Black/Scholes Greeks

$C$ denotes the Black/Scholes formula for the European call option and $P$ denotes the same thing for the European put option on a stock whose price is $S$. $\sigma, t, r, X$ denote the volatility, time to maturity, interest rate, and strike price of the options, respectively. The underlying stock will not pay a dividend for the period $t$ to maturity. Changes in $C$ with respect to an infinitesimal change in $\sigma$ is denoted by the partial derivative $\frac{\partial C}{\partial \sigma}$. Similarly for the other parameters and for changes in $P$.

$$\frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = S\sqrt{t} N'(d_1) > 0,$$

where $N'(x)$ is the standard normal density evaluated at $x$.

$$\frac{\partial C}{\partial t} = \frac{S \sigma N'(d_1)}{2\sqrt{t}} + X r e^{-rt} N(d_2) > 0,$$

$$\frac{\partial P}{\partial t} = \frac{\partial C}{\partial t} - X r e^{-rt} > 0 \text{ if the put is near or out of the money}$$

$$< 0 \text{ if the put is deep in the money}.$$

$$\frac{\partial C}{\partial X} = -e^{-rt} N(d_2) < 0 < \frac{\partial P}{\partial X} = e^{-rt} N(-d_2)$$

$$\frac{\partial C}{\partial S} = N(d_1) > 0 > \frac{\partial P}{\partial S} = N(d_1) - 1.$$

$$\frac{\partial C}{\partial r} = X t e^{-rt} N(d_2) > 0 > \frac{\partial P}{\partial r} = \frac{\partial C}{\partial r} - X t e^{-rt}.$$