The Spot Market
The spot market is the market for immediate delivery (or trade) of a financial asset or a commodity.

The trade occurs immediately between the buyer and the seller:
- The buyer pays cash to the seller;
- The seller hands over the asset.

Forward Contract
Definition:
A forward contract is an agreement between a buyer and a seller, to trade a specific quantity and quality of an asset (or commodity), at a specific time in the future (maturity), at a specific price.

Buyer / long position in the forward contract – the trader who agrees to buy the asset in the future;
Seller / short position in the forward contract – the trader who agrees to sell the asset in the future;
Forward price – the agreed upon asset price.

Time $t = 0$:
Contract is negotiated and the following terms are determined:
- Specification of the asset;
- Quantity and quality;
- Delivery method and date;
- Forward price.

Time $t = T$:
The buyer pays cash to the seller;
The seller hands over the asset.

Pricing Gold Forwards
Example of a forward contract on gold:
- The spot price of gold is $432/oz;
- The one year forward price of gold is $468/oz;
- The annual risk-free rate is 8%.

Is there an opportunity to make arbitrage profits?
Arbitrage Pricing

**Perfect market assumptions:**
- No transaction costs; No taxes; No risk of default; no storage costs.

**Arbitrage pricing:**
- Replicate the present and future payoffs of the forward contract;
- Use the cash-flows of the replicating portfolio to determine the no-arbitrage forward price.

The Cost of Carry Model

- There are two ways to pay cash and end of with gold on date $t = 1$:
  - Buy a forward contract
  - Buy a replicating strategy:
    - Buy gold on the spot market for cash, and carry it into the future (store it);
    - Borrow the cash needed to fund the purchase, and repay the loan on date $t = 1$.

- Note that the interest rate is your "cost of carry":
  - Borrowing cash on date $t = 0$ allows you to carry the gold forward to date $t = 1$;
  - The interest rate is the cost that you pay for carrying gold forward.

Detecting the Arbitrage Opportunity

**Buy a forward contract:**
- $CF_0 = 0$  \quad $CF_1 = -$468$

**Replicating strategy:**
1. Buy gold in the spot market:
   - $CF_0 = -$432
2. Borrow cash at 8%:
   - $CF_0 = +$432  \quad CF_1 = -$466.56$

**Total CF:**
- $CF_0 = 0$  \quad $CF_1 = -$466.56$

Cash and Carry Arbitrage

<table>
<thead>
<tr>
<th>Date $t = 0$</th>
<th>Date $t = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell a forward contract</td>
<td>0</td>
</tr>
<tr>
<td>Deliver gold against forward</td>
<td>$+468.00$</td>
</tr>
<tr>
<td>Buy gold on the spot market</td>
<td>-$432$</td>
</tr>
<tr>
<td>Borrow cash at 8%</td>
<td>$+432$  \quad Repay loan</td>
</tr>
<tr>
<td>Total</td>
<td>0  \quad $+1.44$</td>
</tr>
</tbody>
</table>

Pricing Gold Forwards

Another example of a forward contract on gold:
- The spot price of gold is $432/oz;
- The one year forward price of gold is $464/oz;
- The annual risk-free rate is 8%.

Is there an opportunity to make arbitrage profits?

Detecting the Arbitrage Opportunity

**Buy a forward contract:**
- $CF_0 = 0$  \quad $CF_1 = -$464$

**Replicating strategy:**
1. Buy gold in the spot market:
   - $CF_0 = -$432
2. Borrow cash at 8%:
   - $CF_0 = +$432  \quad CF_1 = -$466.56$

**Total CF:**
- $CF_0 = 0$  \quad $CF_1 = -$466.56$
Reverse Cash and Carry Arbitrage

<table>
<thead>
<tr>
<th>Date t = 0</th>
<th>Date t = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy a forward</td>
<td>0</td>
</tr>
<tr>
<td>contract</td>
<td>Get gold</td>
</tr>
<tr>
<td></td>
<td>against forward</td>
</tr>
<tr>
<td>Sell gold on the</td>
<td>+$432</td>
</tr>
<tr>
<td>spot market</td>
<td></td>
</tr>
<tr>
<td>Lend cash at 8%</td>
<td>-$432</td>
</tr>
<tr>
<td>Collect loan</td>
<td>+$466.56</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>+$2.56</td>
</tr>
</tbody>
</table>

Forward Pricing

The cost-of-carry arbitrage restriction:

\[ F_0 = S_0(1+r_f)^T \]

Where
- \( F_0 \) = forward price
- \( S_0 \) = spot price
- \( r_f \) = risk-free rate for one period
- \( T \) = number of periods to maturity

One derivation:

Use the cost-of-carry model

Another derivation:

A long position in the underlying asset and a short position in the forward contract on that asset is a perfect hedge, and therefore should pay the risk-free rate of return. If the forward contract matures one period from now

\( (F_0 - S_0)S_0 = r_f \) or

\[ F_0 = S_0(1+r_f) \]

If the forward contract matures \( T \) periods from now

\[ F_0 = S_0(1+r_f)^T \]

Cost of Carry Model

Imperfect Markets

Usually the borrowing rate is higher than the lending rate.

Example:
- The spot price of gold is $432/oz;
- The borrowing rate is 9% (annual);
- The lending rate is 8%.

What is the no-arbitrage price range (lower and upper bounds) of the one year forward price of gold?
Cash and Carry Arbitrage

<table>
<thead>
<tr>
<th>Date t = 0</th>
<th>Date t = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell forward contract</td>
<td>0</td>
</tr>
<tr>
<td>Buy gold on the spot market</td>
<td>-$432</td>
</tr>
<tr>
<td>Borrow cash at 9%</td>
<td>+$432</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

Reverse Cash and Carry Arbitrage

<table>
<thead>
<tr>
<th>Date t = 0</th>
<th>Date t = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy forward contract</td>
<td>0</td>
</tr>
<tr>
<td>Sell gold on the spot market</td>
<td>+$432</td>
</tr>
<tr>
<td>Lend cash at 8%</td>
<td>-$432</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

Cost of Carry Model – Imperfect Markets

Cash and carry:  
forward price \leq $470.88

Reverse cash and carry:  
forward price \geq $466.56

The no-arbitrage price range:

\[ LB = S_0(1+r_{f, lend})^T \leq F_0 \leq S_0(1+r_{f, borrow})^T = UB \]

\[ LB = $466.56 \leq \text{forward price} \leq $470.88 = UB \]

Show that if the forward price is $470 you cannot make arbitrage profits.

Cash and Carry

Buy a forward contract:

\[ CF_0 = 0 \quad CF_1 = -$470 \]

Replicating strategy:

1. Buy gold in the spot market:

\[ CF_0 = -$432 \]

2. Borrow cash at 9%:

\[ CF_0 = +$432 \quad CF_1 = -$466.56 \]

Total CF:

\[ CF_0 = 0 \quad CF_1 = -$466.56 \]

Cost of Carry Model

Arbitrage pricing:

We have solved for the forward price taking the spot price and interest rate as given.

We can use the same framework to solve for the spot price or the interest rate.

Limitations:

This model is useful only for contracts on assets that have certain characteristics:

- Storability
- Ease of short selling
- Underlying asset must be traded
Futures Contracts

- The futures contract is a standardized form of the forward contract, which is traded on the exchange:
  - The selection of underlying assets is limited, but there is a liquid secondary market for the contracts;
  - All contract parameters are standardized, except for the futures price which is determined by supply and demand;
  - In order to eliminate counter-party risk, the clearinghouse is always the seller if the trader takes the long position, and the buyer if he takes the short position.

Why Trade Futures?

Speculation: futures offer the opportunity to place "bets on the future spot price of a commodity.

Hedging: futures offer the opportunity to hedge against the risk of the future spot price.

Futures provide leverage: allow for big bets on the value of the underlying asset with little cash (the initial margin).

Creating a Liquid Market

Ex ante:
- The buyer and the seller managed to eliminate the risk of the future spot price
- Both buyer and seller are "winners"

Ex post:
- The futures price is either higher or lower than the contracted price
- One side is a "loser" with an incentive to renege on the futures contract.

Creating a Liquid Market

Eliminating counter-party risk:
- If all traders realize that the "losers" will renege, then no one will trade
- To promote a liquid market, we must remove the incentive to renege (the counter-party risk)
- The individual verification of the "credit worthiness" of every counter-party is costly
- The solution is to establish the clearinghouse

The Clearinghouse

- The clearinghouse is the counter-party to every trade
- As long as the clearinghouse is "credit-worthy" no one faces counter-party risk
- The clearinghouse takes no position – for every buyer there is a seller
- Traders still have an incentive to renege – to promote liquidity, the clearinghouse should make sure that they are credit-worthy.

Viability of the Clearinghouse

- Margin account
  - Initial margin
  - Variation margin
- Marking-to-market
  - Daily settlement
- Daily price limits
Margin Account

- The margin is a security account consisting of cash or near-cash securities
- Note that both the seller and the buyer of the futures contract are exposed to losses
- Initial margin – the initial deposit in the margin account (usually 5%-15% of the value of the contract)
- Marking-to-market – the gains and losses from the daily changes in the futures price are realized on daily basis

Margin Account

- Maintenance (variation) margin – once the value of the account falls below this value the trader receives a margin call
- If the trader does not deposit the required funds in the margin account the broker may close out his position
- Convergence property – the futures price converges to the spot price at maturity
- The sum of all daily settlements is the total profit or loss from the position in the futures contract

Example:
Assume that the current futures price of silver, for delivery in 5 days, is $5.20 per ounce.
Each silver contract on the Commodity Exchange (CMX) calls for purchase and delivery of 5,000 ounces.
Assume that the initial margin is 10% of the value of the contract and the maintenance margin is 8% of that value.

Margin Account (Buyer)

<table>
<thead>
<tr>
<th>Day</th>
<th>Price</th>
<th>Market Value</th>
<th>Profit (loss)</th>
<th>Acct Balance</th>
<th>Cash in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.20</td>
<td>$26,000</td>
<td></td>
<td>$2,600</td>
<td>$2,600</td>
</tr>
<tr>
<td>2</td>
<td>$5.15</td>
<td>$25,750</td>
<td>-$250</td>
<td>$2,350</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$5.09</td>
<td>$25,450</td>
<td>-$300</td>
<td>$2,050</td>
<td>$550</td>
</tr>
<tr>
<td>4</td>
<td>$5.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$5.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Initial margin = 10%; Maintenance margin = 8%
### Margin Account

<table>
<thead>
<tr>
<th>Day</th>
<th>Price</th>
<th>Market Value</th>
<th>Profit (loss)</th>
<th>Acct Balance</th>
<th>Cash in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.20</td>
<td>$26,000</td>
<td>$2,600</td>
<td>$2,600</td>
<td>$2,600</td>
</tr>
<tr>
<td>2</td>
<td>$5.15</td>
<td>$25,750</td>
<td>-$250</td>
<td>$2,350</td>
<td>$2,350</td>
</tr>
<tr>
<td>3</td>
<td>$5.09</td>
<td>$25,450</td>
<td>-$300 ($1,050)</td>
<td>$2,050</td>
<td>$550</td>
</tr>
<tr>
<td>4</td>
<td>$5.10</td>
<td>$25,500</td>
<td>+$50</td>
<td>$2,650</td>
<td>$2,650</td>
</tr>
<tr>
<td>5</td>
<td>$5.12</td>
<td>$25,600</td>
<td>+$100</td>
<td>$2,750</td>
<td>$2,750</td>
</tr>
</tbody>
</table>

### Daily Price Limits
- The maximum amount that a futures price may change in one day is restricted.
- If there is a large change in the value of the underlying asset's spot price, there should also be a large change in the futures price.
- In such cases, the restriction will result in daily changes in the futures price without trade.
- The restriction gives the clearinghouse a chance to weed out traders who may renege.
- On the last month of the futures contract, this restriction is usually removed (convergence property).

### Pricing Futures Contracts
- We have priced forward contracts.
- Futures contracts have:
  - Margin accounts
  - Marking-to-market
  - Price limits
- The no-arbitrage approach to pricing forward contracts carries directly to pricing futures contracts, but it involves more tedious calculations.

### Synthetic Positions with Futures
- **Example:**
  - Institution wants to hold a long position of $130 million in S&P index for one month. Current index level = 1,300; current 1-month delivery futures price is 1,313.
  - Convention: each contract calls for $250 \times \text{level of the index} = 250 \times 1300 = $325,000.
- Thus, the institution can do the following:
  1. Buy 130M/325,000 = 400 contracts.
  2. Invest enough in T-bills to ensure that the payment is covered (need to buy $130 million in bills today, assume r = 10%)
- **Check:**

<table>
<thead>
<tr>
<th></th>
<th>Theoretical Payoff</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic Value</td>
<td>-4,750</td>
<td>-4,750</td>
</tr>
<tr>
<td>Face value of T-Bills</td>
<td>131,300,000</td>
<td>131,300,000</td>
</tr>
<tr>
<td>Total payoff</td>
<td>130,550,000</td>
<td>130,550,000</td>
</tr>
</tbody>
</table>

→ We created a synthetic equity position.

### Synthetic Positions with Futures
- **Synthetic stock purchase**
  - Purchase of the stock index instead of actual shares of stock.
  - Lower transactions costs.
  - Can buy and sell all stocks in the market index quickly and relatively cheaply.

### Hedging Systematic Risk
- **How would a portfolio manager hedge market exposure?**
  - To protect against a decline in level stock prices, short the appropriate number of futures index contracts.
  - Less costly and quicker to use the index contracts.
  - Use the portfolio beta to determine the hedge ratio.
Suppose you manage $30M and want to hedge. How many contracts should you go short?

Portfolio Beta = .8  S&P 500 = 1,000
Decrease = 2.5%  S&P falls to 975
Project loss if market declines by 2.5% = (.8) (2.5) = 2%
2% of $30 million = $600,000

How much does each S&P500 index contract change for a 2.5% change in the index?
$250 * 25 point swing = $6,250

Hedge Ratio: Text Example
H = \frac{\text{Change in the portfolio value}}{\text{Profit on one futures contract}}
= \frac{\$600,000}{\$6,250} = 96 \text{ contracts short}

Practice Problems

BKM Ch. 22
7th Ed.: 3-4, 6-8, 10-12, 16, 20.
8th Ed.: 2-3, 8-9, 11-12, 15
CFA: 1, 5.

Practice problems:
Forward and futures contracts 1-5.