How Firms Manage Risk: The Optimal Mix of Linear and Non-Linear Derivatives

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A major challenge faced by corporate risk managers in formulating a value-maximizing hedging strategy is identifying the appropriate mix of linear and non-linear derivatives. We use the term “linear derivative” to refer to products such as futures, forward, and swap contracts whose payoffs vary in linear fashion with changes in the underlying asset price or reference rate. “Non-linear derivatives” are contracts with option-like payoffs, including caps, floors and swaptions.

A cursory inspection of the corporate derivatives positions summarized in Table 1 shows large variation in U.S. companies’ use of linear and non-linear derivatives. In a sample of 671 non-financial firms that reported using interest rate derivatives, we found 470 companies that reported the use of linear derivatives only, 89 firms using strictly option-like or non-linear derivatives, and 112 firms using some combination of the two groups (see Panel A). We also found similar patterns among corporate users of both currency derivatives and commodity derivatives (see Panels B and C).

These observations give rise to several interesting questions about a company’s choice and mix of hedging instruments. For instance, why do most firms use mainly linear derivatives? Why do a substantially smaller number of firms use only non-linear derivatives? And what factors dictate a use of a combination of the two?

In this paper we provide evidence that suggests that the extent and nature of a company’s business risk are key factors for understanding these questions. Typical textbook presentations of hedging focus on the management of market or price risk and, in so doing, typically assume the associated quantity levels are fixed. For example, to protect revenues, a natural gas producer will sell linear derivatives such as natural gas futures in quantities to cover its projected gas production; a multinational corporation will enter currency forward contracts to eliminate the exchange rate risk associated with its anticipated foreign sales; and to stabilize the cost of its projected jet fuel requirements, an airline will purchase a swap having a corresponding notional quantity. These are reasonable strategies for companies facing little or no uncertainty about the quantities to be hedged. But in practice, in addition to price uncertainty, most companies face other factors contributing to business risk, including the demand and supply uncertainty associated with the quantity of its outputs and inputs. As we attempt to show in this paper, it is this uncertainty about quantity that creates an important role for non-linear derivatives.

The major findings of our study can be summarized as follows:

- For corporations having a low degree of business risk, the optimal hedging portfolio will be composed largely of linear instruments.
- As business risk increases and thus the risk of “overhedging” becomes a greater concern, there should be a substitution of non-linear contracts for linear contracts.
- The degree of substitution between linear and non-linear instruments will be influenced strongly by the correlation between the level of output and prices. A negative correlation will exacerbate the substitution effect (that is, cause the firm to use even more option-like contracts and fewer linear contracts than otherwise) while a positive correlation will dampen it.
Finance theory has identified several ways that hedging can increase firm equity values. For example, by reducing the dispersion of operating cash flows, hedging can be used to reduce both the company’s expected tax liabilities and its direct and indirect costs associated with financial distress and bankruptcy. In the process, it can also help manage any underinvestment problem associated with costly external financing.1

But as we have already noted, reducing the dispersion of operating cash flows presents a dual challenge for corporate managers, who must contend with (at least) two sources of risk. These include not only the traditional market or price risk (“P” risk), but also the associated quantity risk (“Q” risk). Corporate cash flows are affected by the interaction of these two risks in at least two potential contexts. Consider first typical producer firms such as natural gas producers, power generators, mining companies, or even the multinational corporations that “produce” foreign sales. For these firms, dollar revenues can be generally expressed as “P × Q,” where Q represents the number of units sold at a unit price of P. For manufacturing or commodity-using firms where the hedgeable risks are associated with the firm’s inputs, P × Q represents the total input cost.

As quantity and price risk increase, there is greater likelihood that the firm will experience “overhedging” costs from a strategy of matching a linear hedging position to an expected exposure level.2 For the producer firm, these costs can arise if the firm ends up having sold too many linear (e.g., forward) contracts relative to realized output or foreign revenues, and output prices or currency rates rise above expected levels. In such a case, the firm suffers not only a revenue shortfall due to the lower output, but also finds that the revenue benefits from higher prices are more than offset by the losses on its excess linear contract position.

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2. In an earlier issue of this journal, Christopher Culp and Merton Miller analyze the merits of a matching (rollover) hedging strategy. The problem we address differs in that we consider quantity risk, whereas their analysis focuses on the hedging of sales contracts having fixed quantities in which quantity risk did not arise apart from a performance default. See Culp and Miller, “Metallgesellschaft and the Economics of Synthetic Storage,” Journal of Applied Corporate Finance, Vol. 7, No. 4 1995, pp. 62-76.
For this reason, as the risk of overhedging increases, managers are likely to reduce the firm's linear positions. But, to maintain protection against falling prices, the firm will substitute to some extent a quantity of non-linear contracts (e.g., purchase puts). And the reverse is true for manufacturers, in which case overhedging results from going long an excessive number of linear contracts relative to actual input requirements, coupled with a subsequent fall in input prices. In such cases, managers should again reduce their linear hedging position and substitute the purchase of calls to maintain the firm's protection against rising prices.

As two pieces of evidence in support of these quantity risk effects, consider the risk management practices of Merck and American Airlines. Merck has extensive worldwide pharmaceutical operations and sales that expose it to extensive foreign exchange risk. According to recent 10-K filings, the firm's hedging program is aimed at protecting the U.S. dollar value of (1) certain assets and liabilities held in foreign currencies and (2) future cash flows derived from foreign currency-denominated sales. Merck's hedges of its assets and liabilities, whose quantities are largely fixed, are conducted primarily with linear contracts such as currency forward contracts. But with respect to its foreign sales, Merck's policy is to "partially hedge forecasted sales that are expected to occur over its planning cycle, typically no more than three years into the future. The Company will layer in hedges over time, increasing the portion of sales hedged as it gets closer to the expected date of the transaction." Thus, with the resolution of quantity risk with the passage of time, the firm will increase its hedging position. Furthermore, in managing the exposures of those foreign sales with a high degree of quantity risk, Merck states that its policy is primarily to use currency options.

American Airlines uses two basic strategies to manage the commodity price risk associated with its extensive jet fuel requirements and the currency risk associated with its foreign revenues. Like Merck, to protect the dollar value of its cash flows derived from its uncertain foreign currency-denominated revenues, American enters into various currency put option agreements on a number of currencies. But, in the case of jet fuel, since there is relatively little uncertainty about its future requirements (due to its adherence to an advanced flight schedule), American enters into fuel swap (linear) contracts to protect against increases in fuel prices.

A NUMERICAL ILLUSTRATION

In this section of the paper, we illustrate the optimal roles of linear and non-linear derivatives using a simple example involving a small independent oil producer that faces both price and quantity risk with respect to its output. The firm anticipates producing 10 million (mm) barrels of oil during the upcoming year, which are to be sold at an expected market price of $25 per barrel. The firm's variable production costs are $15 per barrel. Thus, the firm's expected revenues and gross profits are $250mm and $100mm, respectively. Fixed production costs are $60mm, expected EBIT is thus $40mm, and interest expenses (I) are $10mm, giving an expected EBT of $30mm.

Though management expects the firm to be profitable, changes in the demand for and price of oil could affect the firm's ability to meet its debt service obligations. Creditor agreements require the firm to maintain a minimum interest coverage ratio, as measured by EBIT/I, of 2.0X. Thus, with interest expense of $10mm, the firm must generate EBIT of at least $20mm. If this condition is violated, the firm incurs a financial distress cost that is an increasing percentage of the extent of the shortfall.

We consider two cases for output levels. In the first case, output is fixed at 10 million barrels, while in the second case demand is uncertain and can vary between 6 and 14 million barrels. To assist in the management of price and quantity risk, the firm has access to both forward and option contracts, each with contract/strike prices of $25 per barrel. To maximize shareholder wealth, managers choose that combination of forward and options contracts that minimizes financial distress costs.

3. In an earlier issue of this journal, Judy Lewent and John Kearney discuss the currency risk management practices at Merck and the firm’s aim to ensure sufficient cash flow for financing its growth strategy and to fund research and development activities. See Lewent and Kearney “Identifying, Measuring, and Hedging Currency Risk at Merck,” *Journal of Applied Corporate Finance*, Vol. 2, No. 4, 1990.

4. René Stulz discusses necessary conditions for corporate hedging programs to be in the interest of shareholders. As used in this example, he offers as one justification the reduction of financial distress costs. See Stulz, “Rethinking Risk Management,” *Journal of Applied Corporate Finance*, Vol. 9 No. 3, pp. 8-24.
Case 1: Hedging in the Absence of Quantity Risk

(a) Linear Contracts: In the simple case of zero output risk, the firm’s hedging decision is straightforward. The optimal linear strategy is to sell a quantity of forward contracts that ensures a minimum guaranteed level of cash flow so as to avoid the triggering of financial distress costs. The firm avoids financial distress when it generates sufficient cash flows to cover its fixed costs ($60mm) and still produce a level of EBIT that will satisfy its coverage ratio requirements ($20mm), or $80mm in total.

Assume for illustration purposes that though prices are projected to be $25, they can range from $15 to $40. Thus, to guarantee a minimum level of cash flow of $80mm, even in the worst case that output prices fall to $15, the firm will sell forward contracts for 8mm barrels.

This example first illustrates that a firm’s optimal hedging position is typically to partially hedge. The hedge needs to ensure only that the firm generates sufficient funds to avoid financial distress costs. Though the firm’s entire production of 10mm barrels is subject to price risk, the firm can hedge as few as 8mm barrels and still avoid financial distress costs. Ignoring transaction costs, the firm is actually indifferent to hedging with between 8mm and 10mm contracts, but hedging with more than 8mm contracts within this range produces zero expected net benefits.

Hedging more than 10mm barrels, however, exposes the firm to the possibility of “overhedging.” Overhedging in this case would arise from selling more linear contracts than realized output. In our example, hedging more than 10mm barrels of output will be suboptimal because the additional forward contract position will actually increase expected distress costs. To see this, note first that if the firm hedges at least 8mm barrels and prices fall, the gains from any additional short forward contracts will not further reduce distress costs since the minimum required funds of $80mm are already ensured. Second, if prices rise, the loss on any additional forward position beyond 10mm barrels will exceed the extra revenues the firm generates from selling output at higher prices. If prices rise far enough, this could trigger financial distress costs. For example, if the firm sells 13 million forwards, financial distress costs are generated if prices rise above $31.67. We refer to this increase in expected financial distress costs due to overhedging with linear contracts as the “cost of overhedging.”

(b) Non-linear Contracts: At the margin, the use of non-linear instruments will produce negligible (or even negative) benefits. For example, if the firm is already at its optimal linear position, substituting non-linear for linear contracts may not be cost-effective for at least two reasons. First, hedging with non-linear or option contracts will generally require a larger number of contracts, which may result in larger transaction costs. Second, unlike linear contracts, option-like contracts generally require payment of up-front option “premiums,” which could end up contributing to financial distress.

To illustrate these points, first recall that the firm’s optimal linear hedging position is 8mm contracts. Now consider the substitution of, say, 1 million long put contracts for the last million forward contracts. The firm’s hedging position will now consist of 7mm short forward contracts and 1mm long puts. Because of the option premium, the put will not provide the same level of net downside protection as did the forward contract. To illustrate, assume a put value of $2 and that output prices fall to say $15. The firm’s gross output profit will be zero, the profits on the 7mm forward contracts will be $70mm, and the net profit on the put will be ($10mm-$2mm) or $8mm. In this case, the firm’s total cash flow before fixed costs will be only $78mm, and the firm will now incur financial distress costs (since this number is less than the required $80mm).

To avoid these costs, the firm must instead purchase puts in the ratio of $10/($10 – $2), or 1.25 puts for every forward contract. In terms of hedging performance, the firm will be indifferent between purchasing the 1.25mm puts and selling the last 1mm of forward contracts. But if transaction costs for the two contract types are roughly the same, then selling forward contracts could be preferred due to the lower number of contracts required.

Now let’s consider the situation where the firm is much closer to experiencing financial distress. For example, suppose the firm’s cost structure and

5. As an alternative to forward contracts in our example, the linear strategies can also be conducted using, for example, NYMEX crude oil futures as well as OTC swap contracts.

6. We later present results using more realistic distributional assumptions for output prices.
coverage ratio requirement require it to generate gross profits of, say, $988mm (instead of $80mm) to avoid financial distress costs. With expected revenues of $100mm, the firm is now much closer to potentially experiencing financial distress. In this case, the firm’s optimal linear strategy would be to sell 9.8mm forward contracts, which will ensure a minimum level of hedged revenue of $988mm (even if prices were to fall to $15). In terms of substituting puts, the maximum quantity that the firm can purchase and still ensure a sufficient level of hedged revenue of at least $988mm is only 1mm puts (at $2 each and thus a total cost of $2mm). But because of the premium payments required by options, purchasing more than 1mm puts will produce hedged revenue of less than $988 if, for example, prices were to remain unchanged and the options expire worthless. Thus, to maintain the same level of downside protection, the firm’s purchase of these 1mm puts must be in place of only 800,000 forward contracts (based on the substitution ratio of 1.25 puts per forward). The resulting hedge position will consist of 1mm puts long and 9mm forwards short. Thus, as this example is meant to show, the closer the firm is to experiencing financial distress, the smaller is the opportunity for using puts. Moreover, as long as the quantity risk is zero, the use of puts will not provide benefits over and above those provided by forwards.

Case 2: Hedging in the Presence of Quantity Risk

(a) Linear Contracts: Introducing quantity uncertainty makes avoidance of overhedging a more difficult task. Assume for now that output levels are uncorrelated with prices and that the firm’s output could vary between 6mm and 14mm barrels. If the firm continues with an 8mm forward contract position, then overhedging costs can arise if prices increase and realized output is between 6mm and 8mm barrels. To avoid completely the risk of overhedging, the firm would have to sell 6mm or fewer forward contracts. The problem, however, is that using less than 8mm contracts exposes the firm to potential distress costs in the event that prices fall. Thus, in this case, the optimal forward contract position will be for a notional quantity of between 6mm and 8mm barrels. The optimal position will be the one that minimizes the sum of the expected distress costs from falling prices and the costs of overhedging from rising prices—a sum that we will henceforth refer to as “total overall expected distress costs.” Towards the end of this example, we provide solutions for the optimal position under varying degrees of quantity risk.

(b) Non-linear Contracts: In cases involving uncertain quantity, there is likely to be an important role for non-linear contracts because of their ability to reduce further a firm’s total overall expected distress costs. Given a firm’s optimal linear position, consider the substitution of a long put in place of the last short forward contract. If prices fall, the put is less effective for reducing expected distress costs because of the option premium. But if prices rise, the put is more advantageous since it will expire worthless instead of generating losses like the forwards. Thus, the substitution of puts for forwards can end up either increasing or reducing total overall expected distress costs. The substitution will be value-increasing only if the reduction in overhedging costs exceeds the increase in expected distress costs, thus reducing total overall expected distress costs. In cases where substitution was expected to increase overall costs, the firm could engage in “reverse substitution” whereby they simultaneously sell additional forward contracts and write puts (which together can be viewed as equivalent to “writing calls”).

In either event, however, the extent of substitution—and hence the likely use of options—is limited. Recall that the optimal linear position was between 6mm and 8mm barrels. And let’s assume that it was 7mm. If either substitution or reverse substitution is optimal, the resulting optimal linear position will not fall below 6mm barrels (a reduction of 1mm) or above 8mm (an addition of 1mm). If the hedge is reduced to 6mm barrels, all costs of overhedging are completely eliminated, while at 8mm barrels the expected distress costs in low-price states cannot be further reduced through additional forward contracts. Thus, based on the 1.25 substitution rate of puts per forward derived earlier, the potential maximum number of puts will be limited to only 1.25mm puts (long or short).

But, as quantity risk further increases, there is a greater opportunity for substitution and hence a

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7. This is also to say that, at the optimum, the marginal expected distress and overhedging costs from adding or subtracting linear contracts are equal.
Like Merck, to protect the dollar value of its cash flows derived from its uncertain foreign currency-denominated revenues, American enters into various currency put option agreements. But, in the case of jet fuel, since there is relatively little uncertainty about its future requirements, American enters into fuel swap (linear) contracts to protect against increases in fuel prices.

larger role for options. To illustrate, assume that output can now range from 4mm to 16mm barrels. The optimal linear position will thus be somewhere between 4mm and 8mm barrels. Assume that it is 6mm. Depending on whether substitution or reverse substitution is optimal, 2mm barrels of forward contract position could be eliminated or added and, therefore, as many as 2.5mm long or short put contracts substituted.

Taking Account of Correlation Effects

Thus far, we have considered how fluctuations in output levels and prices can affect the corporate decision to hedge with linear or option-like contracts. But there is another important factor in determining the optimal hedging position—namely, the correlation between output levels and prices. Depending on the sign of the correlation, revenues will exhibit either greater or lower volatility. A positive correlation will exacerbate fluctuations in revenues since output levels and prices will likely move in the same direction. However, this will cause the overhedging problem to become less severe since the likelihood of a simultaneous decline in output and an increase in price is diminished. Thus, for a given level of quantity risk, the more positive the correlation between output levels and prices, the greater is the optimal linear position and the smaller the non-linear position.

A negative correlation will produce the opposite result, dampening fluctuations in revenues and so producing a “natural hedge” effect. For example, a producer firm that tends to experience higher output in periods of lower prices or vice versa will have its fluctuations in revenues at least partially reduced as compared to the zero correlation case. A second example would be the case of a U.S. exporter selling goods in a foreign country (say, France). Here, a negative correlation between exchange rates and foreign currency-denominated sales implies that the exporter will experience increasing (decreasing) foreign sales at the same time the foreign currency is weakening (strengthening)—that is, when the U.S. dollar/Euro exchange rate is becoming lower (higher).

But if a negative correlation will tend to dampen fluctuations in cash flows, it will also exacerbate the overhedging problem. As mentioned, the overhedging problem for a producer arises precisely when output falls and prices rise. And that is why firms whose quantity and prices tend to be negatively correlated are expected to make less use of linear and more use of option-like instruments.

But this raises the question: what is likely to determine the correlation between a given firm’s output levels and the commodity price whose risk the firm is attempting to manage—and, in the case of exporters, the correlation between exchange rates and foreign sales? In the case of commodity producers (like the oil firm in our example) attempting to hedge output revenue, the correlation between output and prices will depend on whether the firm is more susceptible to demand or supply shocks. Demand shocks will lead to a positive correlation (for example, a surge in the demand for oil will lead, at least in the short run, to price increases) while supply shocks (say, an OPEC agreement to cut supply) will lead to a negative correlation. The resulting correlation will depend on the relative likelihood and strength of the two shocks. An oil producer is likely to have positive correlation because of its greater exposure to demand shocks related to weather and the business cycle. An agriculture producer is likely to have negative correlation because of its greater susceptibility to supply shocks related to weather. An example would be the tendency of excessive rain or drought to lead to poor harvests and higher crop prices.

In the case of exporters, the correlation between exchange rates and foreign sales is heavily influenced by the elasticity of demand. The correlation is positive when the firm’s demand elasticity is high (greater than one). That is, the correlation is positive when an increase in the (U.S. dollar/Euro) exchange rate leads to a percentage drop in the local-currency unit price of the export that is lower than the accompanying percentage increase in the quantity of goods sold. As a result, when the demand elasticity is high, the increase in the exchange rate results in an overall increase in foreign sales. On the other hand, when demand is inelastic (less than one), the correlation between exchange rates and foreign sales is negative.

To illustrate these points, Coca-Cola faces high demand elasticity with respect to its foreign sales because of consumers’ access to a number of substitutes (for example, products of other soft drink competitors and other beverage choices). We thus would expect Coke to have a positive correlation with respect to exchange rates and its foreign sales—that is, strengthening foreign currencies should lead to stronger foreign sales—which in turn should lead
to greater reliance on linear contracts in hedging its FX risk. By contrast, because there are few short-run substitutes for Microsoft’s software products, the company is likely to face relatively inelastic demand (less than one). As a consequence, we would expect Microsoft’s correlation to be negative and that the firm would accordingly place greater reliance on option-like hedging instruments in hedging its FX risk.

### Numerical Solutions

Having illustrated the main factors that corporate hedgers consider when choosing between linear and non-linear derivatives, we now complete our example by solving for the optimal linear and non-linear positions for various assumed levels of quantity risk and correlation values between output and prices. In our calculations, we assume that oil prices are log-normally distributed with a level of market price risk (standard deviation) of 30%. Quantity risk is measured as the standard deviation of output about the expected level of 10 million barrels. In solving for the optimal positions we specify the firm’s financial distress cost function (FDC) as follows:

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\text{FDC} = \begin{cases} 
0 & \text{if EBIT} \geq 20 \text{mm} \\
0.1 \times (20 \text{mm} - \text{EBIT})^2 & \text{if EBIT} < 20 \text{mm}.
\end{cases}
\]

The basic intuition behind this function is that, in the case of a small shortfall in EBIT, financial distress costs are likely to be minor and reflect effects associated with closer monitoring by creditors, a potential credit downgrade, and restrictions placed on financing and dividends. In more severe cases, distress costs grow proportionately, reflecting, for example, the greater possibility of reduced access to or denial of trade credit, key employee turnover, legal expenses, and losses on asset sales.

Figure 1 presents graphically the optimal hedging positions that minimize total overall expected distress costs. Panels A and B show the optimal notional forward and put positions (y-axis), respectively, for various levels of quantity risk (x-axis) and for correlation values equal to −0.25, 0 and +0.25. Consider, for example, the case when the firm’s quantity risk has a standard deviation of 2mm barrels and the correlation between output and oil prices is zero. This implies that there is a 95% probability that output will fall between 6mm and 14mm barrels (which essentially follows the situation presented in Case 2 earlier). In this case, the optimal linear position would be to sell forward contracts on 7.083mm barrels and to purchase puts on 0.364mm barrels.

As can be seen in Panel A, the more positive is the correlation between oil prices and barrels of oil produced and sold, the larger is the optimal linear position for a given level of quantity risk. This result reflects the fact that the overhedging problem becomes less (more) severe as the correlation becomes more positive (negative).

The corresponding optimal put positions are presented in Panel B. As can be seen, the substitution effect is inversely related to the level of correlation. That is, the more positive (negative) the correlation, the lower (greater) the non-linear position. For the negative correlation case, the non-linear position is initially both greater than that observed for the zero correlation case and increases with the degree of quantity risk. Though the put position is still relatively small, the larger non-linear position combined with a lower linear position limits the overhedging problem while still providing protection against declines in prices that could generate financial distress costs.

We also observe that, as the correlation becomes increasingly positive and the recommended use of puts goes down, the optimal put position can even become negative. In this case the firm switches from essentially purchasing put protection to a strategy of put writing (i.e., “reverse substitution”) to generate premia income to add to its revenue stream. To see why, note that the increased linear position shown in Panel A due to the positive correlation, coupled with put writing, is equivalent to writing calls. The revenue from writing options is designed to offset the lower revenue in either low price or low output states, thus reducing potential financial distress costs. In high-price states, the potential losses on the calls are of less concern to the firm because output and hence revenue is likely to be higher due to the positive correlation.

### EMPIRICAL TEST OF CORPORATE HEDGING PRACTICES

We next examine the extent to which the composition of companies’ derivatives holdings conforms to our predictions. We began by constructing a large sample of firms that use currency derivatives. Then, as discussed below, we computed alternative measures of business risk for these firms.
Sample Selection and Description

Our sample initially consisted of all non-financial corporations included in the 1997 “Database of Users of Derivatives” (the last year of its publication), which was published by Swaps Monitor Publications. The database reports both which companies are using derivatives and the extent of their use. From this set of firms, we selected those firms that have foreign currency-denominated sales to allow cleaner tests of our hypotheses. When computing measures of business risk for derivatives-using firms, we used “pre-risk management” cash flows that were not “contaminated” by gains and losses from currency derivatives. More specifically, we constructed two business risk measures based on foreign sales.

Our first measure of business risk was the coefficient of variation of a firm’s foreign sales ($FSCV$), as measured by the standard deviation of foreign sales divided by its mean. To compute this measure, we required that each firm in our sample have seven yearly observations appearing in the Geographic segment file of the COMPUSTAT database spanning the years 1990-1996. After eliminating companies not having the requisite seven years of data, we were left with a sample of 312 firms using currency derivatives. Of these 312 firms, 235 used strictly linear derivatives, 72 used

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8. Under current hedge accounting rules in effect during our sample period, gains or losses from currency derivatives are recorded in the consolidated statement of income as adjustments to revenue. However, for most firms, these gains or losses are not specifically identified as derivatives-related and are commingled with other non-derivatives activities. Thus, the use of measures based on accounting items appearing “below” the revenue-related entries on the income statement such as net operating income or EBITDA could be contaminated.
both linear and non-linear derivatives, and the remaining five companies used only non-linear derivatives.

Our second measure of business risk was intended to take account of the fact that financial data, especially sales, can exhibit systematic trends as companies mature or develop their markets. We thus estimated the following trend equation to account for differences in growth patterns among different companies and industries:

$$\text{Foreign Sales (t) = } \beta_0 + \beta_1 \cdot t + \epsilon_t$$  \hspace{1cm} (2)

where $t$ refers to the years 1 through 7 corresponding to each of our seven years of annual foreign sales data, and $\epsilon_t$ represents the unsystematic variation about the time-adjusted trend level of foreign sales. Following the estimation of equation (2), our second measure of business risk was the coefficient of variation of foreign sales about its trend ($TrFSCV$), computed as follows:

$$TrFSCV = \frac{\sqrt{\sum_{t=1}^{7} \epsilon_t^2 / 5}}{FS}$$

where $\epsilon_t$ is the residual error in period $t$ taken from regression (2), 5 is the number of degrees of freedom associated with the residual variance, and $FS$ is the mean of the dependent variable, foreign sales.\(^9\)

**Univariate Tests**

In Table 2 we report the mean and median estimates of each of our two business risk measures ($FSCV$ and $TrFSCV$) for firms in each of our three derivatives-use groups. Group 1 corresponds to firms that use only linear derivatives, Group 2 refers to users of both linear and non-linear derivatives, and Group 3 to non-linear users only. Our prediction was that the greater a firm’s business risk, the lower would be its use of linear derivatives and the higher its use of non-linear derivatives.

The results reported in Table 2 are consistent with these predictions. For both of the two business risk measures, companies using only linear derivatives (Group 1) had the lowest mean and median levels of business risk followed by those firms using both linear and non-linear derivatives (Group 2), while firms using only non-linear derivatives (Group

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3) had the highest values of business risk. Furthermore, for both business risk measures, the differences in the means and medians of Groups 1 and 2 were statistically significant. (Tests involving Group 3 were generally insignificant due mainly to its small sample size.)

Regression Analysis

We further investigated the effect of business risk on companies’ derivatives positions using a multivariate framework. We defined LIN as equal to a firm’s total notional amount of linear currency derivatives usage expressed as a percentage of the level of its foreign sales. LIN thus proxies for the fraction of a firm’s currency exposure that is hedged with linear currency derivatives. Similarly, we defined NLIN as the firm’s total notional amount of non-linear currency derivatives, again expressed as a percentage of foreign sales. We then regressed each of these two measures of derivatives use on our business risk measures and on a set of control variables discussed next.

Control Variables: We attempted to control for other factors that could influence the relation between a firm’s derivative holdings and its level of business risk, such as firm size, the correlation between a firm’s foreign sales and currency rates, debt, and a firm’s cash holdings. Firm size could influence firm’s use of derivatives for various reasons, including, for example, the transaction costs related to conducting a hedging program. Larger firms may enjoy economies of scale in the initiation and conduct of a hedging program. To control for firm size, we included in each regression the variable SIZE, which was measured as the log of the book value of the firm’s total assets. To control for the correlation between output and currency prices, we included the variable CORR, defined as the correlation between the changes in a firm’s foreign sales with the changes in value of the FRB foreign currency basket index. The correlation is computed using fiscal-year-end values for the period 1990-1996.

Prior research indicates that a company’s debt level is an important determinant of derivatives usage, with greater use of derivatives associated with higher leverage. To control for this factor we included the variable DEBT, defined as the firm’s average debt ratio over the prior three year-end period. To capture any “quadratic” relationships, we also included the variable DEBTSQ, defined to be the square of the firm’s debt ratio. Finally, in practice, the distress costs resulting from any shortfall in cash flow will be tempered to the extent that the firm can draw on its working capital reserves. To control for this possibility, we included the variable CASH, defined as the ratio of cash and short-term investments to total assets measured as of prior year-end.

Regression Results

Tables 3 and 4 present regression results corresponding to our two dependent variables LIN and NLIN, respectively. In each table, the results presented in the first two columns (Models 1 and 2) correspond to the first business risk measure ($FSCV$) while those in the third and fourth columns (Model 3 and 4) make use of the second business risk measure ($TbFSCV$). Information for computing each of the control variables was obtained from the COMPUSTAT database. After eliminating firms for which complete financial data was not available, the estimates reported in Models 1 and 3 (which exclude the control variables) are based on a final sample of 312 firms, while those reported for Models 2 and 4 (which include the control variables) are based on a sample of 274 firms.

Our primary interest focuses on the relation between the two measures of derivatives use and the level of a company’s business risk. We predicted a negative relation between the extent of a firm’s use of linear derivatives and its level of business risk—and the regression findings reported in Table 3 strongly support our predictions. For each of the four model specifications (that is, with and without the inclusion of control variables, and using both measures of business risk), the corporate use of linear derivatives is negatively related to business risk (with statistical significance at the 1 percent level). And, as we also predicted, the results reported in Table 4 indicate a strong positive relation (also at the 1 percent level) between the extent of companies’ use of non-linear derivatives and the level of business risk.

To further illustrate the economic effect that a corporation’s business risk has on its linear and non-linear derivatives usage, consider for example the Tobit regression coefficients on the
business risk measure FSCV.\textsuperscript{10} As shown in Table 3, the regression coefficient for linear derivatives usage (from model 2) is \(-0.267\) while in Table 4 the coefficient for non-linear derivatives (from model 2) is \(0.810\). Given that the mean firm in the sample has a value of FSCV equal to approximately 33.3 percent, if FSCV were to be 10 percent higher (43.3 percent), the coefficients suggest that linear derivatives usage relative to foreign sales would decrease by 2.67 percent while non-linear derivatives usage would increase by 8.1 percent. Similarly, for the second business risk measure TrFSCV (which has a mean of 19.5 percent), the coefficients from model 4 in Tables 3 and 4 are \(-0.280\) and 1.054, respectively. Thus, were TrFSCV to be 10 percent higher (29.5 percent), linear derivatives usage (LIN) would decrease by 2.80 percent while non-linear derivatives usage (NLIN) would increase by 10.54 percent.

**IN CLOSING**

Linear instruments such as futures, forward, and swap contracts will typically dominate a firm’s hedging mix because of their effectiveness in smoothing corporate cash flows. But, as our analysis also shows, the corporate use of such linear products will be replaced by non-linear, or

\begin{table}
\centering
\begin{tabular}{lcccc}
\hline
\textbf{Variable} & \textbf{With FSCV} & & \textbf{With TrFSCV} & \\
 & \textbf{Model 1} & \textbf{Model 2} & \textbf{Model 3} & \textbf{Model 4} \\
\hline
Intercept & 0.318 & 0.562 & 0.289 & 0.509 \\
 & (0.038)*** & (0.122)*** & (0.034)*** & (0.120)*** \\
CASH & \multirow{-2}{*}{-0.187} & \multirow{-2}{*}{-0.232} \\
 & & & & \\
 & & & (0.217) & (0.217) \\
DEBT & \multirow{-2}{*}{-0.229} & \multirow{-2}{*}{-0.192} \\
 & & & & \\
 & & & (0.341) & (0.343) \\
DEBTSQ & 0.069 & 0.050 & & \\
 & (0.341) & (0.452) & & \\
SIZE & \multirow{-2}{*}{-0.036} & \multirow{-2}{*}{-0.035} \\
 & & & & \\
 & & & (0.014)** & (0.014)** \\
CORR & 0.001 & 0.001 & & \\
 & (0.002) & (0.002) & & \\
FSCV & \multirow{-2}{*}{-0.231} & \multirow{-2}{*}{-0.267} & \multirow{-2}{*}{-0.268} & \multirow{-2}{*}{-0.280} \\
 & & & & \\
 & & & (0.088)*** & (0.094)*** \\
 & & & (0.133)** & (0.154)** \\
TrFSCV & & & & \\
 & & & & \\
\end{tabular}
\caption{The Effect of Business Risk on Firms’ Linear Derivatives Usage$^{a}$.}
\end{table}

$^{a}$ The table presents regression results from Tobit model estimations. The sample for Models 1 and 3 includes 312 non-financial firms using currency derivatives comprised of 235 firms using strictly linear derivatives, 72 firms using both linear and non-linear derivatives and 5 firms using only non-linear derivatives. The sample for Models 2 and 4 includes 274 non-financial firms comprised of 203 firms using strictly linear derivatives, 67 firms using both linear and non-linear derivatives and 4 firms using only non-linear derivatives. The dependent variable is LIN, defined as the total notional amount of linear derivatives scaled by foreign sales measured as of fiscal year-end 1996. CASH is defined as the ratio of a firm’s cash and short term investments to total assets as of year-end 1995; DEBT is the average ratio of long-term debt to total assets for the three years 1993-1995; DEBTSQ is the square of DEBT; SIZE is the natural log of the firm’s book value of total assets as of year-end 1995; CORR is the correlation between the annual changes in the FRB foreign currency basket index and the changes in a firm’s annual foreign sales for the seven-year period 1990 through 1996; FSCV is the coefficient of variation of a firm’s annual foreign sales over the 1990-1996 period; and TrFSCV is the coefficient of variation of a firm’s foreign sales about its trend line over the 1990-1996 period. Standard errors are in parentheses.

*Significant at 10 percent level.
**Significant at 5 percent level.
***Significant at 1 percent level.

\textsuperscript{10} Since our two derivatives usage measures, LIN and NLIN, are censored or bounded from below at a value of zero, we use Tobit regression methodology instead of ordinary least squares (OLS) regression. Tobit regression coefficients are interpreted to represent the change (marginal effects) in the dependent variable with respect to the independent variables. However, unlike in the OLS regression model, the coefficients should be evaluated at the means of the independent variables.
For each of the four model specifications, the corporate use of linear derivatives is significantly negatively related to business risk. And, as also predicted, the results indicate a strong positive relationship between the extent of companies’ use of non-linear derivatives and the level of business risk.

<table>
<thead>
<tr>
<th>Variable</th>
<th>With FSCV</th>
<th></th>
<th>With TrFSCV</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
<td>Model 4</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.837</td>
<td>-1.052</td>
<td>-0.792</td>
<td>-1.086</td>
</tr>
<tr>
<td></td>
<td>(0.121)***</td>
<td>(0.266)***</td>
<td>(0.107)***</td>
<td>(0.239)***</td>
</tr>
<tr>
<td>CASH</td>
<td>0.468</td>
<td>0.591</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.505)</td>
<td>(0.492)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEBT</td>
<td>0.549</td>
<td>0.478</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.412)</td>
<td>(0.375)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEBTSQ</td>
<td>-0.892</td>
<td>-0.768</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.528)*</td>
<td>(0.521)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIZE</td>
<td>0.169</td>
<td>0.153</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)***</td>
<td>(0.035)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CORR</td>
<td>-0.003</td>
<td>-0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FSCV</td>
<td>0.703</td>
<td>0.810</td>
<td>1.136</td>
<td>1.054</td>
</tr>
<tr>
<td></td>
<td>(0.202)***</td>
<td>(0.209)***</td>
<td>(0.282)***</td>
<td>(0.266)***</td>
</tr>
<tr>
<td>TrFSCV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.282)***</td>
<td>(0.266)***</td>
</tr>
</tbody>
</table>

a. The table presents regression results from Tobit model estimations. The sample for Models 1 and 3 includes 312 non-financial firms using currency derivatives comprised of 235 firms using strictly linear derivatives, 72 firms using both linear and non-linear derivatives and 5 firms using only non-linear derivatives. The sample for Models 2 and 4 includes 274 non-financial firms composed of 203 firms using strictly linear derivatives, 67 firms using both linear and non-linear derivatives and 4 firms using only non-linear derivatives. The dependent variable is NLIN, defined as the total notional amount of non-linear derivatives scaled by foreign sales measured as of fiscal year-end 1996. CASH is defined as the ratio of a firm’s cash and short term investments to total assets as of year-end 1995; DEBT is the average ratio of long-term debt to total assets for the three years 1993-1995; DEBTSQ is the square of DEBT; SIZE is the natural log of the firm’s book value of total assets as of year-end 1995; CORR is the correlation between the annual changes in the FRB foreign currency basket index and the changes in a firm’s annual foreign sales for the seven-year period 1990 through 1996; FSCV is the coefficient of variation of a firm’s annual foreign sales over the 1990-1996 period; and TrFSCV is the coefficient of variation of a firm’s foreign sales about its trend line over the 1990-1996 period. Standard errors are in parentheses.

*Significant at 10 percent level.
**Significant at 5 percent level.
***Significant at 1 percent level.

option-like derivatives as both price risk and, more important, the quantity risk of the firm increases. Moreover, the extent of this substitution effect between linear and non-linear instruments will be strongly influenced by the level of correlation between a firm’s output and market prices, with lower and negative correlations tending to result in greater use of non-linear derivatives.

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