The Pricing and Valuation of Swaps[1]

I. Introduction

The size and continued growth of the global market for OTC derivative products such as swaps, forwards, and option contracts attests to their increasing and wide-ranging acceptance as essential risk management tools by financial institutions, corporations, municipalities, and government entities. Findings from a recent Bank of International Settlements (BIS) survey indicate that outstanding notional amounts of these products as of mid-year 2008 surpassed $516 trillion or $20.4 trillion in gross market value, which represents the cost of replacing all open contracts at prevailing market prices. Of these totals, interest rate swaps alone accounted for $357 trillion in notional amount or $8.1 trillion in gross market value.

In this chapter we focus on this important component of the market for derivatives—swaps—and provide a primer on how they are priced and valued. While our emphasis is largely on interest rate swaps, the framework we present is applicable to a wide array of swaps including those based on currencies and commodities.[2] In addition, we provide a number of examples to illustrate such applications. The tools presented should prove useful to students of these markets having interests in trading, sales, or financial statement reporting.

A general description of a swap is that they are bilateral contracts between counterparties who agree to exchange a series of cash flows at periodic dates. The cash flows can be either fixed or floating and are typically determined by multiplying a specified notional principal amount by a referenced rate or price.

To illustrate briefly a few common types of swaps that we examine in greater detail below, a plain-vanilla 'fixed for floating' interest rate swap would require one party to pay a series of fixed payments based on a fixed rate of interest applied to a specified notional principal amount, while the counterparty would make variable or floating payments based on a Libor interest rate applied to the same notional amount. A commodity swap involving say aviation jet fuel would have one party agreeing to make a series of fixed payments based on a notional amount specified in gallons multiplied by a fixed price per gallon, while the counterparty would make a series of floating payments based on an index of jet fuel prices taken from a specific geographic region. Similarly, in a currency swap the counterparties agree to exchange two series of interest payments, each denominated in a different currency, with the added distinction that the respective principal amounts are also exchanged at maturity, and possibly at origination.

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[2] For a review and analysis of other popular swap structures including credit default swaps, equity swaps, and total return swaps, see, for example, Bomfim (2005), Chance and Rich (1998), Chance and Brooks (2007), Choudhry (2004), and Kolb and Overdahl (2007).

Illustration 1: An end user swap application

To motivate our framework for pricing and valuing swaps, we first provide a hypothetical scenario involving a swap transaction. The CFO of company ABC seeks to obtain $40 million in debt financing to fund needed capital expenditures. The CFO prefers medium-term, 10-year financing at a fixed rate to provide protection against unexpected rising interest rates. The CFO faces something of a dilemma. Although the company currently enjoys an investment grade rating of BBB, the CFO believes that its financial well-being will noticeably improve over the next couple of years to possibly an A credit rating, thus allowing it to obtain debt financing on more attractive terms. As an interim solution, the CFO considers issuing a shorter-term, floating rate note and concurrently entering a longer-term pay fixed, receive floating interest rate swap to maintain interest rate risk protection.

The CFO contacts a swap dealer who provides an indicative quote schedule, a portion of which is reported in Table 1. For various swap tenors (i.e., the term of the swap), bid and ask "all-in" rates are shown.[3] We assume in the table that all quotes are presented on a semi-annual, actual/365 basis versus 6-month Libor.[4]

[Table 1 about here]

Consider the quote for a swap having a 10-year tenor reported to be 6.35 bid, 6.50 ask. End users paying fixed (and thus receiving floating) would make semi-annual payments to the dealer based on a 6.50% annualized rate (ask rate) and using an actual/365 day count convention. End users receiving fixed would receive payments from the dealer based on the 6.35% bid rate. The difference in the bid and ask rates of 15 basis points represents the dealer's gross compensation for engaging in this market making activity. The floating rate based on 6-month Libor would be calculated using an actual/360 day count convention. Typically, the floating rate is quoted "flat" without a spread.

Upon finalization, the company issues a 3-year, $40 million floating rate note having a quoted rate of Libor plus 100 basis points and concurrently enters a 10-year pay fixed, receive floating swap having $40 million in notional amount (see Figure 1). The net effect of this set of transactions results in the company paying a synthetic fixed rate of 6.50 plus 1.00% or 7.50% for the first 3 years. For the remaining 7 years, the company would pay 7.50% less the reduction in its credit spread observed after 3 years when it will need to roll over its maturing note.

[Figure 1 about here]

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[3] Alternatively, quotes may be presented in terms of a swap spread, an amount to be added to the yield of a Treasury instrument having a comparable tenor. The swap spread should not be confused with the bid-ask spread of the swap quote.

[4] We later discuss and consider other common day count conventions.

On a final note, the CFO would also execute an ISDA Master Agreement with the swap dealer, if one is not already in place. This agreement along with its supporting schedules and addenda serves to document many of the terms and conditions governing the swap for the purpose of mitigating credit and legal risks. Once the agreement is in place, these issues need not be repeatedly negotiated upon additional transactions between the counterparties.[5]

II. A framework for pricing and valuation

We next provide a framework for understanding swap pricing and valuation accompanied by a simple numerical example. Later we describe procedures for applying this framework when using actual market data. In swap terminology, the price of a swap differs from the value of the swap. The swap "price" refers to an interest rate, specifically, the interest rate used to determine the fixed rate payments of the swap. To begin, consider two bonds where the first bond has a fixed rate coupon while the second bond features a floating rate coupon. Values for the fixed rate bond, B^{Fix}, and the floating rate bond, B^{Flt}, are determined as follows:

$$B^{Fix} = \sum_{t=1}^{n} \frac{\overline{C}}{(1+{}_0R_t)^t} + \frac{F}{(1+{}_0R_n)^n} \tag{1}$$

$$B^{Flt} = \sum_{t=1}^{n} \frac{\widetilde{C}_t}{(1+{}_0R_t)^t} + \frac{F}{(1+{}_0R_n)^n} \tag{2}$$

In the above expressions, F denotes the face or notional amount of each bond, $\overline{C}$ is the fixed rate coupon, $\widetilde{C}_t$ is the floating rate coupon associated with period t, and ${}_0R_t$ is the rate on a zero coupon bond having a maturity t. Note that all cash flows are discounted by a unique zero coupon rate corresponding to the specific timing of the cash flow.

Next, define V to be the value of a swap. The value of a 'receive fixed, pay floating' swap can be expressed as a portfolio consisting of a long position in a fixed rate bond and a short position in a floating rate bond. Thus, the value of the swap can be expressed as the difference in equations (1) and (2) as follows:

$$V = B^{Fix} - B^{Flt} \tag{3}$$

Similarly, the value of a 'pay fixed, receive floating' swap can be expressed as the difference in equations (2) and (1) as follows:

$$V = B^{Flt} - B^{Fix} \tag{4}$$

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[5] For additional discussion regarding the role of the ISDA Master Agreement, see Gerald D. Gay and Joanne Medero (1996).

To price the swap, we recognize two key points: (1) at its inception, the value of a fairly priced swap is zero; and (2) the value of a floating rate bond at either issuance or upon any reset date is its par or face amount. For discussion purposes, we assume the par amount equals $1. Thus, using either equation (3) or (4), we have:

$$V = B^{Fix} - B^{Flt} = \$0;$$

$$B^{Fix} - \$1 = \$0; \text{ thus,}$$

$$B^{Fix} = \$1 \tag{5}$$

Expression (5) provides the key insight into pricing a swap. The "price" of a swap (sometimes referred to as the par value swap rate) will be the coupon rate that makes the fixed rate bond have a value equal to that of the floating rate bond, and thus causes the initial swap value to equal zero.

Illustration 2: A simple example

Consider an example wherein we seek to price a one-year (360 day) swap having semi-annual payments at 180-day intervals and a $1 notional amount. As we discuss later in greater detail, we use Libor interest rates to discount cash flows and follow money market conventions by using an actual/360 day count convention. Since cash flows occur at two dates (180 and 360 days), we assume that $_0L_{180} = 6.00\%$ and $_0L_{360} = 7.00\%$. To price the swap, we simply solve for the fixed coupon rate $\overline{C}$ that causes B^{Fix} to sell at par, according to the following expression:

$$B^{Fix} = \frac{\overline{C} * \dfrac{180}{360}}{(1+0.06 * \dfrac{180}{360})} + \frac{\overline{C} * \dfrac{180}{360} + 1}{(1+0.07 * \dfrac{360}{360})} = 1$$

Solving for $\overline{C}$, we find its value to equal 0.0686. The price of the swap is thus 0.0686 or 6.86%.[6] The fixed rate payer will therefore make a fixed payment each period of $0.0343 per $1 of notional value.

For the floating rate payer, standard convention is that floating payments are based on Libor rates observed at the start of the period rather than in-arrears. Thus, the initial floating payment will be 6.00% x 180/360 x $1 or $0.0300. The amount of the second floating payment will be determined at the next settlement date when a new value of $_0L_{180}$ is observed.

Next, we consider how to value a swap at times following origination. Both the passage of time and changing market interest rates can cause the swap to take on positive or negative

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[6] In practice, 6.86% would be the mid-rate around which a swap dealer would establish a bid-ask quote.

value. Assume in this same example that 10 days elapse during which market interest rates appeared to have risen. The two remaining settlement dates are now in 170 and 350 days, respectively, thus we require zero coupon discount rates corresponding to 170 and 350 days. Assume that $_0L_{170} = 6.50\%$ and $_0L_{350} = 7.50\%$. To solve for the new value of the swap we first estimate the new values of the two component bonds, B^{Fix} and B^{Flt}.

Consider the floating rate bond. Since only 10 days have elapsed, the $.0300 coupon to be paid at the next settlement date is still in effect. Also, upon the next settlement date, a new floating rate will be selected such that the remaining value of the floating rate bond is restored to par. The value of B^{Flt} is determined as follows:

$$B^{Flt} = \frac{0.0300 + 1}{(1 + 0.0650 * \frac{170}{360})} = 0.99933$$

For the fixed rate bond, there are two remaining coupons of $.0343 along with the repayment of the principal amount. The value of B^{Fix} is determined as follows:

$$B^{Fix} = \frac{0.0343}{(1 + 0.0650 * \frac{170}{360})} + \frac{0.0343 + 1}{(1 + 0.0750 * \frac{350}{360})} = 0.99729$$

Thus, the value of the swap, assuming 'pay fixed, receive floating', is equal to:

$$V = B^{Flt} - B^{Fix} = 0.99933 - 0.99729 = \$0.00204$$

For the same swap having instead a notional amount of $50 million, the value of the swap would be 50mm times $0.00204 or +$102,000. This is the value to the fixed rate payer. To the fixed rate receiver, a 'receive fixed, pay floating' swap would have a value of -$102,000. The change in value of $102,000 also represents the profit/loss on the swap over the 10 day interval due to the change in interest rates.[7]

III. Steps for swap pricing

In this section we extend the above procedures to apply to market data and to reflect industry conventions. But first, we consider credit concerns. Common practice in swap pricing is to use

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[7] For financial reporting purposes according to FAS 133, if day 10 corresponded to an end of period reporting date, an end user would record the 'pay fixed, receiving floating' swap as a $102,000 asset, while the counterparty would record the swap as a $102,000 liability. The change in value of the swap since origination would also have income statement implications depending on the swap's intended purpose.

Libor-based rates to discount cash flows and to determine floating payments. Libor reflects a rate at which high quality borrowers, typically A and AA rated commercial banks, can obtain financing in capital markets. While most swap counterparties are of investment grade quality, significant differentials in their credit quality can exist. Rather than make adjustments to the swap price or rate, these differences are typically accounted for through non-price means as specified in master agreements.

As mentioned earlier, most swaps will be executed under a master agreement, which contains a number of provisions aimed at mitigating credit and legal risk. Master agreements typically specify terms that allow counterparties to engage in payment netting and to conduct both (1) upfront risk assessment through the provision of documents concerning credit risk as well as through representations concerning enforceability; and (2) ongoing risk assessment through the periodic provision of documents, maintenance of covenants, use of collateral, and mark-to-market margining.

We next discuss the following steps for pricing and valuing swaps.

1. Obtain market inputs

2. Make convexity adjustments to implied futures rates

3. Build the zero curve

4. Identify relevant swap features

5. Price/value the swap

1. Obtain market inputs

Because we will be computing the present value of swap cash flows, for each payment date, we require a discount rate of a corresponding maturity. These rates will be drawn from a Libor-based, zero coupon discount curve, which henceforth we refer to as the zero curve. Since most of the rates comprising this curve are not readily available, they must be estimated using market data. For rates at the short end of the zero curve, Libor spot rates are directly observable and easy to obtain. For the middle and longer term portions of the zero curve, the rates can be derived from forward rate agreements (FRAs) or, as illustrated below, Eurodollar futures.

Eurodollar futures, traded on the Chicago Mercantile Exchange, are among the world's most liquid and heavily traded futures due in large part to their central role in the pricing and hedging of interest rate swaps. Eurodollar futures are in essence cash-settled futures on three-month Libor-based deposit rates. Quotes on Eurodollar futures prices are readily available for maturities extending out 10 years. Prices are quoted such that to find the associated implied futures rate, one must first subtract the observed futures price from 100.00. Consider a

Eurodollar futures quoted at 94.00. To find its associated implied futures rate, we subtract this price from 100.00 and obtain a rate of 6.00%.[8]

For use in the calculations to follow, we present in Table 2 a set of sample spot Libor rates and Eurodollar futures prices observed on December 18, 2007. One, three, and six month Libor spot rates are given. In addition, we report Eurodollar futures prices and corresponding implied futures rates, beginning with the June 2008 contract and extending to the September 2011 contract. In practice, the termination of trading of a Eurodollar futures contract is two business days prior to the third Wednesday of the contract month; we abstract somewhat and assume for purposes of our analysis that this date will fall on the 18th day of each contract month. Also, given that the 3-month implied futures rate underlying the September 2011 contract extends from September 18, 2011 to December 18, 2011, the market data reported in Table 2 permits us to price and value swaps extending out four years. To price swaps having even longer tenor, we would simply add futures having more distant maturities.

[Table 2 about here]

2. Make convexity adjustments to implied futures rates

We use implied Eurodollar futures rates to proxy for the forward rates that are used to construct the zero curve. However, due to the daily resettlement feature common to futures markets (and not to forward markets), the implied futures rate is likely to be an upward biased estimate of the desired forward rate. To see why this is the case, consider the following illustration.

Assume that a Eurodollar futures contract is trading at 95.00 (implied futures rate of 5%). For each contract there will be a long and short position. If the futures price increases 100 basis points to 96.00 (implied futures rate of 4%), then, following daily resettlement, the long position will show a credit or profit of $2,500 (100 basis points times $25 per basis point). These funds are then invested elsewhere to earn 4%. On the other hand, the account of the short position will show a debit or loss of $2,500, which is financed at 4%. Consider instead the result of a 100 basis point decline in price to 94.00 (implied futures rate of 6%). In this case, the long position incurs a loss of $2,500, which is financed at 6%. The short position earns a $2,500 profit, which is invested at 6%.

Assuming that positive and negative price changes are equally likely, the above situation clearly favors the short position. This is because short (long) profits and thus invests when rates are relatively higher (lower), and loses and thus borrows when rates are relatively lower (higher). Longs recognize this and thus require compensation to induce them to trade. Thus, shorts will agree to a lower equilibrium price than they would otherwise in the absence of daily resettlement. As a result of the lowered futures price, the implied futures rate becomes an

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[8] The standard contract size for Eurodollar futures is $1 million notional. Thus, a one basis point change in price corresponds to a $1,000,000 x .0001 x 90/360 = $25 change in contract value.

upward biased proxy for the desired forward rate. This bias will become more severe (1) the longer the time to the futures maturity date because of the greater number of daily resettlements; and (2) the longer the duration or price sensitivity of the asset underlying the futures contract.

To correct for the bias, a "convexity adjustment" is applied to the futures rate. A reasonable and simple approximation for the convexity adjustment is given by:[9]

Forward rate = Futures rate – (.5) x σ^2 x T1 x T2   (6)

In the above expression, σ is the annualized standard deviation of the change in the short-term interest rate, T1 is the time (expressed in years) to the futures maturity date, and T2 is the time to the maturity date of the implied futures rate. We report in Table 2 the results of convexity adjustments to all the implied futures rates assuming a value of σ of 1%. To see the potential magnitude of the convexity adjustment, consider first the September 2008 contract reported at a price of 96.31 (implied futures rate of 3.69%). This contract matures in 275 days while the implied futures rate extends out another 91 days to day 366. Thus,

Forward rate = .036900 - .5 x $.01^2$ x 275/365 x 366/365

= .036900 - .000038 = .036862.

The convexity adjustment in this instance is quite small at .38 basis points. Consider, however, the convexity adjustment for a longer dated futures such as the September 2011 contract quoted at 95.31 (implied futures rate of 4.69). Thus,

Forward rate = .046900 - .5 x $.01^2$ x 1370/365 x 1461/365

= .046900 - .000751 = .046149.

In this instance, the convexity adjustment was a more significant 7.51 basis points. For a Eurodollar futures maturing in 8 years (not shown in Table 2) the adjustment would grow to .5 x 01^2 x 8 x 8.25 = .0033 or 33 basis points. Thus, it is important to make the convexity adjustment to implied futures rates when pricing swaps, especially swaps of longer maturities.

3. Build the zero curve

To construct the zero curve, we use a bootstrapping technique in which short-term Libor spot rates are combined with forward rates to produce Libor-based, discount rates of longer maturities. Given that Eurodollar futures contract maturities extend out 10 years, it is possible to use this technique to produce rates of similar maturity.

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[9] See Hull (2008), pp. 136-138, as well as Ron (2000) who provides a more elaborate estimation for the convexity adjustment. Also, Gupta and Subrahmanyam (2000) conduct an empirical investigation into the extent that the market has correctly incorporated over time the convexity adjustment into observed swap rates.

Consider the spot and forward rates reported in Table 2. We see that the 6-month Libor spot rate ($_0L_{183}$) is 4.8250%, and extends out 183 days to June 18, 2008. The forward rate associated with the June 2008 futures contract is 3.9281% ($_{183}\ell_{92}$) and commences on June 18 and extends 92 days to September 18.[10] Thus, we can compute a 275-day Libor-based discount rate by linking the two rates as follows:

$$(1 + {_0L_{275}} \times 275/360) = (1 + {_0L_{183}} \times 183/360) \times (1 + {_{183}\ell_{92}} \times 92/360)$$

$$(1 + {_0L_{275}} \times 275/360) = (1 + 0.048250 \times 183/360) \times (1 + .039281 \times 92/360)$$

$$_0L_{275} = .045572 \text{ or } 4.5572\%.$$

Using this 275-day rate, we can similarly calculate a 366-day Libor rate by linking it to the 91-day forward rate associated with the September 2008 futures contract as follows:

$$(1 + {_0L_{366}} \times 366/360) = (1 + {_0L_{275}} \times 275/360) \times (1 + {_{275}\ell_{91}} \times 91/360)$$

$$(1 + {_0L_{366}} \times 366/360) = (1 + 0.045572 \times 275/360) \times (1 + .036862 \times 91/360)$$

$$_0L_{366} = .043725 \text{ or } 4.3725\%.$$

We repeat this process in an iterative manner to produce a set of Libor-based discount rates extending out essentially four years and at three month intervals. Given the data available in the table, the longest term discount rate ($_0L_{1461}$) is computed as:

$$(1 + {_0L_{1461}} \times 1461/360) = (1 + {_0L_{1370}} \times 1370/360) \times (1 + {_{1370}\ell_{91}} \times 91/360)$$

$$(1 + {_0L_{1461}} \times 1461/360) = (1 + 0.044361 \times 1370/360) \times (1 + .046149 \times 91/360)$$

$$_0L_{1461} = .044957 \text{ or } 4.4957\%.$$

Following these procedures, we report in Table 3 our set of discount rates representing the zero curve.

[Table 3 about here]

Before concluding this discussion, we note two situations where it will be necessary to interpolate between rates. First, we assumed a start date (December 18, 2007) in which the 6-month Libor spot rate extended exactly to a futures maturity date (June 18, 2008), thus allowing us to link immediately to a forward rate. In practice, this will usually not be the case and one will need to interpolate between two spot rates surrounding a futures maturity date in order to

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[10] In our notation we use an upper case "L" to denote a Libor-based spot rate and a lower case "ℓ" to denote a Libor-based forward rate.

link to the first forward rate. To illustrate, assume the start date is February 1, 2008. The 3-month Libor rate will extend to 90 days to May 1 while the 6-month rate will extend 182 days to August 1. What is required however is a spot rate extending 138 days to June 18. Assume that the 3 and 6-month Libor rates are 5.00% and 6.00%, respectively. Through linear interpolation, we estimate the 138-day rate as $5.00 + (6.00 - 5.00) \times (138 - 90)/(182 - 90) = 5.52\%$. If one had access to 4 and 5 month Libor spot rates extending to June 1 and July 1, respectively, a more accurate interpolation is then possible.

Second, the computed rates comprising the zero curve had maturities corresponding to the futures maturity dates. More than likely, these dates will not be the same as the swap payment dates. Thus, one will need to interpolate between the computed zero curve rates to produce additional discount rates that correspond to the swap payment dates.[11]

4. Identify relevant swap features

Key information required to price a swap includes its tenor, settlement frequency, payment dates, and day count conventions. To value an existing swap, one also needs to know the notional value of the swap, the swap rate, the floating rate in effect for the current payment period, and whether the swap is to be valued from the either the fixed rate payer or receiver perspective.

Swap settlement frequencies of quarterly and semi-annual are most common, while monthly and annual frequencies are also observed. The day count convention addresses how days in the payment period will be counted and how many days are assumed to be in a year. For purposes of calculating the fixed rate interest payment, the three most common day count conventions used in U.S. dollar swaps are Actual/360, Actual/365, and 30/360. The numerator in each refers to the number of days assumed to be in the payment period while the denominator specifies the number of days assumed to be in a year. 'Actual', as its name suggests, refers to the actual number of days in the period. The 30/360 convention, sometimes referred to as 'bond basis', assumes that every month contains 30 days regardless of the actual number of days in the month. For purposes of calculating the floating interest payment, the Actual/360 convention is typically used since it will be based on a Libor interest rate.

5. Price/value the swap

Following the completion of the above steps, one is prepared to price a swap to be originated or to value an existing swap. Using the information presented in Table 3, we illustrate the pricing of a swap followed by the valuation of an existing swap.

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[11] For additional discussion and detail regarding curve construction, see Overdahl, Schachter, and Lang (1997).

Illustration 3: Pricing an interest rate swap

Consider a swap having a three-year tenor, semi-annual payments, and an Actual/365 day count convention. The payment dates thus fall on June 18 and December 18 of each year. To price the swap, we find the value $\overline{C}$ in the following expression that causes a bond with a face value of $1 to sell for par.

$$\$1 = \frac{\overline{C}*183/365}{(1+0.048250*\frac{183}{360})} + \frac{\overline{C}*183/365}{(1+0.043725*\frac{366}{360})} + \frac{\overline{C}*182/365}{(1+0.041694*\frac{548}{360})} +$$

$$\frac{\overline{C}*183/365}{(1+0.041313*\frac{731}{360})} + \frac{\overline{C}*182/365}{(1+0.041786*\frac{913}{360})} + \frac{\overline{C}*183/365+1}{(1+0.042694*\frac{1096}{360})} \qquad (7)$$

Solving for $\overline{C}$, the swap rate is 4.1145%. Note that for each coupon payment, we apply an actual/365 day count convention and a unique discount rate. Table 4 reports this and other swap rates for tenors ranging from one to four years under each of the various day count conventions.

[Table 4 about here]

Illustration 4: Valuing an existing interest rate swap

Consider the valuation of an existing swap again using the rate information presented in Table 3. Assume that a corporation entered a 'pay fixed/receive floating' swap a few years ago at a swap rate of 2.00% (Actual/365) when rates were significantly lower than currently. Assume also that the swap has a notional amount of $40 million, makes semi-annual payments on March 18 and September 18 of each year, and is set to mature on March 18, 2009. Hence, there are three payment dates remaining. The 6-month Libor rate in effect for the current period is 5.4200%, which was last reset on September 18, 2007.

To value the swap, we value a portfolio of two bonds in which we assume the swap holder is long a floating rate bond and short a fixed rate bond. As discussed earlier, the value of the floating rate bond will be restored to its par amount of $1 upon the next reset date. Thus, to value the floating rate bond, we simply find the present value of the sum of $1 plus the next semi-annual floating rate payment of .0542 x 182/360 = .02740 to be received in 91 days on March 18, 2008. Thus,

$$B^{Flt} = \frac{0.02740+1}{(1+0.049263*\frac{91}{360})} = 1.014764$$

For the fixed rate bond, we find the present value of the three remaining semi-annual payments plus the $1 face value. Thus,

$$B^{Fix} = \frac{.0200*182/365}{(1+0.049263*\frac{91}{360})} + \frac{0.0200*184/365}{(1+0.045572*\frac{275}{360})} + \frac{0.0200*181/365+1}{(1+0.042483*\frac{456}{360})} = 0.977940$$

The value of the swap is thus:

$$V = B^{Flt} - B^{Fix} = [1.014764 - 0.977940] \text{ x } \$40 \text{ mm}$$

$$= 0.036824 \text{ x } \$40 \text{ mm} = \$1,472,960.$$

The valuation of this swap could serve a number of purposes. For example, for purposes of financial statement reporting, the corporation would record the swap as an asset valued at $1,472,960, while its counterparty would record the same value as a liability. Also, if the two parties agreed to terminate the swap, the corporation would receive a payment of $1,472,960 from the counterparty.

IV. Other swaps

The framework above can be easily extended to swaps other than interest rate swaps. We next discuss such applications to currency and commodity swaps.

(1) Currency swaps

The interest rate swaps considered above can be thought of as 'single currency' swaps as all cash flows were denominated in the same currency, e.g., U.S. dollars. Importantly, the procedures we reviewed are equally applicable when working with 'single currency' interest rates swaps denominated in currencies other than the U.S. dollar.

Currency swaps are in essence interest rate swaps wherein the two series of cash flows exchanged between counterparties are denominated in typically two different currencies. The interest payments can be in a fixed-for-floating, fixed-for-fixed, or floating-for-floating format.[12] Floating rates are typically expressed in terms of a Libor rate based on a specified currency.[13] An added distinction is that the principal amounts in a currency swap are not merely notional, but rather are typically actually exchanged at maturity and may also be exchanged when the swap is originated.

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[12] See Kolb and Overdahl (2007) for additional analysis of various currency swap structures.

[13] In addition to the U.S. dollar, Libor rates are specified in the several other currency denominations including the Australian dollar, British pound sterling, Canadian dollar, Danish krone, Euro, Japanese yen, New Zealand dollar, Swedish krona, and Swiss franc.

The procedures for pricing and valuing currency swaps become intuitive when considered in the context of our earlier discussion of interest rate swaps. Regardless of the currency denomination of any 'single currency' swap, the following should hold. First, the swap can be viewed as a portfolio of two bonds, a fixed rate bond and a floating rate bond, wherein one of the bonds is held long and the other is held short. Second, the initial value of the swap is zero since the value of the fixed rate bond equals that of the floating rate bond. Third, the fixed swap rate, when set correctly, is the coupon rate that makes the fixed rate bond sell at par. Thus, from a value perspective, at origination one would be indifferent between holding long or short either bond comprising the swap since both bonds have identical value equal to their par or notional principal amounts.

In a currency swap, one party is in essence long one of the two bonds comprising a 'single currency' interest rate swap of one currency, and is short one of the two bonds comprising a 'single currency' swap of a second currency. The counterparty's position is opposite. To make each party indifferent as to the two bonds they hold representing the two legs of the currency swap, the bonds' par or notional principal amounts are set to reflect the current spot exchange rate. This in turn will lead to an initial value of the swap equal to zero. Thus, at the swap's origination, the two principal amounts are such that:

$$B_0^{Dom} = B_0^{For} \times S_0 \tag{8}$$

where B_0^{Dom} is the initial value or principal amount of the bond having cash flows (fixed or floating) expressed in the domestic currency, B_0^{For} is the initial value or principal amount of the bond having cash flows (fixed or floating) expressed in the foreign currency, and S_0 is the current spot exchange rate (Dom/For). Again, note that at the origination of the swap, the value of each bond will equal their respective par or principal amounts.

Illustration 5: Pricing and valuing a currency swap

To illustrate the pricing of a currency swap, consider the following market information from Wednesday, December 26, 2007 involving the U.S. dollar and Swiss franc. For five-year U.S. dollar swaps having semi-annual payments, the swap rate was 4.38%, the five-year Swiss franc swap rate was 3.05%, and the spot exchange rate was .8687 (US/SF). Now consider a currency swap in which the U.S. dollar leg has a principal amount of $20 million. At the current exchange rate, this is equivalent to approximately SF23.02 million. A number of currency swap structures are then possible, e.g., fixed for fixed, fixed for floating, or floating for floating. If properly priced and configured, each swap would entail the following cash flows between the counterparties, Party One and Party Two:

(a) At origination, the counterparties may exchange the two principal amounts; if so, Party One would send $20 million to Party Two, while receiving SF23.02 million from Party Two.

(b) Party One would receive from Party Two U.S. dollar interest payments based on a 4.38% fixed rate over the life of the swap or, alternatively, the U.S. dollar Libor floating rate. Either rate would be applied to a principal amount of $20 million.

(c) Party Two would receive from Party One Swiss franc interest payments based on a 3.05% fixed rate over the life of the swap or, alternatively, the SF Libor floating rate. Either rate would be applied to a principal amount of SF23.02 million.

(d) At maturity, and regardless of whether there was an initial exchange, the counterparties would exchange the original principal amounts. Party One would receive $20 million from Party Two, and send SF23.02 million to Party Two.

To value the swap at times subsequent to origination, note that a currency swap can take on positive or negative value depending on changes in the spot exchange rate and in the interest rates associated with each currency. Thus, one simply finds the present value of each respective set of cash flows using the revised zero curve for that currency and accounts for the prevailing exchange rate. The value of a 'receive domestic/pay foreign' swap at any time t (V_t) expressed in terms of the domestic currency is thus equal to:

$$V_t = B_t^{Dom} - \{B_t^{For} \times S_t\} \tag{9}$$

while the value of a 'pay domestic/receive foreign' swap equals:

$$V_t = \{B_t^{For} \times S_t\} - B_t^{Dom} \tag{10}$$

(2) Commodity swaps

Finally, we consider the pricing and valuation of commodity swaps. Note upon substituting equations (1) and (2) for B^{Fix} and B^{Flt}, respectively, into equation (3) and simplifying, the value of a swap can also be written as:

$$V = \sum_{t=1}^{n} \frac{\overline{C} - \tilde{C}}{(1 + {_0}R_t)^t} \tag{11}$$

This equation is appealing as it expresses the swap's value in terms of the portfolio value of a series of N forward contracts. Equation (11) is especially useful for pricing a commodity swap in that one simply solves for the fixed price of the commodity $\overline{C}$ that makes the overall value of the swap equal zero. That is, one solves for the value of $\overline{C}$ that makes not the value of each of the N forward contracts equal zero, but rather the sum of the value of all N forward contracts.

Before applying equation (11), one must first build a zero curve as previously shown. In addition, one must also obtain a set of forward prices for the commodity corresponding to each

payment date of the swap. Typically, these can be taken from the commodity's forward curve or futures prices can be used.

Illustration 6: Pricing a commodity swap

We again assume that the current date is December 18, 2007, which allows us to make use of the zero curve information presented in Table 4. We consider an end-user of crude oil that wishes to fix its supply costs over the next two years. The end-user approaches a commodity swap dealer and is presented with the following term sheet.

Commodity:	Crude Oil (West Texas Intermediate)
Notional Amount:	100,000 Barrels
Agreed Fix Price:	$87.59/Barrel
Agreed Oil Price Index:	"Oil-WTI-Platt's Oilgram"
Term:	2 years
Settlement Basis:	Cash Settlement, Semi-annual
Payment Dates:	June 18 and December 18

To determine whether the quoted $87.59 swap price reflects current market conditions, the end-user observes that current NYMEX WTI crude oil futures prices are in their typical backwardation pattern. Futures prices corresponding to the next four semi-annual dates are: $89.50 (June 2008), $88.00 (December 2008), $86.75 (June 2009), and $86.00 (December 2009). Applying equation (11), we solve for the value of $\overline{C}$ in the following expression that causes the overall value of the swap to equal zero.

$$0 = \frac{\overline{C} - \$89.50}{(1 + 0.048250 * \frac{183}{360})} + \frac{\overline{C} - \$88.00}{(1 + 0.043725 * \frac{366}{360})} + \frac{\overline{C} - \$86.75}{(1 + 0.041694 * \frac{548}{360})} + \frac{\overline{C} - \$86.00}{(1 + 0.041313 * \frac{731}{360})}$$

Solving, we find that $\overline{C}$ equals $87.59 and conclude that the swap is priced correctly.

As we mentioned at the outset, the market for OTC derivatives continues to grow rapidly reflecting their value and acceptance as important risk management tools. Focusing on the largest segment of this market, swaps, we have attempted to present a simple framework for facilitating an understanding of their pricing and valuation, relying mainly on time value of money concepts and noting a few market conventions.

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Figure 1
Illustration of Swap Transaction Fund Flows for Company ABC

Swap Dealer		ABC		Note Holders
	$\widetilde{L}$ →		$\widetilde{L}+1.0\%$ →	
	← 6.50%		← $ 40 million	
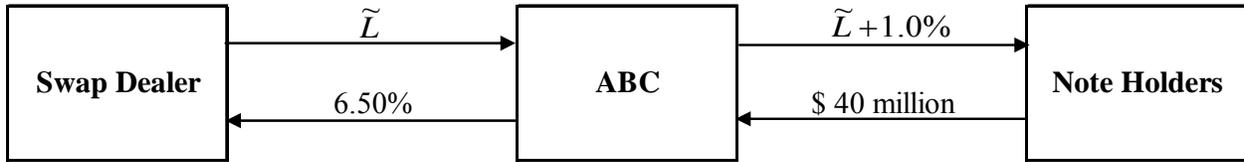
ABC's Net Financing Cost:

To Creditors	$<\widetilde{L}+1.0\%>$
To Swap Dealer	$<6.50\%>$
From Swap Dealer	$+\widetilde{L}$
	_____
Net	**7.50%**
	_____

Table 1
Indicative Dealer Swap Quote Schedule
(Rates quoted on a semi-annual, actual /365 basis versus 6-month Libor)

Tenor (years)	Swap Rate (%)	
	Bid	*Ask*
1	5.75	5.80
2	5.42	5.99
3	6.05	6.12
5	6.15	6.25
7	6.25	6.35
10	6.35	6.50

Table 2
Market Data for the Zero Curve Construction
(for December 18, 2007)

(a) Libor Spot Rate Information

Term	Maturity	Days	Rate (%)
1-month	18-Jan-08	31	4.9488
3-month	18-Mar-08	91	4.9263
6-month	18-Jun-08	183	4.8250

(b) Eurodollar Futures Information

Contract	Price	Start Date	Days (T1)	End Date	Days (T2)	Days (T1 -T2)	Futures Rate (%)	Convexity Adjustment	Forward Rate (%)
Jun 2008	96.070	18-Jun-08	183	18-Sep-08	275	92	3.9300	0.0019	3.9281
Sep 2008	96.310	18-Sep-08	275	18-Dec-08	366	91	3.6900	0.0038	3.6862
Dec 2008	96.410	18-Dec-08	366	18-Mar-09	456	90	3.5900	0.0063	3.5837
Mar 2009	96.405	18-Mar-09	456	18-Jun-09	548	92	3.5950	0.0094	3.5856
Jun 2009	96.295	18-Jun-09	548	18-Sep-09	640	92	3.7050	0.0132	3.6918
Sep 2009	96.155	18-Sep-09	640	18-Dec-09	731	91	3.8450	0.0176	3.8274
Dec 2009	96.025	18-Dec-09	731	18-Mar-10	821	90	3.9750	0.0225	3.9525
Mar 2010	95.905	18-Mar-10	821	18-Jun-10	913	92	4.0950	0.0281	4.0669
Jun 2010	95.770	18-Jun-10	913	18-Sep-10	1005	92	4.2300	0.0344	4.1956
Sep 2010	95.660	18-Sep-10	1005	18-Dec-10	1096	91	4.3400	0.0413	4.2987
Dec 2010	95.560	18-Dec-10	1096	18-Mar-11	1186	90	4.4400	0.0488	4.3912
Mar 2011	95.480	18-Mar-11	1186	18-Jun-11	1278	92	4.5200	0.0569	4.4631
Jun 2011	95.395	18-Jun-11	1278	18-Sep-11	1370	92	4.6050	0.0657	4.5393
Sep 2011	95.310	18-Sep-11	1370	18-Dec-11	1461	91	4.6900	0.0751	4.6149

Table 3
Zero Curve Rate Information
(for December 18, 2007)

Maturity	Term (days)	Rate (%)
18-Jan-08	31	4.9488
18-Mar-08	91	4.9263
18-Jun-08	183	4.8250
18-Sep-08	275	4.5572
18-Dec-08	366	4.3725
18-Mar-09	456	4.2483
18-Jun-09	548	4.1694
18-Sep-09	640	4.1345
18-Dec-09	731	4.1313
18-Mar-10	821	4.1480
18-Jun-10	913	4.1786
18-Sep-10	1005	4.2209
18-Dec-10	1096	4.2694
18-Mar-11	1186	4.3219
18-Jun-11	1278	4.3778
18-Sep-11	1370	4.4361
18-Dec-11	1461	4.4957

Table 4
Swap Rates (%)
(Rates based on market data from December 18, 2007)

Tenor (Years)	Day Count Convention		
	Actual/360	*Actual/365*	*30/360*
1	4.3304	4.3906	4.4026
2	4.0159	4.0717	4.0773
3	4.0582	4.1145	4.1183
4	4.1696	4.2275	4.2304