2/19/2010

FI3300
Corporate Finance

Spring Semester 2010
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NY Times Article - details

It was April 2006, a moment when the perpetual rise of real estate was considered practically a law of physics. Mr. Koellmann was 23, a management consultant new to Miami. Financially cautious by nature, he bought a small, plain one-bedroom apartment for $215,000, much less than his agent told him he could afford. He put down 20 percent and received a fixed-rate loan from Countrywide Financial.

Not quite four years later, apartments in the building are selling in foreclosure for $90,000.

"There is no financial sense in staying," Mr. Koellmann said. With the $1,500 he is paying each month for his mortgage, taxes and insurance, he could rent a nicer place on the beach, one with a gym, security and valet parking.

NY Times Article - questions

1) Write down the transaction on April 2006 (buying the condo) as a balance sheet: point out the assets, liabilities and equity and their value.

2) Assuming that by February 2010 Benjamin Koellmann paid 16% of the principal and the value of the condo does not change ($215,000); write down the same balance sheet for February 2010. What is the value of equity?

3) Assuming that by February 2010 Benjamin Koellmann paid 16% of the principal and the value of the condo is marked-to-market ($90,000); write down the same balance sheet for February 2010. What is the value of equity?

4) How should we determine the financial value of the condo? How is the monthly CF of $1,500 related to this value?

Where are we?

Learning objectives

- Present the Time Value of Money (TVM) concept
- Define and demonstrate compounding and discounting
- Define and describe each variable in the Present-Value Future-Value (PV-FV) equation and use it to solve for:
  - The PV if the interest rate (r) and the FV are known
  - The FV if the interest rate (r) and the PV are known
  - The interest rate (r) if the PV and FV are known
- Present the value additivity principal and use it to solve multi-period valuation problems

Preferences: assumption 1

Magnitude: investors prefer to have more money rather than less:

$100 are "better" than $80

Value($100 today) > Value($80 today)
Preferences: assumption 2

Timing: investors prefer to get the money today rather than the same sum in the future:

$100 now are "better" than $100 one year from now

Value ($100 today) > Value ($100 one year from now)

Why? 1. Positive rate of return on investment 2. Option value

Time Value of Money - implications

If

Value ($100 today) > Value ($100 one year from now)

Then

Value ($100 today) + Value ($100 one year from now) ≠ Value ($200 today)

Note: if we set Value ($100 today) = $100

then Value ($100 one year from now) < $100

We need a common basis to compare (or add) cash flows (CFs) received in different points in time

Time Value of Money: basics

Deposit: $100 in a savings account today
The annual interest rate: r = 10%

How much will you get one year from now?

100 + 10 = 110  
Principal + Interest = Future Value

100 + 100(0.1) = 110

100 x (1+0.10) = 110

How much will you get in two years?

Date t=0: deposit $100 for one year, r=10%

Date t=1: get principal + interest 

$100+$10 = $100 x (1 + 0.1) = $110 

deposit $110 for another year

Date t=2: get principal + compounded interest 

$110 x (1 + 0.1) = $121

Summary:

$100 x (1 + 0.1) x (1 + 0.1) = $100 x (1 + 0.1)^2 = $121

Compounding

$100 FV

1  0

r = 10%

FV = $100 x (1+0.1)^1 = $110

$100 FV

1  2

r = 10%

FV = $100 x (1+0.1)^2 = $121

Present Value - Future Value Formula

FV = Future Value

T = number of periods

FV = PV x (1 + r)^T

PV = Present Value

r = interest rate for one period
Time Value of Money - Compounding

Starting point: CF in the present, say PV=$100
Annual interest rate: r > 0, say r=10%
Wanted: CF's value on date T in the future, T=12

\[ FV = PV \times (1 + r)^T \]

Discounting

Starting point: CF in the future: FV=$100, T=12
Annual interest rate: r > 0, say r=10%
Wanted: CF's value in the present (today)

\[ PV = \frac{FV}{(1 + r)^T} \]

Note that:

The interest rate r is always positive: 
\( r > 0 \)

\((1 + r)\) is always greater than one: 
\((1 + r) > 1\)

\(1/(1 + r)\) is always less than one: 
\(1/(1 + r) < 1\)

The PV of a CF is always less than its FV: 
\( PV(CF) < FV(CF) \)

The Future Value and r

\( FV = PV \times (1+r) \)

- \( r = 5\% \) \( FV = \$100 \times 1.05 = \$105 \)
- \( r = 10\% \) \( FV = \$100 \times 1.10 = \$110 \)
- \( r = 20\% \) \( FV = \$100 \times 1.20 = \$120 \)
- \( r = 50\% \) \( FV = \$100 \times 1.50 = \$150 \)

If \( T \) and the PV are fixed (\( T=1, \ PV=\$100 \)) then

As \( r \) (↑) increases the FV (↑) decreases

The Present Value and r

\( PV = \frac{FV}{1+r} \)

- \( r = 5\% \) \( PV = \$100/1.05 = \$95.24 \)
- \( r = 10\% \) \( PV = \$100/1.10 = \$90.91 \)
- \( r = 20\% \) \( PV = \$100/1.20 = \$83.33 \)
- \( r = 50\% \) \( PV = \$100/1.50 = \$66.67 \)
The Present Value and $r$

<table>
<thead>
<tr>
<th>PV</th>
<th>FV=$100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$r = 5%$</td>
<td>$FV = $100 = $95.24 \times 1.05$</td>
</tr>
<tr>
<td>$r = 10%$</td>
<td>$FV = $100 = $90.91 \times 1.10$</td>
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<td>$r = 50%$</td>
<td>$FV = $100 = $66.67 \times 1.50$</td>
</tr>
</tbody>
</table>

If $T$ and the FV are fixed ($T=1$, FV=$100$) then

As $r$ ($\uparrow$) increases the PV ($\downarrow$) decreases

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The Present Value and $T$

<table>
<thead>
<tr>
<th>PV</th>
<th>$100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$r = 10%$, $T=1$: $PV = $100/1.10 = $90.91$</td>
<td></td>
</tr>
</tbody>
</table>

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Time Value of Money - Additivity

What is the value of the following CF stream: $100$ today and $100$ one year from now

Answer:

$100$ today + $100$ one year from now ≠ $200$

We cannot just add up $CF_0$ and $CF_1$

(CF$_0$ in the present and CF$_1$ in the future)

There are two options:

1. Calculate the Present Value
2. Calculate the Future Value

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Time Value of Money - Additivity

1. Calculate $PV($100 one year from now$)$ and add the two present values = PV (CF stream)

FV=$100$

$r=10\%$ a year

$T=1$ year

$PV($100 one year from now$) = $100 / (1.1)^1 = $90.91$

$PV$ of the CF Stream = $100 \times $90.91 = $190.91$

We can add up $PV(CF_0)$ and $PV(CF_1)$

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Time Value of Money - Additivity

2. Calculate $FV($100 today$)$ and add the two future values = FV (CF Stream)

PV=$100$

$r=10\%$ a year

$T=1$ year

$FV($100 one year from now$) = $100 \times (1.1) = $110$

$FV$ of the CF Stream = $100 \times $110 = $210$

We can add up $FV(CF_0)$ and $FV(CF_1)$
Time Value of Money - Additivity

Note that the two answers are equivalent:

\[ PV(CF \ Stream) = $190.91 \quad r=10\% \text{ a year} \]
\[ FV(CF \ Stream) = $210 \quad T=1 \text{ year} \]

\[ FV = PV \times (1 + r)^T = $190.91 \times (1 + 0.1)^1 = $210 \checkmark \]
\[ PV = \frac{FV}{(1 + r)^T} = \frac{$210}{(1 + 0.1)^1} = $190.91 \checkmark \]

Multiple Cash Flows - 1

1. Multiple CFs on the same date:

\[ PV(CF \ Stream) = $250 \quad r=10\% \text{ a year} \]
\[ FV(CF \ Stream) = $100 \quad T=1 \text{ year} \]

\[ FV = PV \times (1 + r)^T = $250 \times (1 + 0.1)^1 = $210 \]
\[ PV = \frac{FV}{(1 + r)^T} = \frac{$210}{(1 + 0.1)^1} = $190.91 \]

Step 1: add up the CFs
\[ CF_1 = $100 + $250 = $350 \]
Step 2: calculate the Present Value
\[ PV(CF_1) = \frac{$350}{(1.10)} = $318.18 \]

Multiple Cash Flows - 2

2. Multiple CFs on different dates (CF Stream):

\[ PV \quad CF_1 = $100 \quad CF_2 = $250 \quad r = 10\% \]
\[ 0 \quad 1 \quad 2 \]

Step 1: Calculate the PV of CF_1
\[ PV(CF_1) = \frac{$100}{(1.10)} = $90.91 \]
Step 2: Calculate the PV of CF_2
\[ PV(CF_2) = \frac{$250}{(1.10)^2} = $206.61 \]
Step 3: Add up the Present Values
\[ PV(CF \ Stream) = PV(CF_1) + PV(CF_2) = $297.52 \]

Textbook Examples 1

Find the FV (T=1):
You require $1,700 to buy a computer and the bank is offering a loan at an interest rate of 14%. If you plan to repay the loan after one year, how much will you have to pay the bank?

Challenge: if the store offers 10% discount on cash or full price in one year, do you prefer a bank loan or store credit? 13% discount?

Textbook Examples 2

Find the PV (T=1):
What is the present value of $16,000 to be received at the end of one year if the interest rate is 10%?

Textbook Examples 3

Find r (T=1):
The bank promises to pay you $28,400 in one year if you deposit $27,000 today. What is the interest rate on your deposit?
Textbook Examples 4

Find the FV (T>1):
You plan to lend $11,000, for two years at an interest rate of 8% a year. How much do you expect to get at the end of the second year?

If the loan is for five years, how much do you expect to get?

Textbook Examples 5

Find the PV (T>1):
You get a chance to invest in a project that promises a payment of $28,650 at the end of the second year. If your required rate of return is 12%, how much should you invest in this project?

You suspect that the project will be delayed and you will get the payment at the end of the third year, what is the value of the project?

Textbook Examples 6

Find r (T>1):
You are considering the following opportunity: Invest $31,500 today and get $39,700 after two years. What is the return on this investment?

You suspect that the project will delayed and you will get the payment at the end of the third year, what is return on this investment?

Textbook Examples 7

Find the PV, Value Additivity Principle:
You invest in two projects:
Project 1 will pay $5,500 after one year and Project 2 will pay $12,100 after one year.

If your required rate of return is 10% (for both projects), how much did you invest in both projects?

Textbook Examples 8

Find the PV, Value Additivity Principle:
You plan to withdraw $3,200 from your account in one year and $7,300 in two years. If the bank pays 6% interest a year, how much shuld you deposit in your account today?

Textbook Examples 9

Find the FV, Value Additivity Principle:
You deposit $5,000 in the bank today and you make a second deposit of $4,000 in one year. If the bank pays 6% interest per year, how much will you have in your account in one year? in two years?
Summary

- Time value of money
- Discounting and compounding
- Single and multi-period problems
- Single and multiple CF problems
- The value additivity principle
- Finding the PV, FV and interest rate $r$
- The PV - interest rate ($r$) relation
- The PV - time ($T$) relation