{ \frac{1}{1 + r} \right\}^{10} \]

\[ .25^{1/10} = 1 / (1 + r) \]

\[ r = .1487 \text{ or } 14.87\% \]

**72.** The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

\[ \text{Effective 2-year rate} = \left[ 1 + \left( .11 / 365 \right) \right]^{365(2)} - 1 = .2460 \text{ or } 24.60\% \]

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

\[ PV = \frac{$7,500}{.2460} = $30,483.41 \]
This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

\[ PV = \$30,483.41(1 + .11/365)^{365} = \$34,027.40 \]

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

\[ PV = \$30,483.41 / (1 + .11/365)^{2(365)} = \$24,464.32 \]

73. To solve for the PVA due:

\[
PVA = \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \ldots + \frac{C}{(1 + r)^t}
\]

\[
PVA_{due} = C + \frac{C}{(1 + r)} + \ldots + \frac{C}{(1 + r)^{t-1}}
\]

\[
PVA_{due} = (1 + r) \left( \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \ldots + \frac{C}{(1 + r)^{t-1}} \right)
\]

\[
PVA_{due} = (1 + r) \cdot PVA
\]

And the FVA due is:

\[
FVA = C + C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^t
\]

\[
FVA_{due} = C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^{t-1}
\]

\[
FVA_{due} = (1 + r) \left[ C + C(1 + r) + \ldots + C(1 + r)^{t-1} \right]
\]

\[
FVA_{due} = (1 + r) \cdot FVA
\]

74. We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = \$1,000,000/(1 + .10)^{30} \]

\[ PV = \$57,308.55 \]

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

\[
PV = C \left\{ \left[ 1 - \left( \frac{1 + g}{1 + r} \right)^t \right] / (r - g) \right\}
\]

\[
\$57,308.55 = C \left\{ \left[ 1 - \left( \frac{1 + .03}{1 + .10} \right)^{30} \right] / (.10 - .03) \right\}
\]

\[ C = \$4,659.79 \]
This is the amount you need to save next year. So, the percentage of your salary is:

Percentage of salary = $4,659.79/$55,000
Percentage of salary = .0847 or 8.47%

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

75. a. The APR is the interest rate per week times 52 weeks in a year, so:

\[ APR = 52(8\%) = 416\% \]

\[ EAR = (1 + .08)^{52} - 1 = 53.7060 \text{ or } 5,370.60\% \]

b. In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With an 8 percent discount, you would receive $9.20 for every $10 in principal, so the weekly interest rate would be:

\[ \frac{10}{9.20} = 1 + r \]

\[ r = (10 / 9.20) - 1 = .0870 \text{ or } 8.70\% \]

Note the dollar amount we use is irrelevant. In other words, we could use $0.92 and $1, $92 and $100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

\[ APR = 52(8.70\%) = 452.17\% \]

\[ EAR = (1 + .0870)^{52} - 1 = 75.3894 \text{ or } 7,538.94\% \]

c. Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

\[ PVA = 68.92 = 25\left\{ \frac{1 - [1 / (1 + r)]^{4}}{r} \right\} \]

Using a spreadsheet, trial and error, or a financial calculator, we find:

\[ r = 16.75\% \text{ per week} \]

\[ APR = 52(16.75\%) = 871.00\% \]

\[ EAR = 1.1675^{52} - 1 = 3142.1572 \text{ or } 314,215.72\% \]
76. To answer this, we need to diagram the perpetuity cash flows, which are: (Note, the subscripts are only to differentiate when the cash flows begin. The cash flows are all the same amount.)

\[
\begin{array}{cccccc}
\ldots & C_3 & C_2 & C_1 \\
C_1 & C_2 & C_1 \\
\end{array}
\]

Thus, each of the increased cash flows is a perpetuity in itself. So, we can write the cash flows stream as:

\[
\frac{C_1}{R} \quad \frac{C_2}{R} \quad \frac{C_3}{R} \quad \frac{C_4}{R} \quad \ldots
\]

So, we can write the cash flows as the present value of a perpetuity, and a perpetuity of:

\[
\frac{C_2}{R} \quad \frac{C_3}{R} \quad \frac{C_4}{R} \quad \ldots
\]

The present value of this perpetuity is:

\[
PV = \frac{C}{R} / R = \frac{C}{R^2}
\]

So, the present value equation of a perpetuity that increases by \( C \) each period is:

\[
PV = \frac{C}{R} + \frac{C}{R^2}
\]

77. We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum as:

\[
FV = PV(1 + R)^t
\]

\[
\$2 = \$1(1 + R)^t
\]

Solving for \( t \), we find:

\[
\ln(2) = t[\ln(1 + R)]
\]

\[
t = \ln(2) / \ln(1 + R)
\]

Since \( R \) is expressed as a percentage in this case, we can write the expression as:

\[
t = \ln(2) / \ln(1 + R/100)
\]
To simplify the equation, we can make use of a Taylor Series expansion:

\[ \ln(1 + R) = R - \frac{R^2}{2} + \frac{R^3}{3} - \ldots \]

Since \( R \) is small, we can truncate the series after the first term:

\[ \ln(1 + R) = \ R \]

Combine this with the solution for the doubling expression:

\[ t = \frac{\ln(2)}{(R/100)} \]
\[ t = \frac{100\ln(2)}{R} \]
\[ t = 69.3147 / R \]

This is the exact (approximate) expression. Since 69.3147 is not easily divisible, and we are only concerned with an approximation, 72 is substituted.

**78.** We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum with continuously compounded interest as:

\[ \$1 = \$2e^{Rt} \]
\[ 2 = e^{Rt} \]
\[ Rt = \ln(2) \]
\[ Rt = .693147 \]
\[ t = .691347 / R \]

Since we are using interest rates while the equation uses decimal form, to make the equation correct with percentages, we can multiply by 100:

\[ t = 69.1347 / R \]
Calculator Solutions

1. 

<table>
<thead>
<tr>
<th></th>
<th>CF0</th>
<th>C01</th>
<th>F01</th>
<th>C02</th>
<th>F02</th>
<th>C03</th>
<th>F03</th>
<th>C04</th>
<th>F04</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0</td>
<td>$1,100</td>
<td>1</td>
<td>$720</td>
<td>1</td>
<td>$940</td>
<td>1</td>
<td>$1,160</td>
<td>1</td>
</tr>
<tr>
<td>I = 10</td>
<td>NPV CPT</td>
<td>$3,093.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CF0</td>
<td>C01</td>
<td>F01</td>
<td>C02</td>
<td>F02</td>
<td>C03</td>
<td>F03</td>
<td>C04</td>
<td>F04</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$1,100</td>
<td>1</td>
<td>$720</td>
<td>1</td>
<td>$940</td>
<td>1</td>
<td>$1,160</td>
<td>1</td>
</tr>
<tr>
<td>I = 18</td>
<td>NPV CPT</td>
<td>$2,619.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CF0</td>
<td>C01</td>
<td>F01</td>
<td>C02</td>
<td>F02</td>
<td>C03</td>
<td>F03</td>
<td>C04</td>
<td>F04</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$1,100</td>
<td>1</td>
<td>$720</td>
<td>1</td>
<td>$940</td>
<td>1</td>
<td>$1,160</td>
<td>1</td>
</tr>
<tr>
<td>I = 24</td>
<td>NPV CPT</td>
<td>$2,339.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. 

Enter 8 5% $7,000
Solve for PV $7,000 PMT FV $45,242.49

Enter 5 5% $9,000
Solve for PV $9,000 PMT FV $38,965.29

Enter 8 22% $7,000
Solve for PV $7,000 PMT FV $25,334.87

Enter 5 22% $9,000
Solve for PV $9,000 PMT FV $25,772.76

3. 

Enter 3 8% $700
Solve for PV $700 PMT FV $881.80

Enter 2 8% $950
Solve for PV $950 PMT FV $1,108.08

Enter 1 8% $1,200
Solve for PV $1,200 PMT FV $1,296.00

FV = $881.80 + 1,108.08 + 1,296 + 1,300 = $4,585.88
Enter 3 11% $700

N I/Y PV PMT FV

Solve for $957.34

Enter 2 11% $950

N I/Y PV PMT FV

Solve for $1,170.50

Enter 1 11% $1,200

N I/Y PV PMT FV

Solve for $1,332.00

FV = $957.34 + 1,170.50 + 1,332 + 1,300 = $4,759.84

Enter 3 24% $700

N I/Y PV PMT FV

Solve for $1,334.64

Enter 2 24% $950

N I/Y PV PMT FV

Solve for $1,460.72

Enter 1 24% $1,200

N I/Y PV PMT FV

Solve for $1,488.00

FV = $1,334.64 + 1,460.72 + 1,488 + 1,300 = $5,583.36

4. Enter 15 8% $4,600

N I/Y PV PMT FV

Solve for $39,373.60

Enter 40 8% $4,600

N I/Y PV PMT FV

Solve for $54,853.22

Enter 75 8% $4,600

N I/Y PV PMT FV

Solve for $57,320.99
<table>
<thead>
<tr>
<th>Enter</th>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>Solve for</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>15</td>
<td>8.25%</td>
<td>$28,000</td>
<td></td>
<td></td>
<td>$3,321.33</td>
</tr>
<tr>
<td>6.</td>
<td>8</td>
<td>8.5%</td>
<td>$65,000</td>
<td></td>
<td></td>
<td>$366,546.89</td>
</tr>
<tr>
<td>7.</td>
<td>20</td>
<td>10.5%</td>
<td>$3,000</td>
<td></td>
<td></td>
<td>$181,892.42</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>10.5%</td>
<td>$3,000</td>
<td></td>
<td></td>
<td>$1,521,754.74</td>
</tr>
<tr>
<td>8.</td>
<td>10</td>
<td>6.5%</td>
<td>$80,000</td>
<td></td>
<td></td>
<td>$5,928.38</td>
</tr>
<tr>
<td>9.</td>
<td>7</td>
<td>8%</td>
<td>$30,000</td>
<td></td>
<td></td>
<td>$5,762.17</td>
</tr>
<tr>
<td>12.</td>
<td>7%</td>
<td>EFF</td>
<td>4</td>
<td></td>
<td></td>
<td>7.19%</td>
</tr>
<tr>
<td>13.</td>
<td>18%</td>
<td>EFF</td>
<td>12</td>
<td></td>
<td></td>
<td>19.56%</td>
</tr>
<tr>
<td>14.</td>
<td>10%</td>
<td>EFF</td>
<td>365</td>
<td></td>
<td></td>
<td>10.52%</td>
</tr>
<tr>
<td>15.</td>
<td>12.2%</td>
<td>EFF</td>
<td>2</td>
<td></td>
<td></td>
<td>11.85%</td>
</tr>
</tbody>
</table>
Enter NOM 9.4% EFF C/Y
Solve for 9.02%

Enter NOM 8.6% EFF C/Y
Solve for 8.26%

14.
Enter NOM 13.1% EFF C/Y
Solve for 13.92%

Enter NOM 13.4% EFF C/Y
Solve for 13.85%

15.
Enter NOM 14% EFF C/Y
Solve for 13.11%

16.
Enter $20 \times 2$ 9.6%/2 $\times 365$ $1,400$ N I/Y PV PMT FV
Solve for $9,132.28$

17.
Enter $5 \times 365$ 8.4% / 365 $6,000$ N I/Y PV PMT FV
Solve for $9,131.33$

Enter $10 \times 365$ 8.4% / 365 $6,000$ N I/Y PV PMT FV
Solve for $13,896.86$

Enter $20 \times 365$ 8.4% / 365 $6,000$ N I/Y PV PMT FV
Solve for $32,187.11$

18.
Enter $6 \times 365$ 11% / 365 $45,000$ N I/Y PV PMT FV
Solve for $23,260.62$
19. Enter 300% 12
Solve for 1,355.19%

20. Enter 60 7.4% / 12 $61,800
Solve for $1,235.41

21. Enter 7.4% 12
Solve for 7.66%

22. Enter 1,733.33% 52
Solve for 313,916,515.69%

23. Enter 22.86% 12
Solve for 25.41%

24. Enter 30 × 12 10% / 12 $250
Solve for $565,121.98

25. Enter 10.00% 12
Solve for 10.47%

26. Enter 4 × 4 0.75% $1,500
Solve for $22,536.47
27. Enter 11.00% NOM EFF C/Y Solve for 11.46%

\[
\begin{align*}
&\text{CF}_0: \$0 \\
&\text{C}_1: \$900, \text{F}_1: 1 \\
&\text{C}_2: \$850, \text{F}_2: 1 \\
&\text{C}_3: \$0, \text{F}_3: 1 \\
&\text{C}_4: \$1,140, \text{F}_4: 1 \\
\end{align*}
\]

I = 11.46%
NPV CPT $2,230.20

28. Enter 18% NOM EFF C/Y Solve for 17.26%

\[
\frac{17.26\%}{2} = 8.63\%
\]

\[
\begin{align*}
&\text{CF}_0: \$0 \\
&\text{C}_1: \$2,800, \text{F}_1: 1 \\
&\text{C}_2: \$0, \text{F}_2: 1 \\
&\text{C}_3: \$5,600, \text{F}_3: 1 \\
&\text{C}_4: \$1,940, \text{F}_4: 1 \\
\end{align*}
\]

I = 8.45%
NPV CPT $8,374.62

30. Enter 18% NOM EFF C/Y Solve for 16.90%

\[
\frac{16.90\%}{4} = 4.22\%
\]

Enter 18% NOM EFF C/Y Solve for 16.67%

\[
\frac{16.67\%}{12} = 1.39\%
\]
31. Enter 6 2.50% / 12 $5,000
   N I/Y PV PMT FV
   Solve for $5,062.83

Enter 6 17% / 12 $5,062.83
   N I/Y PV PMT FV
   Solve for $5,508.70
   $5,508.70 – 5,000 = $508.70

32. Stock account:
Enter 360 12% / 12 $600
   N I/Y PV PMT FV
   Solve for $2,096,978.48

   Bond account:
Enter 360 7% / 12 $300
   N I/Y PV PMT FV
   Solve for $365,991.30
   Savings at retirement = $2,096,978.48 + 365,991.30 = $2,462,969.78

Enter 300 9% / 12 $2,462,969.78
   N I/Y PV PMT FV
   Solve for $20,669.15

33. Enter 12 1.08% $1
   N I/Y PV PMT FV
   Solve for $1.14

Enter 24 1.08% $1
   N I/Y PV PMT FV
   Solve for $1.29

34. Enter 480 11% / 12 $1,000,000
   N I/Y PV PMT FV
   Solve for $116.28

Enter 360 11% / 12 $1,000,000
   N I/Y PV PMT FV
   Solve for $356.57
<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
<th>Calculations</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.</td>
<td>Enter 240 11% / 12 $1,000,000 Solve for ( PV = $1,155.22 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36.</td>
<td>Enter 12 / 3 ±$1 $4 Solve for ( PMT = 41.42% )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36.</td>
<td>Enter 10.00% 12 NOM EFF C/Y Solve for ( I = 10.47% )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enter 2 10.47% $90,000 Solv for ( FV = $155,215.98 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39.</td>
<td>Enter 10 10% $7,000 Solv for ( PV = $43,011.97 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enter 10 5% $7,000 Solv for ( PV = $54,052.14 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enter 10 15% $7,000 Solv for ( PV = $35,131.38 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.</td>
<td>Enter 9% / 12 ±$225 $20,000 Solv for ( I = 68.37 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
41.
Enter $55,000 \pm $1,120

Solve for $0.682\% \times 12 = 8.18\%$

42.
Enter $6.8\% / 12 \times $1,100$

Solve for $220,000 - 168,731.02 = $51,268.98$

Enter $51,268.98$

Solve for $392,025.82$

43.

<table>
<thead>
<tr>
<th>CF0</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>$1,500$</td>
</tr>
<tr>
<td>F01</td>
<td>1</td>
</tr>
<tr>
<td>C02</td>
<td>$0$</td>
</tr>
<tr>
<td>F02</td>
<td>1</td>
</tr>
<tr>
<td>C03</td>
<td>$1,800$</td>
</tr>
<tr>
<td>F03</td>
<td>1</td>
</tr>
<tr>
<td>C04</td>
<td>$2,400$</td>
</tr>
<tr>
<td>F04</td>
<td>1</td>
</tr>
</tbody>
</table>

$I = 10\%$

NPV CPT $4,355.24$

PV of missing CF $= 6,785 - 4,355.24 = $2,429.76$

Value of missing CF:

Enter $2 \times 10\% \times $2,429.76$

Solve for $2,940.02$
44. 

\[
\begin{array}{c|c}
C01 & \$1,400,000 \\
F01 & 1 \\
C02 & \$1,800,000 \\
F02 & 1 \\
C03 & \$2,200,000 \\
F03 & 1 \\
C04 & \$2,600,000 \\
F04 & 1 \\
C05 & \$3,000,000 \\
F05 & 1 \\
C06 & \$3,400,000 \\
F06 & 1 \\
C07 & \$3,800,000 \\
F07 & 1 \\
C08 & \$4,200,000 \\
F08 & 1 \\
C09 & \$4,600,000 \\
F09 & 1 \\
C10 & \$5,000,000 \\
\end{array}
\]

I = 9%

NPV CPT
$19,733,830.26

45. 
Enter 360 .80($2,400,000) \pm$13,000

\[
\begin{array}{c}
N \\
I/Y \\
PV \\
PMT \\
FV \\
\end{array}
\]

Solve for

APR = 0.598% \times 12 = 7.17%

Enter 7.17% EFF 12

\[
\begin{array}{c}
NOM \\
EFF \\
C/Y \\
\end{array}
\]

Solve for

7.42%

46. 
Enter 3 13% $145,000

\[
\begin{array}{c}
N \\
I/Y \\
PV \\
PMT \\
FV \\
\end{array}
\]

Solve for

Profit = $100,492.27 – 94,000 = $6,492.27

Enter 3 \pm$94,000 $145,000

\[
\begin{array}{c}
N \\
I/Y \\
PV \\
PMT \\
FV \\
\end{array}
\]

Solve for

15.54%
47. Enter 17 10% $2,000
Solve for $16,043.11

Enter 8 10% $16,043.11
Solve for $7,484.23

48. Enter 84 13% / 12 $1,500
Solve for $82,453.99

Enter 96 10% / 12 $1,500
Solve for $98,852.23

Enter 84 13% / 12 $98,852.23
Solve for $39,985.62

$82,453.99 + 39,985.62 = $122,439.62

49. Enter 15 × 12 9.5%/12 $1,000
Solve for $395,984.63

FV = $395,984.63 = PV e^{-0.09(15)}, PV = $395,984.63e^{-1.35} = $102,645.83

50. PV@ t = 14: $5,000 / 0.057 = $87,719.30

Enter 7 5.7% $87,719.30
Solve for $59,507.30

51. Enter 12 $20,000 ±$1,916.67
Solve for 2.219%

APR = 2.219% × 12 = 26.62%

Enter 26.62% NOM 12 EFF 30.12% C/Y
52. Monthly rate = \( \frac{.10}{12} = .0083 \); semiannual rate = \( (1.0083)^6 - 1 = 5.11\% \)

Enter 10 5.11% $6,000
Solve for PV $46,094.33

Enter 8 5.11% $46,094.33
Solve for PV $30,949.21

Enter 12 5.11% $46,094.33
Solve for PV $25,360.08

Enter 18 5.11% $46,094.33
Solve for PV $18,810.58

53. 

a. Enter 8 9.5% $950
Solve for PV $5,161.76

b. 2\textsuperscript{nd} BGN 2\textsuperscript{nd} SET
Enter 8 9.5% $950
Solve for PV $5,652.13

54. 2\textsuperscript{nd} BGN 2\textsuperscript{nd} SET
Enter 60 8.15% / 12 $61,000
Solve for PMT $1,232.87

57. Pre-retirement APR:
Enter NOM 11% EFF 12
Solve for EFF 10.48%

Post-retirement APR:
Enter NOM 8% EFF 12
Solve for EFF 7.72%
At retirement, he needs:

Enter 240 7.72% / 12 $20,000 $750,000
Solve for

$2,602,465.76

In 10 years, his savings will be worth:

Enter 120 10.48% / 12 $2,000
Solve for

$421,180.66

After purchasing the cabin, he will have: $421,180.66 – 325,000 = $96,180.66

Each month between years 10 and 30, he needs to save:

Enter 240 10.48% / 12 $96,180.66 $2,602,465.76
Solve for

$2,259.65

58. PV of purchase:

Enter 36 8% / 12 $15,000
Solve for

$11,808.82

$28,000 – 11,808.82 = $16,191.18

59. PV of lease:

Enter 36 8% / 12 $380
Solve for

$12,126.49 + 1 = $12,127.49

Lease the car.

You would be indifferent when the PV of the two cash flows are equal. The present value of the purchase decision must be $12,127.49. Since the difference in the two cash flows is $28,000 – 12,127.49 = $15,872.51, this must be the present value of the future resale price of the car. The break-even resale price of the car is:

Enter 36 8% / 12 $15,872.51
Solve for

$20,161.86

59.
New contract value = $36,764,432.45 + 750,000 = $37,514,432.45

PV of payments = $37,514,432.45 – 9,000,000 = $28,514,432.45
Effective quarterly rate = \([1 + (0.055/365)]^{91.25} – 1 = 0.01384 \text{ or } 1.384\%\)

### 60.

Enter: 
- 1  $17,200
- 16.28%

Solve for: 
- PMT
- FV
- ±$20,000

### 61.

Enter: 
- 9% 12

Solve for: 
- NOM
- EFF
- C/Y
- 8.65%

Enter: 
- 12 8.65% / 12  $44,000 / 12

Solve for: 
- N
- I/Y
- PV
- PMT
- FV
- $45,786.76

Enter: 
- 1 9% $45,786.76

Solve for: 
- N
- I/Y
- PV
- PMT
- FV
- $49,907.57
Enter 12 8.65% / 12 $46,000 / 12
Solve for $47,867.98

Enter 60 8.65% / 12 $49,000 / 12
Solve for $198,332.55

Award = $49,907.57 + 47,867.98 + 198,332.55 + 100,000 + 20,000 = $416,108.10

62. Enter 1 $9,700 ± $10,900
Solve for 12.37%

63. Enter 1 $9,800 ± $11,200
Solve for 14.29%

64. Refundable fee: With the $1,500 application fee, you will need to borrow $221,500 to have $220,000 after deducting the fee. Solve for the payment under these circumstances.

Enter 30 × 12 7.20% / 12 $221,500
Solve for $1,503.52

Enter 30 × 12 $220,000 ± $1,503.52
Solve for 0.6057%

APR = 0.6057% × 12 = 7.27%

Enter 7.27% EFF 12
Solve for 7.52%

Without refundable fee: APR = 7.20%

Enter 7.20% EFF 12
Solve for 7.44%
65. Enter $1,000
\[ \text{N} \quad \text{I/Y} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \]
Solve for 2.13%
\[ \text{APR} = 2.13\% \times 12 = 25.60\% \]

Enter 25.60% \[ \text{NOM} \quad \text{EFF} \quad \text{C/Y} \]
Solve for 28.83%

66. What she needs at age 65:
Enter $90,000
\[ \text{N} \quad \text{I/Y} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \]
Solve for $883,633.27

\[ \begin{align*}
a. & \quad \text{Enter} \quad 30 \quad 8\% \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
& \quad \text{Solve for} \quad \text{FV} \quad \text{PMT} \quad \text{PV} \\
& \quad \text{Enter} \quad 30 \quad 8\% \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
& \quad \text{Solve for} \quad \text{FV} \quad \text{PMT} \quad \text{PV} \\
b. & \quad \text{Enter} \quad 30 \quad 8\% \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
& \quad \text{Solve for} \quad \text{FV} \quad \text{PMT} \quad \text{PV} \\
c. & \quad \text{Enter} \quad 10 \quad 8\% \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
& \quad \text{Solve for} \quad \text{FV} \quad \text{PMT} \quad \text{PV} \end{align*} \]

At 65, she is short: $883,633.27 – $53,973.12 = $829,660.15
Enter $829,660.15
\[ \text{N} \quad \text{I/Y} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \]
Solve for $7,323.77
Her employer will contribute $1,500 per year, so she must contribute:
$7,323.77 – 1,500 = $5,823.77 per year

67. Without fee:
Enter $10,000
\[ \text{N} \quad \text{I/Y} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \]
Solve for 94.35
Enter  8.2% / 12 $10,000 ±$200
Solve for  61.39
With fee:
Enter  8.2% / 12 $10,200 ±$200
Solve for  62.92
68. Value at Year 6:
Enter  5 11% $800
Solve for  $1,348.05
Enter  4 11% $800
Solve for  $1,214.46
Enter  3 11% $900
Solve for  $1,230.87
Enter  2 11% $900
Solve for  $1,108.89
Enter  1 11% $1,000
Solve for  $1,110
So, at Year 5, the value is: $1,348.05 + 1,214.46 + 1,230.87 + 1,108.89 + 1,100 + 1,000 = $7,012.26
At Year 65, the value is:
Enter  59 7% $7,012.26
Solve for  $379,752.76
The policy is not worth buying; the future value of the deposits is $379,752.76 but the policy contract will pay off $350,000.
69.
Enter $30 \times 12$ 8.5% / 12 $450,000
Solve for $N$ I/Y PV PMT FV
$3,460.11$
Enter $22 \times 12$ 8.5% / 12 $3,460.11$
Solve for $N$ I/Y PV PMT FV $412,701.01$

70.

75.  

a. APR = 8% $\times 52 = 416$

Enter 416% EFF 52
Solve for NOM EFF C/Y 5,370.60%

b. Enter 1 I/Y $9.20$ PMT PMT $\pm$10.00
Solve for N I/Y PV PMT FV 8.70%

APR = 8.70% $\times 52 = 452.17$

Enter 452.17% EFF 52
Solve for NOM EFF C/Y 7,538.94%

c. Enter 4 I/Y $68.92$ PMT PMT $\pm$25
Solve for N I/Y PV PMT FV 16.75%

APR = 16.75% $\times 52 = 871.00$

Enter 871.00% EFF 52
Solve for NOM EFF C/Y 314,215.72%