The Net Present Value (NPV)

Write down the CF stream
- cash inflows are positive
- cash outflows are negative

Use the risk-adjusted cost of capital to calculate
\[ NPV = PV (CF \text{ stream}) \]

Note: we say "net" present value because we subtract the PV of cash outflows (costs, investment) from the PV of cash inflows (benefits).

The Internal Rate of Return (IRR)

Write down the CF stream
- cash inflows and cash outflows (investment)

Set NPV = 0 and solve for the cost of capital (r):
\[ NPV = C_F^1 + \frac{C_F^2}{(1 + r)^1} + \frac{C_F^3}{(1 + r)^2} + \ldots + \frac{C_F^n}{(1 + r)^n} = 0 \]

or, if the project is conventional and \( C_F^1 \) is the investment (the price) then
\[ \text{Price} = C_F^1 + \frac{C_F^2}{(1 + r)^1} + \frac{C_F^3}{(1 + r)^2} + \ldots + \frac{C_F^n}{(1 + r)^n} \]

Note: use a trial-and-error algorithm to find IRR.

IRR and Conventional Projects

Conventional (normal) project:
1. Starts with an investment: outflows, one or more negative CFs
2. Ends with inflows, positive CFs
3. There is only one sign change - only one transition from negative to positive CFs

USE the IRR rule only for conventional projects!
If projects are not conventional use the NPV rule!
Consider two mutually exclusive projects A & B, with the following annual cash flows.

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF(A)</td>
<td>-3,000</td>
<td>1,000</td>
<td>2,000</td>
</tr>
<tr>
<td>CF(B)</td>
<td>-3,000</td>
<td>1,900</td>
<td>1,900</td>
</tr>
</tbody>
</table>

If the annual discount rate is 10%, which project is better?
Which rule should we use, NPV or IRR?
Note: conventional projects & same scale!

Suppose that your firm is trying to decide between two machines, that will do the same job:

**Machine A** costs $90,000, will last ten years and require operating costs of $5,000 per year. At the end of ten years it will be scrapped for $10,000 (note: treat $10,000 as the salvage value).

**Machine B** costs $60,000, will last seven years and require operating costs of $6,000 per year. At the end of seven years it will be scrapped for $5,000 (i.e., $5,000 salvage value).
Which machine is a better machine if the annual discount rate is 10%?

PV(A: r=10%, T=10) = $116,867.41
Equivalent Annual Series:

Find A's Equivalent Annual Series (EASₐ)

Example: Textbook Problem 11.9

1. The two projects are mutually exclusive - we don't need both machines, only one!
2. How should we compare mutually exclusive projects with different lifetimes? - Can we use the NPV rule?

Note: we assume that
- the firm lives forever
- projects may be repeated forever

PV(A: r=10%, T=10) = $116,867.41
EASₐ = $19,019.631

EASₐ = \[ \frac{EAS}{1 - \frac{1}{(1+r)^T}} \]
Find B’s Equivalent Annual Series (EAS_B)

Original CF stream:
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-5,000</td>
<td>60,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
</tr>
</tbody>
</table>
| 0        | 1        | 2        | ...      | ...      | ...      | T=7

PV(r=10%, T=7) = $86,644.72

Equivalent Annual Series:
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>EAS</td>
<td>EAS</td>
<td>...</td>
<td>...</td>
<td>EAS</td>
</tr>
</tbody>
</table>
| 0        | 1        | 2        | ...      | ...      | T=7

$EAS_B = \frac{EAS}{r} \left[1 - \left(\frac{1}{1+r}\right)^T\right]$  \Rightarrow  $EAS_B = \$17,797.30$

Example: Textbook Problem 11.9

3. Maximizing shareholder wealth implies:
- maximizing benefits (inflows)
- minimizing costs (outflows)

$EAS_A = \$19,019.63 > \$17,797.30 = EAS_B$

Which project has the lowest annual cost?
Accept project _______

One more EAS example

Consider two mutually exclusive projects A & B, with the following annual cash flows.

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
<th>t=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF(A)</td>
<td>-12,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
</tr>
<tr>
<td>CF(B)</td>
<td>-18,000</td>
<td>7,000</td>
<td>7,000</td>
<td>7,000</td>
</tr>
</tbody>
</table>

The annual discount rate is 10%, and the projects can be repeated indefinitely.
Which project is better?

Summary: NPV vs IRR

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$NPV = PV(CF\ stream) = PV(cash\ inflows) - PV(cash\ outflows)$</td>
</tr>
<tr>
<td></td>
<td>Accept project if $NPV &gt; 0$</td>
</tr>
<tr>
<td>Note</td>
<td>the NPV rule has to be modified if mutually exclusive projects + unequal lifetimes - Use Equivalent Annual Series</td>
</tr>
<tr>
<td>IRR</td>
<td>$IRR = \frac{NPV}{-\text{initial\ investment}}$</td>
</tr>
<tr>
<td></td>
<td>Accept project if $IRR &gt; \text{cost\ of\ capital, } r$</td>
</tr>
</tbody>
</table>

1. IRR doesn’t work if the project is not conventional (normal)
2. When projects are mutually exclusive, IRR may not be consistent with maximizing shareholders wealth