Consider a bond that has face (or par or maturity) value of $M$, with $T$ periods to maturity. If the coupon rate of interest is $r$ per period, then $C = rM$ is the coupon interest paid per period.
Let $y$ denote the yield to maturity of the bond (YTM) and let $P$ be the current price of the bond. By definition, the YTM $y$ solves the equation

$$P = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{M}{(1+y)^T}.$$ 

This is equation (3.4) on page 37 in (F) (we use $T$ for the number of
periods, while Fabozzi frequently uses n).

We want to establish the relationship between $P$ and $M$ that reflects the relationship between $r$ and $y$.

SPECIAL CASE $(r = y)$. If $r = y$, then $P = M$. To see this, suppose that $T = 1$. Then paying $P$ now for the bond entitles you to the face value $M$ in one
period plus interest of $C$ for one period. So the cash flow in one period is $M + C$. Recognizing that $C = rM$ and $y = r$, we get that

$$P = \frac{M+C}{1+y} = \frac{M+rM}{1+r} = M.$$ 

Thus the claim is valid for $T = 1$.

Suppose now that $T = 2$. The cash flow at maturity is again $M + C$ and its present value one period before maturity is $M$, as we have just seen.
Thus one period from now the bond will pay interest of $C$ and will have cash flows worth $M$. Again we have cash flow worth $M + C$ in one period. Present valuing this at $y = r$ again gives $M$. Thus the claim is valid for $T = 2$.

Suppose that $T = 3$. In one period we will have an interest payment of $C$ and we will own a bond with two
periods to maturity. That bond will be worth \( M \) if \( y = r \) as we have just seen. Thus we will have cash flows worth \( M + C \) in one period. The present value of these cash flows now when \( y = r \) is \( M \) as demonstrated above. This same argument now applies for any value of \( T \) and hence the claim that \( P = M \) when \( r = y \) is true for any value of \( T \).
There is a pleasant consequence of this result, namely the formula for the value of an ordinary annuity. Let $PVAF(r, T)$ (present value annuity factor) denote the value of an ordinary annuity of $1.00 per period for $T$ periods at interest rate $r$. Then $rMPVAF(r, T)$ is the present value of the interest on a bond with coupon
interest rate of $r$, yield to maturity $y = r$, and face value of $M$. We know that the value of such a bond now must be $P = M$. Thus

$$r \text{MPVAF}(r, T) + \frac{M}{(1+r)^T} = M.$$  

Dividing through by $M$, we get that

$$r \text{PVAF}(r, T) + \frac{1}{(1+r)^T} = 1.$$  

Solving for $\text{PVAF}(r, T)$ gives
PVAF(r, T) = \frac{1 - \frac{1}{(1+r)^T}}{r},

which is the formula (2.5), page 18 in (F).

GENERAL CASE. We now use this formula to establish the relationship for general $r$ and $y$. For a $T$ period bond with coupon rate $r$, face value of $M$, and yield $y$, the interest coupon $C = rM$ is a $T$ period ordinary annuity
whose present value is CPVAF(y, T).

Substituting this into the pricing equation gives,

\[
P = \text{CPVAF}(y, T) + \frac{M}{(1+y)^T}
\]

\[
= C \left( \frac{1 - \frac{1}{(1+y)^T}}{y} \right) + \frac{M}{(1+y)^T}
\]

\[
= rM \left( \frac{1 - \frac{1}{(1+y)^T}}{y} \right) + \frac{M}{(1+y)^T}
\]
\[ M \left[ \frac{r}{y} \left( 1 - \frac{1}{(1+y)^T} \right) + \frac{1}{(1+y)^T} \right] \]
\[ = M s, \]

where \( s = \left[ \frac{r}{y} \left( 1 - \frac{1}{(1+y)^T} \right) + \frac{1}{(1+y)^T} \right] \). Note that the second equality above is equation (4.9) page 64 in (F) for \( M = 100 \).
Let \( q = \frac{1}{(1+y)^T} \). Then clearly

\[ 0 < q < 1, \]

and we have that \( s = q \cdot 1 + (1-q) \frac{r}{y} \). Thus if \( r > y \), then \( s > 1 \), and \( P = Ms > M \), i.e., the bond is selling at a premium to par. If \( r < y \), then \( s < 1 \), and \( P = Ms < M \), i.e., the bond is selling at a discount to par. Finally, if \( r = y \), then \( s = 1 \), and \( P = Ms = M \), i.e., the bond is selling at
par, as we demonstrated above in the SPECIAL CASE.